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Multihadron decays of new mesons

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We discuss the hadronic decays of the new $I=0$ mesons seen in e^+e^-: J (ψ) or ψ' with $G=-$and P_c (χ) or X with $G=+$. We present some isospin inequalities for $I=0$ pure pionic final states, and a discussion of $K\bar{K}$ and η, η' fractions. We also present a statistical-model analysis of pion final states, and conclude that a large fraction of hadronic J (ψ) decays contain something besides pions and $K\bar{K}$—probably η and η', possibly radiative modes.

I. INTRODUCTION

There is now plenty of evidence that the new resonances $J(\psi)(3.1)$[1,2] and $\psi'(3.7)$[3] are hadrons, with $J^{PC}=1^{--}$, odd G parity, and isospin zero. Isospin-violating decays take place via $J\to 1\gamma\to$ hadrons. Recently, even-C states have been found at 3.4 and 3.5 GeV, reached via $\psi'\to P_c\,(\chi)+\gamma$.[4,5,6] There is also some evidence for a state at 2.8 GeV, $J\to X(2.8)+\gamma$, $X\to\gamma\gamma$ and $p\bar{p}$.[6,7] The former states have the decay modes (3.4), $(3.5)\to 2(\pi^+\pi^-),\to 3(\pi^+\pi^-)$.[5] This shows that they have even G and even I. Evidence that $(3.4)\to\pi^+\pi^-$ and $K\bar{K}$ indicates $I=0$ (see Ref. 5); so would a $p\bar{p}$ final state.

In view of the present experimental situation, it is useful to study the hadronic decays of J, ψ', and χ, using the fact that they have $I=0$, or assuming it. That is what we will do here. First, we present rigorous isospin bounds according to the methods of Llewellyn Smith and Pais,[8] also extending the old isospin statistical model of Pais[9] to a larger number of pions. To this we add a few remarks on final states with $K\bar{K}$, η, and η'. The rest of the paper contains a discussion of the present experimental data.

We hope that this material will be useful to experimentalists and offer a partial view of the final hadron states in the decay of the new particles seen in e^+e^-. This may be of importance in view of the very small hadronic width of these mesons. Perhaps a study of final states will help us understand the mechansim which suppresses the hadronic width.

II. PIONS, KAONS, η, and η'

A. Pions

In this sub section we present isospin bounds and a statistical model for pure $I=0$ pionic final states. The method goes back to a very useful paper of Pais.[9] We review his method briefly. For such bounds to be useful in practice, final state pions must not arise from η or η' decay. We will discuss this at the end of the next subsection. We will assume here that events containing $K\bar{K}$, η, η' and nucleon-antinucleon pairs can be segregated out and treated separately.

Isospin-0 states with N pions can be labelled by three numbers N_1, N_2, N_3, where $N_1\geq N_2\geq N_3\geq 0$, N_1-N_3 even, N_2-N_3 even, $N=N_1+N_2+N_3$.[10] These numbers designate a Young tableau of the unitary group in three dimensions U(3), and constitute a labeling of all unitary representations of this group. All the consequences of isospin conservation for $I=0$ decay to pions then follow from the fact that the isospin group $SU(2)_I$ is a subgroup of this U(3). Labeling a charge partition of m_c ($\pi^+\pi^-$) pairs and m_0 π^0 mesons by (m_c, m_0), the probability that this will be found in a state $(N_1N_2N_3)$ is given by a Pais coefficient[9] $[N_1N_2N_3\,|\,m_cm_0]$, normalized so that $\sum_{m_c m_0}[N_1N_2N_3\,|\,m_cm_0]=1$ for fixed $N=2m_c+m_0$. These coefficients can be calculated by combinatoric methods and by establishing nontrivial identities between them. The branching ratio Γ for a state containing only N pions is

$$\Gamma(2m_c\pi^c, m_0\pi^0)=\frac{1}{\Delta}\sum_{N_i}\alpha(N_1N_2N_3)[N_1N_2N_3\,|\,m_cm_0], \tag{1}$$

where

$$\alpha(N_1N_2N_3)=\rho(N_1N_2N_3)K(N_1N_2N_3),$$

$$\rho(N_1N_2N_3)=\frac{N!\,(N_1-N_2+1)(N_1-N_3+2)(N_2-N_3+1)}{(N_1+2)!\,(N_2+1)!\,(N_3)!}, \tag{2}$$

and the non-negative $K(N_1N_2N_3)$ take care of the (unknown) dynamics; $\rho(N_1N_2N_3)$ is the dimensionality of the $(N_1N_2N_3)$ representation of S_3, and simply counts the number of available states (measures the isospin phase space). The normalizations are

$$\Delta = \sum_{N_i} \alpha(N_1N_2N_3),$$
$$\sum_{m_c, m_0} \Gamma(2m_c\pi^c, m_0\pi^0) = 1, \tag{3}$$

where $N = 2m_c + m_0$ and the sum N_i is over all $I=0$ partitions of fixed $N = N_1 + N_2 + N_3$. Because of the linearity of (1), the problem of finding bounds for Γ reduces to that of finding that $[N_1N_2N_3 | m_c m_0]$ which is largest or smallest for a given N. The bound corresponds to $K(N_1N_2N_3) = 1$ for that partition and to zero for all others. The bounds for N odd can be applied to $J, \psi' \to (N - m_0)\pi^c m_0 \pi^0$, $m_0 = 1, 3 \ldots$, or to single-photon e^+e^- annihilation to pions; the bounds for N even can be applied to $\chi_i \to (N - m_0)\pi^c m_0\pi^0$, $m_0 = 0, 2 \ldots$, and to e^+e^-. The bounds for N odd are

$$\left.\begin{matrix} N=3: & 1 \\ N=5: & \frac{2}{3} \\ N=7: & \frac{1}{3} \\ N\geq 9: & 0 \end{matrix}\right\} \leq \Gamma((N-1)\pi^c, 1\pi^0) \leq \frac{2^{N-3}\left[\left(\frac{N-3}{3}\right)!\right]^2}{(N-2)!}, \quad N \geq 3,$$
$$\left.\begin{matrix} 5\leq N \leq 13: & \dfrac{2^{N-4}\left[\left(\frac{N-3}{2}\right)!\right]^2}{(N-2)!} \\ N \geq 15: & 0 \end{matrix}\right\} \leq \Gamma((N-3)\pi^c, 3\pi^0) \leq \begin{cases} \frac{1}{3}, & N=5, \\ \frac{2}{3}, & N=7, \\ \dfrac{2^{N-9}\left[\left(\frac{N-9}{2}\right)!\right]^2}{(N-8)!}, & N\geq 9. \end{cases} \tag{4}$$

The bounds for N even are

$$\left.\begin{matrix} N=2: & \frac{2}{3} \\ N=4: & \frac{1}{3} \\ N\geq 6: & 0 \end{matrix}\right\} \leq \Gamma(N\pi^c) \leq \frac{2^{N}\left[\left(\frac{N}{2}\right)!\right]^2}{(N+1)!}, \quad N \geq 2,$$
$$\left.\begin{matrix} 2\leq N \leq 10: & \dfrac{2^{N-1}\left[\left(\frac{N}{2}\right)!\right]^2}{(N+1)!} \\ N \geq 12: & 0 \end{matrix}\right\} \leq \Gamma((N-2)\pi^c, 2\pi^0) \leq \begin{cases} \frac{1}{3}, & N=2, \\ \frac{2}{3}, & N=4, \\ \dfrac{2^{N-6}\left[\left(\frac{N-6}{2}\right)!\right]^2}{(N-5)!}, & N\geq 6. \end{cases} \tag{5}$$

The existence of nonzero lower bounds for low N may be especially useful.[11] Note that in the limit $N - 3m_0 \to \infty$ the branching ratios fulfill

$$0 \leq \Gamma((N - m_0)\pi^c, m_0\pi^0) \leq \left[\frac{\pi}{2}(N - 3m_0)\right]^{1/2}.$$

We mention that the isospin bounds arise when all but one certain $K(N_1N_2N_3)$ are zero. The opposite extreme is to assume that they are all equal. This produces an isospin statistical model for fixed N, with each $[N_1N_2N_3 | m_c m_0]$ weighted by the number of states $\rho(N_1N_2N_3)$ corresponding to this class.[9] Dynamical effects will give deviations from this model. Being unable to find a general formula, we extended the $[N_1N_2N_3 | m_c m_0]$ tables of Pais ($N \leq 8$) up to $N = 13$ for odd N and to $N = 10$ for even N. The results (including $N \leq 8$ for completeness) can be found in Tables I(a) and I(b).

It is less obvious how to construct a statistical model for the pion distribution in N. The simplest possibility, which we will use, is to assume a Poisson distribution,

$$P(N) = k^{-1} \frac{\langle N\rangle^N}{N!} e^{-\langle N\rangle}, \tag{6}$$

for N odd or even, $2 \leq N \leq 22$, normalizing by $\sum P(N) = 1$ with the sum over even or odd N as the case requires. We will use this to estimate the total pion branching ratio.

B. Kaons, η, and η'

The situation for final states with $K\bar{K}$ is more obscure. We can obtain a few simple isospin relations for such final states by exploiting the fact that the initial state has $I=0$.[12] There are then two amplitudes for inclusively produced $K\bar{K}$ with $I=0$ and $I=1$. Eliminating the $I=0,1$ interference term we find the inclusive rate $d\sigma(abX)$

$$d\sigma(K^0\bar{K}^0X^0)+d\sigma(K^+K^-X^0)\geq d\sigma(K^+\bar{K}^0X^-), \qquad (7)$$

and bounding the interference term,

$$[d\sigma(K^0\bar{K}^0X^0)-d\sigma(K^+K^-X^0)]^2\leq 4d\sigma(K^+\bar{K}^0X^-)[d\sigma(K^0\bar{K}^0X^0)+d\sigma(K^+K^-X^0)-d\sigma(K^+\bar{K}^0X^-)]. \qquad (8)$$

These are summed over the internal variables of X but not over $K\bar{K}$ momenta. For pure $I=1K\bar{K}$, (7) is an equality and of course $d\sigma(K^0\bar{K}^0X^0)=d\sigma(K^+K^-X^0)$ for vanishing isoscalar/isovector interference.

It is possible to do better for exclusive final states. For $\bar{K}K\pi$ (or nucleon-antinucleon-pion) we have pure $I=1$ and[13]

$$2\bar{K}^0K^0\pi^0=2K^+K^-\pi^0$$
$$=K^+\bar{K}^0\pi^-. \qquad (9)$$

For $K\bar{K}\pi\pi$ we can get strong relations only after integrating over all $\pi\pi$ momenta. It is also possible to hold the $\pi\pi$ mass fixed (not integrated out), so long as the c.m. $\pi\pi$ angles are integrated over. The isoscalar/isovector interference then vanishes; this is because $I=0$ and $I=1$ correspond to different $\pi\pi$ angular momenta. Then

$$K^+\bar{K}^0\pi^-\pi^0=K^-K^0\pi^+\pi^0,$$
$$K^+K^-\pi^+\pi^-=\bar{K}^0K^0\pi^+\pi^-, \qquad (10)$$
$$K^+K^-\pi^0\pi^0=\bar{K}^0K^0\pi^0\pi^0,$$
$$2K^+K^-\pi^+\pi^-=4K^+K^-\pi^0\pi^0+K^+\bar{K}^0\pi^-\pi^0. \qquad (10')$$

This last relation enables us to get nearly unmeasurable modes like $K^+K^-\pi^0\pi^0$ and $K^0\bar{K}^0\pi^0\pi^0$ from measurable ones.

Note that if we again appeal to statistical considerations it appears reasonable to weight the isovector : isoscalar ratios by 3:1 and to ignore the interference of the two. Then

$$d\sigma(\bar{K}^0K^0X^0)=d\sigma(K^+K^-X^0)$$
$$=d\sigma(K^+\bar{K}^0X^-)$$
$$=d\sigma(K^-K^0X^+). \qquad (11)$$

Inclusive isospin relations can also be derived for other combinations of final-state particles using the zero isospin of the initial state. Just as an example, consider inclusive $K\pi$ states with isospin $\frac{1}{2}$ amplitude a and $I=\frac{3}{2}$ amplitude b. Then

$$d\sigma(K^+\pi^-)=\tfrac{1}{3}|a|^2+\tfrac{1}{12}|b|^2+\tfrac{1}{3}\mathrm{Re}ab^*,$$
$$d\sigma(K^+\pi^0)=\tfrac{1}{6}|a|^2+\tfrac{1}{6}|b|^2+\tfrac{1}{3}\mathrm{Re}ab^*,$$
$$d\sigma(K^0\pi^+)=\tfrac{1}{3}|a|^2+\tfrac{1}{12}|b|^2-\tfrac{1}{3}\mathrm{Re}ab^*, \qquad (12)$$
$$d\sigma(K^0\pi^0)=\tfrac{1}{6}|a|^2+\tfrac{1}{6}|b|^2-\tfrac{1}{3}\mathrm{Re}ab^*,$$
$$d\sigma(K^+\pi^+)=d\sigma(K^0\pi^-)=\tfrac{1}{4}|b|^2,$$

from which we obtain the equalities

$$d\sigma(K^+\pi^-)+d\sigma(K^0\pi^0)=d\sigma(K^+\pi^0)+d\sigma(K^0\pi^+),$$
$$2[d\sigma(K^+\pi^0)+d\sigma(K^0\pi^0)] \qquad (13)$$
$$=2d\sigma(K^+\pi^+)+d\sigma(K^+\pi^-)+d\sigma(K^0\pi^+).$$

Of course we can go back to (12) and derive inequalities for inclusive rates or for different charge partitions of exclusive channels. In deriving relations for exclusive channels [e.g. Eq. (10′)], it is necessary to remember to include combinatorial factors for identical particles.

In order to obtain relations for final states with η or η', we need to make assumptions beyond isospin invariance. For this reason we will make only a few brief comments, and these are model-dependent.

It is widely believed that the new mesons seen so far in e^+e^- and in hadron collisions are composites of new heavy quarks, and that these states are SU(3) singlets [ignoring SU(3) breaking]. If this is so, and if we can ignore $J\to 1\gamma\to$ hadrons in first approximation, we expect all octet mesons to be produced with equal rate, and[14]

$$d\sigma(K^+X^-)=d\sigma(K_s^0X^0)$$
$$=d\sigma(\eta X^0) \qquad (14)$$

(pure octet η). If in addition η' is an SU(3) singlet built of the familiar u,d,s quarks only,

$$d\sigma(\eta X)=d\sigma(\eta' X). \qquad (15)$$

In applying this, remember that $\eta'\to\eta\pi\pi$ is a prominent decay. Even if SU(3) breaking suppresses production of heavy pseudoscalars K,η,η' relative to pions by a large factor, (14) and (15) may still be good to 20–40% typical for mass splitting corrections. However, it is unlikely that phase space alone will suppress single η or η' production relative to $K\bar{K}$.

Under the same assumptions as (13), plus an ideally mixed ϕ, we have

$$\frac{\Gamma(\eta'\phi)}{\Gamma(\eta\phi)}=\frac{1}{2}. \tag{16}$$

Since η' can in principle have a small admixture of SU(3)-singlet heavy quarks, (16) may be wrong by a significant factor. If this admixture is at the $\lesssim 10\%$ level we would expect only $\lesssim 20\%$ effects on (16) and (15). It is of evident importance to look for η and η' in J and ψ' decays.[15]

III. EXPERIMENT

Although new data on the decays of J, ψ', and χ might lead to changes in any detailed phenomenological analysis, we think it worthwhile to present at least a brief discussion of J, ψ', and χ decays. Experimentally, the decays $J\rightarrow 2m_c\pi^c 1\pi^0$ have been inferred for $m_c=1,2,3,4$ from the missing momentum and mass recoiling against $m_c(\pi^+\pi^-)$. Resonances (e.g. $\omega\pi^+\pi^-$ and $\rho 3\pi$) have been identified for $m_c=2$.[16] The experimentally measured branching ratios are $\sim\leq 1.9\%$ for $m_c=1$, $4\pm1\%$ for $m_c=2$, $2.9\pm0.7\%$ for $m_c=3$, and $0.9\pm0.3\%$ for $m_c=4$.[16] We now assume that the $G=-$ states are reached through an isospin-conserving process and that, further, one may apply the isospin statistical model for fixed $N=7,9$ (the total 5π rate is given by the isospin conservation alone). From Table I(a) we infer from the experimentally measured $\Gamma((N-1)\pi^c,1\pi^0)$ the following total N pion branching ratios: 6% for $N=5$, 7% for $N=7$, and 3.7% for $N=9$. In order to obtain the total pion branching ratio of J, we correct for unmeasured decays with $N\geq 11$ using the Poisson distribution, Eq. (6), for N odd, $3\leq N\leq 22$. Fitting the numbers just quoted we find $\langle N\rangle=7$, and the entries in Table II (in percent). This gives us a total branching ratio $J\rightarrow$ (odd number of pions)

$$\Gamma(J\rightarrow N\pi,\text{odd})\approx 23\%. \tag{17}$$

The error in this number is at least ±5%. Notice that for these decays $\langle N_{ch}\rangle\approx 4.7$. Also, the as yet unobserved modes $J\rightarrow 10\pi^c 1\pi^0$ and $12\pi^c 1\pi^0$ are predicted to be very small: 0.3% and 0.05%.

Because some of this may be useful for the χ states, we also present the same statistical-model expectation for $G=+$ pion decays of χ, using $\langle N\rangle=7.5$ and normalizing the total to 100% (i.e. branching ratios normalized to the total of all pion isospin-conserving rates, not to the total widths). This is shown in Table III.

It is interesting (if somewhat risky) to attempt to find the hadronic branching ratio ($86\pm2\%$) of the J by adding together different classes of events. A possibly large contribution comes from hadronic events with $\geq 1K\bar{K}$ pair (plus pions). To estimate this we can use a couple of simple observations. First, if not more than one $K\bar{K}$ pair is present and if we count events with K_SK_S twice, then the fraction of hadronic events with $K\bar{K}$ is just equal to the fraction with a K^- plus the fraction with a K_S. This is all that has been measured directly so far. Going back to Sec. IIB, $K^-=K_S$ for a statistical distribution. The fraction of purely hadronic events in which K^- has momentum $p\leq 0.7$ GeV at J is $\approx 14\%$.[17] Since corrections due to K^- with $p>0.7$ GeV and events with $2K\bar{K}$ work in opposite

TABLE I. Statistical-model branching ratios $\Gamma((N-m_0)\pi^c, m_0\pi^0)$.

N \ m_0 (N odd)	1	3	5	7	9	11
3	1					
5	$\frac{2}{3}$	$\frac{1}{3}$				
7	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{1}{12}$			
9	$\frac{14}{58}$	$\frac{31}{58}$	$\frac{12}{58}$	$\frac{1}{58}$		
11	$\frac{126}{951}$	$\frac{448}{951}$	$\frac{314}{951}$	$\frac{60}{951}$	$\frac{3}{951}$	
13	$\frac{396}{5649}$	$\frac{2070}{5649}$	$\frac{2300}{5649}$	$\frac{790}{5649}$	$\frac{90}{5649}$	$\frac{3}{5649}$

N \ m_0 (N even)	0	2	4	6	8	10
2	$\frac{2}{3}$	$\frac{1}{3}$				
4	$\frac{6}{15}$	$\frac{8}{15}$	$\frac{1}{15}$			
6	$\frac{20}{105}$	$\frac{66}{105}$	$\frac{18}{105}$	$\frac{1}{105}$		
8	$\frac{70}{819}$	$\frac{440}{819}$	$\frac{276}{819}$	$\frac{32}{819}$	$\frac{1}{819}$	
10	$\frac{252}{6633}$	$\frac{2590}{6633}$	$\frac{2960}{6633}$	$\frac{780}{6633}$	$\frac{50}{6633}$	$\frac{1}{6633}$

TABLE II. Statistical-model branching ratios (in percent) $\Gamma((N-m_0)\pi^c, m_0\pi^0)$ for G-conserving decays of J.

N \ m_0	1	3	5	7	9	11	Γ_N^{total}
3	2.4						2.4
5	4.0	2.0					6 (input)
7	3.0	3.5	0.6				7
9	1.0	2.5	1.0	0.1			4.6
11	0.26	0.94	0.66	0.13	0.01		2
13	0.05	0.26	0.29	0.10	0.01	0.0004	0.7
							$\sum_{N\geq 15}\Gamma_N^{\text{total}}=0.2$

TABLE III. Statistical-model branching ratios (in percent) for $G=+$, N even, $\Gamma((N-m_0)\pi^c, m_0\pi^0)$ normalized to $\sum_N \Gamma_N^{\text{total}} = 100\%$.

$N \backslash m_0$	0	2	4	6	8	10	Γ_N^{total}
2	2	1					3
4	6	8	1				15
6	5	17	5	0.3			27
8	2	15	9	1	0.03		27
10	0.7	7	8	2	0.1	0.003	18
						$\sum_{N\geq 12} \Gamma_N^{\text{total}}$	$=10$

directions, we take the fraction of all events with $K\bar{K}$ to be simply $2\times(14\%)\times 0.86 = 24\%$. The actual fraction could be a bit larger. This includes decays $J\to 1\gamma\to$ hadrons with a $K\bar{K}$ pair. The smallest identifiable branching ratio is for events with $N\bar{N}$. If we take twice the fraction containing $\bar{p}$ with momentum $p<1$ GeV we have $\approx 6\%$ for this. We estimate the fraction $J\to N\pi$ (N even) to be 6%, based on the modes $J\to 4\pi^c, 6\pi^c$ and the same statistical model as before, but now for N even (keeping $\langle N\rangle = 7$). In this way we can obtain

$$\Gamma(J\to\text{pions}) + \Gamma(J\to\text{pions}+\geq 1K\bar{K}) + \Gamma(J\to\text{pions}+N\bar{N}) \approx 29\% + 24\% + 6\% = 59\%, \quad (18)$$

excluding production of η, η' and also excluding possible radiative decays [e.g. $J\to\gamma\eta'$, $\gamma X(2.8)$]. The error in (18) is surely large (±10%, say), and hard to estimate. The decays $\gamma\eta, \gamma\eta'$ are negligible[6,7] but $\gamma X(2.8)$ probably is not. The most likely candidates for most of the marginally "missing" 27% are decays of the type $J\to(\eta$ or $\eta')$+pions → more pions, plus possible photons from η or η' decays. We have already remarked that the fraction of events with $\eta+\eta'$ could be as large as that with $K\bar{K}$. Notice that G-parity–conserving decays of J lead to final states (Npions) + (η or η') with N odd. Thus, since η or η' decays always contain ≥ 1 neutral π^0 or γ, the final pion state in this case has always ≥ 2 neutrals. On the other hand, the presence of η and η' can only lead to final states with charged pions and one neutral through the 1γ decay of J. This source of contamination is surely small, relative to G-conserving decays of J.

If it is possible to separate out the direct decay of $\psi' \to (N-1)\pi^c 1\pi^0$, subtracting those that arise from $\psi'\to J\pi\pi$, it should be possible to use these channels to guess the direct decay branching ratio for $\psi'\to$ hadrons.

It is difficult to make any definite statements concerning the $C=+$ states at 3.4 and 3.5 GeV at present. We only observe here that for the meson at 3.4 GeV, the branching ratios for ψ' into a photon plus $\pi^+\pi^- + K^+K^-$, $4\pi^\pm$, or $6\pi^\pm$, quoted to be[5] $(0.13\pm 0.05)\%$, $(0.14\pm 0.07)\%$, and 0.1%, are consistent with Table III (assuming $\pi^+\pi^- = K^+K^-$). We can use this to guess at a total pion branching ratio $\psi'\to\gamma+\chi(3.4)\to\gamma$+pions of $(2\pm 1)\%$. Since kaon channels have been seen, we feel safe in multiplying this by $\approx 86\%/23\% = 3.7$ (the ratio of all hadrons to all pions at the J) so as to estimate a branching ratio $\psi'\to\gamma+\chi(3.4)\to\gamma$+hadrons of $(7\pm 3)\%$. The error is solely experimental. A similar number emerges for $P_c(3.5)$. Then the quoted branching ratio $\psi'\to P_c\gamma\to J+\gamma\gamma = (4\pm 2)\%$,[6] and the fact that $(P_c'\to J+\gamma)/(P_c\to J+\gamma)$ is roughly 2 events/6 events $=\frac{1}{3}$,[6] indicates that the hadronic decays of $P_c'(3.4)$ are 70–90% of the total $P_c'(3.4)$ decay rate, if $P_c' = \chi(3.4)$. For the $P_c(3.5)$, the estimated range is 50–70%. Of course, this all assumes that only G-conserving hadronic or $J+\gamma$ decays occur.

Note added in proof. We wish to thank P. Jasselette for calling our attention to his extensive tables of Pais coefficients.[18]

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