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Broken $SU(4)$ Relations for Particle Production at Large Angles

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It is pointed out that inclusive distributions of π, K and η at large angles have the same dependence on transverse mass ($m_\perp$) in a wide range of $m_\perp$ at ISR energy. K/π and η/π ratios for the same value of $m_\perp$ are consistent with broken $SU(3)$ relations derived from the tensor dominated Pomeron scheme. This idea is extended to $SU(4)$ to derive asymptotic relations between inclusive distributions of charmed particles and non-charmed particles. From these relations we estimate $d\sigma(pp\to D^\pm X)/dy$ and $d\sigma(pp\to\phi_c X)/dy$ of order of 1~5 μb and 10~100 nb, respectively. The consequences of sizable cross-sections for production of charmed particles are discussed in connection with lepton production at large angles in hadron-hadron scattering.

§ 1. Introduction

Among theoretical models for the recently discovered narrow resonances ψ and ψ' at masses 3.1 and 3.7 GeV,[1] one attractive interpretation is that they are $J^P=1^-$ bound states of charmed quark and anti-quark in the quartet scheme.[2] This suggestion has given stimulus to the study of production mechanisms of charmed particles. In the Regge model approach high energy scattering processes are governed by Pomeron exchange. The tensor dominated Pomeron model with a low intercept of the f_c ($c\bar{c}$ 2^+ meson) trajectory implies that the Pomeron coupling to charmed quarks is suppressed by one order of magnitude relative to the Pomeron coupling to nucleon quarks.[3)~6)] The prediction of strong suppression of ψ and ψ' photo-production is one of its important consequences and had some support from a Fermi NAL experiment.[7]

In this paper we apply the idea of tensor dominated Pomeron to Mueller-Regge model in order to relate inclusive distributions of charmed particles at large angles in high energy hadron-hadron reactions to those of non-charmed particles. First we test tensor dominance relations derived in the $SU(3)$ framework by Yazaki[8] with data on π, K and η production in a wide range of transverse mass ($m_\perp$) at ISR

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energy.[9),10)] We find that K/π and η/π ratios for the same value of $m_\perp$ are approximately constant from $m_\perp=0.5$ to 5 GeV; broken $SU(3)$ predictions for these ratios are valid within an accuracy of $\sim 20\%$. Then we estimate inclusive distributions of charmed pseudoscalar mesons using $SU(4)$ tensor dominance relations and ISR data on π and K distributions. Our estimate gives $d\sigma(pp\rightarrow D^\pm X)/dy=1\sim 4\,\mu$b and $d\sigma(pp\rightarrow F^\pm X)/dy=0.5\sim 1.5\,\mu$b. For vector meson production we introduce the assumption of helicity independence and additive quark model relation between double Pomeron vertices for π and ρ production. Given distributions of pseudoscalar production, then we can evaluate distributions of vector meson production. We obtain $d\sigma(pp\rightarrow\phi_c X)/dy=30\sim 80$ nb.

In § 2, $SU(4)$ tensor dominance relations in the central region are derived and estimated numerically. We compare the $SU(3)$ part of these relations with data in § 3. The results of § 2 are used in § 4 to estimate cross-sections for charmed particle production at ISR energy. In § 5 we discuss consequences of charmed particle production on lepton production in hadron-hadron scattering. Concluding remarks are given in § 6.

§ 2. $SU(4)$ tensor dominance relations in the central region

Let us consider an inclusive reaction

$$a+b\rightarrow c+X, \tag{1}$$

where X denotes anything. We shall use one-particle distributions defined by

$$f_{ab,c}(s,y,p_\perp)=E\frac{d^3\sigma_{ab\rightarrow cX}}{d\boldsymbol{p}^3}, \tag{2}$$

where E and p are the energy and momentum of the particle c, y and $p_\perp$ being the rapidity and transverse momentum respectively.

In the Mueller-Regge model, the one-particle distribution in the central region at high energy is expressed in terms of double Regge exchange

$$f_{ab,c}(s,y,p_\perp)=\sum_{i,j}F_{ab,c}^{i,j}(m_\perp)\,(\cosh(y-y_a))^{\alpha_i-1}(\cosh(y-y_b))^{\alpha_j-1}, \tag{3}$$

where α_i and α_j are the intercepts of the Regge trajectories exchanged in the $a\bar{a}$ and $b\bar{b}$ channels, respectively. $F_{ab,c}^{i,j}$ represents the product of the external Regge residues and the i-j central vertex. It is a function of transverse mass $m_\perp=\sqrt{p_\perp{}^2+m_c^2}$. At sufficiently high energy the expression (3) is governed by double Pomeron exchange and is reduced to a function of a single variable $m_\perp$

$$f_{ab,c}(s,y,p_\perp)\sim F_{ab,c}^{P,P}(m_\perp). \tag{4}$$

Suppose we know the internal symmetry property of the double Pomeron vertex. Then we can relate asymptotic cross-sections for production of different particles in the same multiplet of the internal symmetry group. Yazaki has applied the scheme of f and f' dominated Pomeron[11)] to the inclusive reaction in the certral region and derived the formula relating the double Pomeron vertex to f-f, f-f' and f'-f' central vertices,[8)]

$$F_{ab,c}^{P,P}(m_\perp)=\sum_{i,j,k,l}\beta_a{}^i(0)\frac{1}{1-\alpha_i}B_{ij}{}^P(0)\frac{1}{1-\alpha_j}V_c{}^{j,k}(m_\perp)$$

$$\times\frac{1}{1-\alpha_k}B_{kl}{}^P(0)\frac{1}{1-\alpha_l}\beta_b{}^l(0), \tag{5}$$

where the summation is over all vacuum trajectories f, f' and their daughters. β is the external Regge residue. $B_{ij}{}^P$ is the Pomeron bubble and $V_c^{jk}(m_\perp)$ is the j-k central vertex.

Consider production of a penta-decimet meson. If we repeat the same argument in the $SU(4)$ framework as Yazaki did in the $SU(3)$ framework, we get the same formula as Eq. (5) except that the sum now includes f_c ($c\bar{c}$ memeber of 2^+ hexa-decimet) and its daughters in addition to f, f' and their daughters. Thus the double Pomeron vertices are related to the tensor-tensor central vertices, where tensor represents f, f' and f_c. We make the same additional assumptions as in the case of $SU(3)$ symmetry in Refs. 10) and 8):

(1) The leading f, f' and f_c contributions dominate the summation over vacuum trajectories.

(2) Symmetry breaking affects masses and trajectories but leaves all couplings $SU(4)$ symmetric. The f, f' and f_c mesons are such that they are $I=0$ members of an ideal hexa-decimet. The Pomeron bubble $B_{ij}{}^P$ is coupled to f, f' and f_c like an $SU(4)$ singlet. The central vertex V_c^{jk} preserves $SU(4)$ symmetry for the same value of $m_\perp$.

With these approximations $SU(4)$ breaking of the double Pomeron exchange is parametrized in terms of the breaking of the tensor trajectories

$$r_1=\frac{1-\alpha_f}{1-\alpha_{f'}}, \qquad r_2=\frac{1-\alpha_f}{1-\alpha_{f_c}}. \tag{6}$$

We now apply the model of tensor dominated Pomeron to production of pseudoscalar mesons. It gives the following asymptotic relations for fixed value of $m_\perp$,

$$f_{ab,\pi}:f_{ab,K}:f_{ab,\eta}:f_{ab,\eta'}:f_{ab,D}:f_{ab,F}$$

$$=1:r_1:\sigma^2+\tau^2r_1{}^2:\tau^2+\sigma^2r_1{}^2:r_2:r_1r_2, \tag{7}$$

where σ and τ are given in terms of octet-singlet mixing angle θ,

$$\sigma=\sqrt{1/3}\cos\theta+\sqrt{2/3}\sin\theta, \qquad \tau=\sqrt{1/3}\sin\theta-\sqrt{2/3}\cos\theta. \tag{8}$$

The mixture of $c\bar{c}$ component in η and η' is presumably small and hence is ignored, while the η_c cross-section is sensitive to a small mixture of $N\bar{N}$ and $\lambda\bar{\lambda}$ components and is not given above.

For production of hexa-decimet vector mesons, asymptotic relations for the same value of $m_\perp$ become very simple,

$$f_{ab,\rho}:f_{ab,\omega}:f_{ab,K^*}:f_{ab,\phi}:f_{ab,D^*}:f_{ab,F^*}:f_{ab,\phi_c}$$
$$=1:1:r_1:r_1^2:r_2:r_1r_2:r_2^2\,. \tag{9}$$

(The $SU(3)$ part of Eq. (9) as well as the π/K ratio in Eq. (7) has already been given in Ref. 8).)

It is of interest to note that we can also derive the same relations as Eqs. (7) and (9), if we assume additive quark model relations for double Pomeron vertices at the same value of $m_\perp$ instead of the same value of $p_\perp$.[12] In this picture the parameters r_1 and r_2 are Pomeron couplings to λ and c quarks, respectively, relative to Pomeron coupling to nucleon quarks (see Fig. 1). However, they are not related to the f, f' and f_c intercepts by Eq. (6) any more but are free parameters.

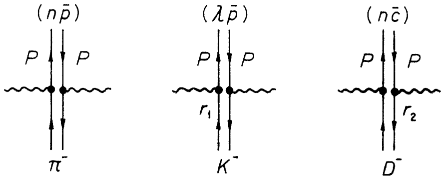


Fig. 1. Additive quark model diagrams of double Pomeron vertices for π^+, K^+ and D^+ productions.

To estimate numerically the ratios (7) and (9) we shall take a model of exchange degenerate linear ϕ_c-f_c trajectory with daughters spaced by two units of angular momentum.[5),6)] This gives $\alpha_{f_c}\simeq-3.8$ and $r_2=0.085\sim0.13$. As for r_1, linear $\phi-f$ trajectory gives $r_1=0.5\sim0.7$. Taking account of $(1+r_1)/2=\sigma_{KN}/\sigma_{\pi N}=0.75\sim0.80$, we choose $r_1=0.5\sim0.6$. For pseudoscalar meson production we take the octet-singlet mixing angle $\theta=10°$. This gives

$$f_{ab,\pi}:f_{ab,K}:f_{ab,\eta}:f_{ab,\eta'}:f_{ab,D}:f_{ab,F}$$
$$=1:0.5\sim0.6:0.63\sim0.68:0.62\sim0.67:0.08\sim0.13:0.04\sim0.08\,. \tag{10}$$

For vector meson production we have

$$f_{ab,\rho}:f_{ab,\omega}:f_{ab,K^*}:f_{ab,\phi}:f_{ab,D^*}:f_{ab,F^*}:f_{ab,\phi_c}$$
$$=1:1:0.5\sim0.6:0.25\sim0.36:0.08\sim0.13:0.04\sim0.08:0.007\sim0.016\,. \tag{11}$$

§ 3. Test of broken $SU(3)$ relations

Detailed experimental studies of inclusive reactions in the central region have been done at CERN ISR[9),10)] since Yazaki's proposal. We now have an opportunity of checking the $SU(3)$ part of the relations (7) in a wide range of $m_\perp$. Thus we can test the basic assumption of f and f' dominated Pomeron as well as the supplementary assumption of $SU(3)$ symmetry of couplings. ISR data indicate that one-particle distributions are still increasing in their energy range, although the rate of increase is much smaller than at PS energy. This implies that terms other than the double Pomeron exchange may possibly be not negligible. Since no reasonable way of separating the double Pomeron exchange from secondary terms is known, we had better use data at as high energy as possible. Fortunately, at $\sqrt{s}=53$ GeV there are data on π, K and η production with accurate relative normalization.[9),10)]

In Fig. 2(a) we plot one-particle distributions for these three reactions at $\theta_{\text{c.m.}}\simeq 90°$ as functions of $m_\perp$. For $\pi^\pm$ and $K^\pm$ distributions we have taken the average of positive charge and negative charge cross-sections in order to eliminate $C=-1$ exchange. The π and K cross-sections vary more than six orders magnitude from $m_\perp=0.5$ to 5 GeV. We note that in spite of this large variation of the individual cross-sections, the K and π distributions have approximately the same $m_\perp$ dependence all the way out to $m_\perp\simeq 5$ GeV. More quantitatively, the K/π ratio is more or less constant and is ~ 0.4, which is close to the theoretical value $0.5\sim 0.6$ (see Fig. 2(b)). Using data at lower ISR energies we get a similar conclusion. η and π^0 distributions also have the same $m_\perp$ dependence. The η/π^0 ratio is consistent with the predicted value $0.63\sim 0.68$ (see Fig. 2(c)), although relatively large errors on the η cross sections do not allow us to draw a more quantitative conclusion.

We note in passing that the data on $\pi^\pm$ and π^0 cross-sections used in Fig. 2(a) are subject to contamination from decay products of K_s^0, η and η' mesons. Making corrections for this contamination would largely reduce the discrepancy between the experimental K/π and η/π ratios and the predictions seen in Figs. 1(b) and 1(c) (it is likely that the contamination is $5\sim 10\%$). Further measurement of η production at smaller $p_\perp$, i.e., $p_\perp \lesssim 1.5$ GeV, as well as η' production will be very useful to test the broken $SU(3)$ relations more thoroughly. We give in Fig. 3 predictions for the $p_\perp$ dependence of η and η' production at $\theta_{\text{c.m.}}\simeq 90°$ relative to π production at higher ISR energies.

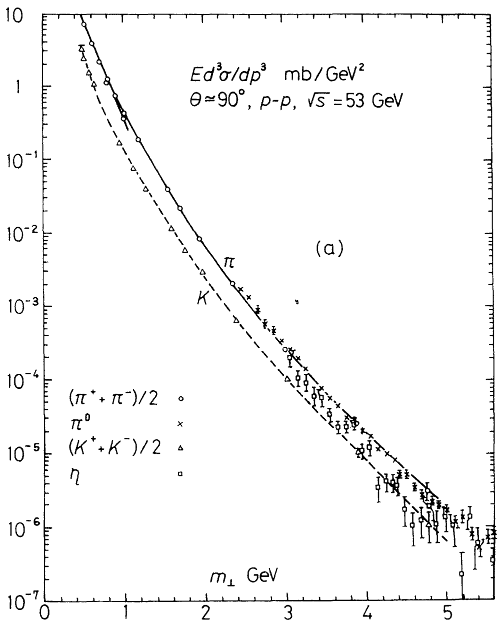
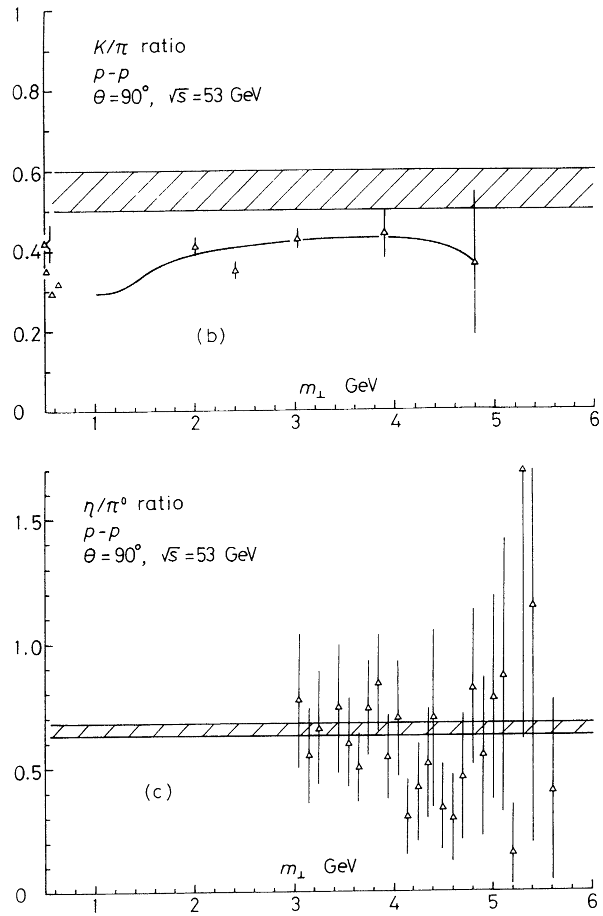


Fig. 2(a). One-particle distributions as functions of $m_\perp$ for π (circle and cross), K (triangle) and η (square) productions at $\theta_{\text{c.m.}}\simeq 90°$ in p-p scattering at $\sqrt{s}=53$ GeV. The solid and dashed lines are smooth interpolations of $\pi^\pm$ and $K^\pm$ data points. Data are from Refs. 9) and 10).



Figs. 2(b), (c). K/π and η/π^0 ratios for the same value of $m_\perp$ calculated from the π, K and η distributions given in Fig. 2(a). The predicted ratios, Eq (10), are shown by shaded area for comparison.

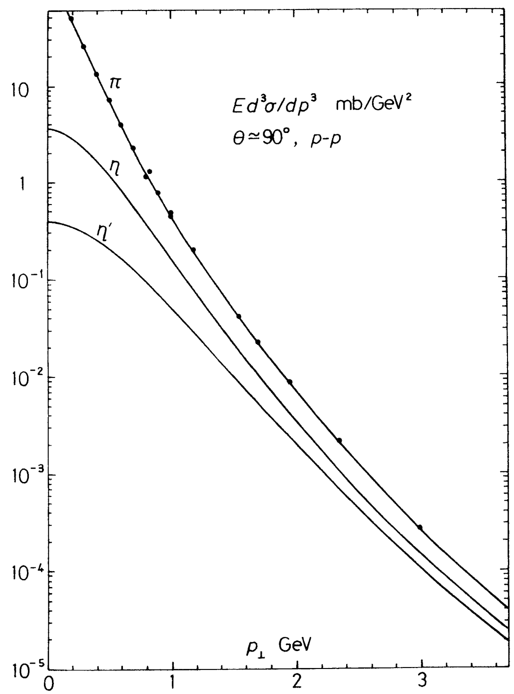


Fig. 3. The predictions of the $p_\perp$ dependence for η and η' production at $\theta_{c.m.} \simeq 90°$ relative to π distribution at higher ISR energies. Data on π distribution at $\sqrt{s} = 53\,\text{GeV}$[9] are plotted.

§ 4. Estimate of cross-sections for charmed particle production

The above analysis of K/π and η/π ratios has shown that the $SU(3)$ part of the tensor dominance relations is valid within an accuracy of $\sim 20\%$ in a wide range of $m_\perp$. Now we want to use the broken $SU(4)$ relations (7) and (9), or (10) and (11), to estimate the order of magnitude as well as $p_\perp$ dependence of inclusive cross-sections for charmed particle production.

Figure 4(a) shows the prediction for the $m_\perp$ dependence of inclusive D and F production (average of positive charge and negative charge cross-sections) at $\theta_{\text{c.m.}} \simeq 90°$ in p-p reaction. They are calculated from the ratio (10) and the experimental values of π and K cross-sections at $\sqrt{s} = 53$ GeV. When we go from the $m_\perp$ variable to the $p_\perp$ variable, D and F distributions depend on their masses. In Fig. 4(b) are given two sets of $p_\perp$ dependence calculated by using $m_D = 2.20$ GeV and $m_F = 2.25$ GeV (I), and $m_D = 2.00$ GeV and $m_F = 2.05$ GeV (II), respectively. At small $p_\perp$, D and F distributions are suppressed by several orders of magnitude relative to π and K distributions. This suppression is mainly due to thier masses being large. The predicted D/π and F/K ratios increase rapidly as

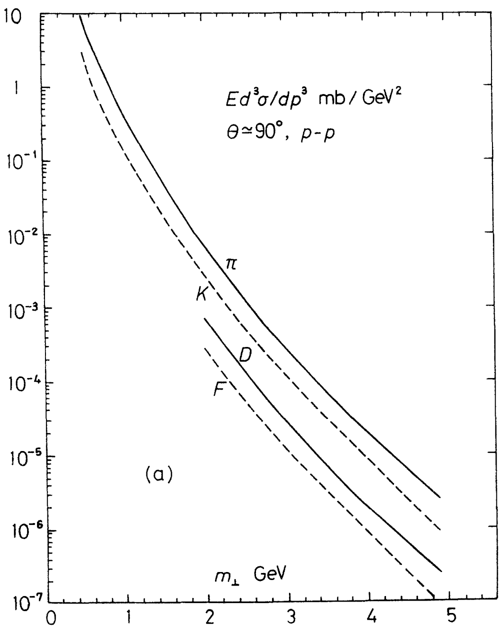


Fig. 4(a). The predictions of the $m_\perp$ dependence of D (solid line) and F (dashed line) production at $\theta_{\text{c.m.}} \simeq 90°$ relative to π (solid line) and K (dashed line) production at higher ISR energies. Data on π (solid line) and K (dashed line) distributions[9] at $\sqrt{s} = 53$ GeV are used as input.

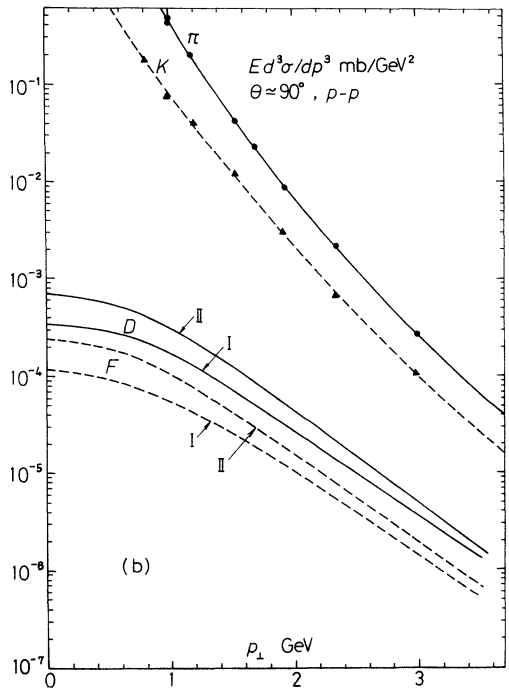


Fig. 4 (b). The $p_\perp$ dependence of the same quantities as given in Fig. 4 (a). The lines I correspond to $m_D = 2.20$ GeV and $m_F = 2.25$ GeV. The lines II correspond to $m_D = 2.00$ GeV and $m_F = 2.05$ GeV.

$p_\perp$ becomes large and are $1\sim2\%$ at $p_\perp\simeq 3$ GeV/c.

Integrating the D and F distributions in Fig. 3(b) over $p_\perp$ we obtain

$$d\sigma(pp\to DX)/dy=1\sim2\ \mu\text{b}, \quad d\sigma(pp\to FX)/dy=0.5\sim0.8\ \mu\text{b (I)},$$
$$d\sigma(pp\to DX)/dy=2\sim4\ \mu\text{b}, \quad d\sigma(pp\to FX)/dy=0.8\sim1.3\ \mu\text{b (II)}. \qquad (12)$$

As for vector meson production, unfortunately there is no measurement of $p_\perp$ dependence of either ρ or K^* production at sufficiently high energy. We introduce (i) helicity indepedndence of double Pomeron central vertex for vector meson production and (ii) an additive quark model relation between he double Pomeron vertices for π and ρ production. Namely, we assume

$$\sum_{\text{spin}} F^{P\text{-}P}_{ab,\rho}(m_\perp)=3F^{P\text{-}P}_{ab,\pi}(m_\perp), \qquad (13)$$

or at high energy

$$\sum_{\text{spin}} f_{ab,\rho}(s,y,p_\perp)\sim 3f_{ab,\pi}(s,y,p_\perp'). \quad (m_{\rho\perp}=m_{\pi\perp}) \qquad (14)$$

The experimental test of the relation (14) is not unambiguous. The correct way may be to compare ρ distribution with π distribution after making correction of pions from decays of vector mesons. If this would be the case, one should rewrite Eq. (14) as

$$\sum_{\text{spin}} f_{ab,\rho}(s,y,p_\perp)\sim 3(1-\varepsilon(p_\perp'))\times f_{ab,\pi}(s,y,p_\perp'), \quad (m_{\rho\perp}=m_{\pi\perp}) \qquad (15)$$

where ε is the fraction of vector meson-mediated pion distribution. Experimental analysis at $p_L=24$ GeV/c[13] suggests that the fraction ε may be as large as $\sim30\%$. It would be even larger at higher energy.

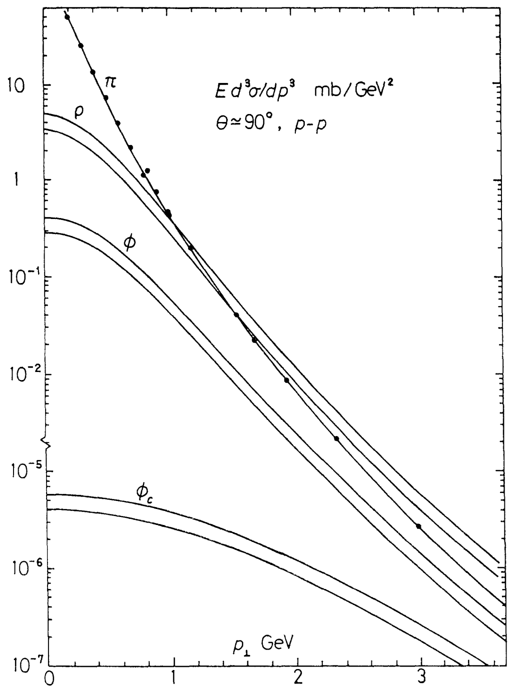


Fig. 5. The predictions of the $p_\perp$ dependence for ρ, ϕ and ϕ_c productions at $\theta_{\text{c.m.}}\simeq90°$ relative to the π distribution at higher ISR energies. The upper lines are the predictions using Eq. (14) and the lower lines are the predictions using Eq. (15) with $\varepsilon=0.3$.

In Fig. 5 we present the prediction for one particle distributions of $\rho(\omega)$, ϕ and ϕ_c at $\theta_{\text{c.m.}}\simeq90°$ in p-p reaction relative to the π distribution at $\sqrt{s}=53$ GeV. The upper curves represent the prediction using Eq. (14) as input. The lower curves give the prediction taking account of the above-mentioned correction with $\varepsilon=0.3$. We see that the assumption (13) implies that the asymptotic ρ/π ratio increases with $p_\perp$ and is larger than 1 for $p_\perp \gtrsim 1.2\,\mathrm{GeV}/c$. There is experimental evidence[13), 14)] that the ρ/π ratio increases rapidly with $p_\perp$. Predicted cross-sections integrated over $p_\perp$ for charmed vector meson production (without correction of decays of vector mesons) are

$$d\sigma(pp\to D^*X)/dy = 4\sim 7\ \mu\mathrm{b}\,,$$
$$d\sigma(pp\to F^*X)/dy = 1\sim 2\ \mu\mathrm{b}\,,$$
$$d\sigma(pp\to \phi_c X)/dy = 30\sim 80\ \mathrm{nb}\,. \tag{16}$$

We have used $m_{D^*}=2.20$ GeV, $m_{F^*}=2.25$ GeV and $m_{\phi_c}=3.10$ GeV. Here and hereafter we take Eq. (14). Recent measurement at the ISR indicates that the ϕ_c production cross-section is indeed of order 100 nb.[15)]

If we assume the universal slope $\alpha'_{f_c} \simeq 1.0\,\mathrm{GeV}^{-2}$ [3)] instead of our model of the f_c trajectory with daughters spaced by two units of angular momentum, we have $r_2 \simeq 0.05$. For this value of r_2 predicted cross-sections for production of particles with non-zero charm and ϕ_c are smaller by factors of two and four, respectively, than those given above.

The present scheme of tensor dominated Pomeron invokes several approximations which are not very well founded. Among others, the f_c intercept is not known experimentally and $SU(4)$ symmetry of the Pomeron bubble B_{ij} may be badly broken.[16)] For phenomenological purpose it may be sensible to leave the $SU(4)$ breaking parameter r_2 free and to be determined from data on total cross-sections (see Ref. 5) in this connection). Table I gives prediction for cross-sections for charmed particle production at $\sqrt{s}=53$ GeV in terms of the parameter r_2

Table Ia. Prediction of inclusive cross-sections $d\sigma/dy$ for production of charmed pseudoscalar mesons in p-p reactions at higher ISR energies. They are calculated from the asymptotic relations (7) (the third row) and (10) (the last row) and using π and K distributions at $\sqrt{s}=53$ GeV.[9)]

$pp\to DX$		$pp\to FX$	
$m_D=2.20$ GeV	$m_D=2.00$ GeV	$m_F=2.25$ GeV	$m_F=2.05$ GeV
$17\cdot r_2\ \mu$b	$27\cdot r_2\ \mu$b	$5.7\cdot r_2\ \mu$b	$9.3\cdot r_2\ \mu$b
$1\sim 2\ \mu$b	$2\sim 4\ \mu$b	$0.5\sim 0.8\ \mu$b	$0.8\sim 1.3\ \mu$b

Table Ib. Prediction of $d\sigma/dy$ for production of vector mesons $\rho(\omega)$, ϕ and charmed vector mesons. ρ cross-section is calculated from the relation (14) and using π distribution at $\sqrt{s}=53$ GeV as input. ϕ and charmed vector meson cross-sections are then calculated from the asymptotic relations (9) (the second row) and (11) (the last row) and using the ρ distribution as input.

$pp\to\rho X$	$pp\to\phi X$	$pp\to D^*X$	$pp\to F^*X$	$pp\to\phi_c X$
5.7 mb	$2.2\cdot r_1^2$ mb	$50\cdot r_2\ \mu$b	$17\cdot r_2\ \mu$b	$4.7\cdot r_2^2\ \mu$b
5.7 mb	$0.5\sim 0.8$ mb	$4\sim 7\ \mu$b	$1\sim 2\ \mu$b	$30\sim 80$ nb

together with a summary of the preceding numerical estimate. In these calculations corrections for the input π distribution due to decays of K, η, η' and vector mesons have been ignored.

§ 5. Consequence on lepton production at large angles

One of the simplest methods of testing our prediction for vector meson production would be to measure cross-section for lepton pair production at large angles around resonance masses in high energy hadron-hadron scattering. Denoting the branching ratio of $V^0 \to l^+l^-$ by $B(V^0 \to l^+l^-)$ we have

$$d\sigma(ab \to V^0X \,[V^0 \to l^+l^-])/dy = B(V^0 \to l^+l^-)\, d\sigma(ab \to V^0X)/dy\,.$$

At $\sqrt{s} \simeq 50$ GeV, where we have done our estimate of ρ, ω, ϕ and ϕ_c productions (Table Ib), we get

$$d\sigma(ab \to \rho^0X \,[\rho^0 \to l^+l^-])/dy + d\sigma(ab \to \omega X \,[\omega \to l^+l^-])/dy : d\sigma(ab \to \phi X \,[\phi \to l^+l^-])/dy : d\sigma(ab \to \phi_c X \,[\phi_c \to l^+l^-])/dy$$

$$= 1 : 0.25 \sim 0.35 : 0.003 \sim 0.008\,, \qquad (17)$$

where we have used $B(\rho^0 \to l^+l^-) = 4.5 \times 10^{-5}$, $B(\omega \to l^+l^-) = 7.5 \times 10^{-5}$, $B(\phi \to l^+l^-) = 3.0 \times 10^{-4}$ and $B(\phi_c \to l^+l^-) = 7.0 \times 10^{-2}$. As seen from Fig. 5, the contribution of ϕ_c to lepton pair production becomes more important as the $p_\perp$ of produced vector mesons increases.

The estimate in § 4 implies that D and F mesons and their vector counter parts are produced rather copiously; their production cross-sections are of order of $1 \sim 10\,\mu$b, one to two orders of magnitude larger than ϕ_c production cross-section. Particles with non-zero charm have leptonic and semi-leptonic decay modes such as $D \to l^+ + \nu +$ hadrons and $\bar{D} \to l^- + \bar{\nu} +$ hadrons. This seems to suggest that leptons among the decay products of D, F, D^* and F^* are responsible, at least partly, for unexpectedly large cross-sections for $l^\pm$ production at large angles in p-p scattering observed at Serpukhov, Fermi NAL and ISR recently.[17] How important these contributions to $l^\pm$ production would depend on the branching ratio of leptonic and semi-leptonic decays. Since the particles with non-zero charm cannot be produced singly, there is a certain probability that the leptons of their decay products would be observed in pair. However, if the leptonic branching ratio is small, as widely believed,[18] these leptons would be observed singly most of the time. To pursue this interpretation of the copious lepton production in a quantitative way and compare it with data[17] we need to estimate the $p_\perp$ dependence of the child leptons. This requires knowledge of the decay properties of the parent charmed particles and is beyond the scope of the present paper.

§ 6. Concluding remarks

It has been widely believed that the energy dependence of inclusive distributions at large angles between PS and ISR energies is largely due to kinematical effect, which dies away at high energy.[19] Our numerical analyses of inclusive distributions in §§ 3~5 rest on the assumption that the double Pomeron exchange is a fair approximation at higher ISR energies. At lower ISR energies our estimate may perhaps give a bit too large cross-sections for charmed particle production. We expect, however, that they give roughly correct magnitude relative to π and K production at the same value of $m_\perp$. For 2 GeV/$c \leq p_\perp \leq 3$ GeV/c, π distributions at $\sqrt{s}=31$ GeV are 20~40% lower than those at $\sqrt{s}=53$ GeV.[20] It is conceivable, therefore, that cross-sections for charmed particle production at $\sqrt{s}\simeq 30$ GeV would be lower than given in § 4 by a similar amount.

There have recently been attempts to construct a unified description of inclusive processes in the whole $p_\perp$ region at high energy.[21] These models are reduced to conventional Mueller-Regge model at small $p_\perp$ and to constituent-interchange model in the large $p_\perp$ limit. In this picture energy dependence of inclusive distributions is of dynamical origin and should be described correctly by assuming scaling in the variable $x_\perp=2p_\perp/\sqrt{s}$. We have shown in § 3 that the broken $SU(3)$ relations in the central region are in good accord with experiment out to large $m_\perp$. This indicates a possibility that these $SU(3)$ (and $SU(4)$) relations derived in the Mueller-Regge model can be extended to the large $p_\perp$ region in the above-mentioned approach and give symmetry constraints on constituent interchange interactions.

Recently Kwiencinski and Roberts have made estimates of cross-sections for inclusive D and F production at large angle in tensor-dominated Pomeron schemes.[22] They have considered a few possible Pomeron coupling schemes depending on the degree of symmetry breaking introduced by the loop integration in the Pomeron bubble B^P_{ij}. After the completion of the present work the author was informed that Kinoshita et al. had estimated $p_\perp$ and energy dependence of charmed particle production in a constituent rearrangement model.[23]

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