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PARAMETRIZATION OF THE q^2 DEPENDENCE OF $\gamma_v p$ TOTAL CROSS SECTIONS IN THE RESONANCE REGION

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All existing data on $\gamma_V p$ total cross sections in the resonance region are fitted in the absolute value of the three-momentum transfers $|\boldsymbol{q}|$ independently for small bins of W in the range $1.11 \leqslant W \leqslant 1.99$ GeV. The data are divided into three ranges of the polarization $\epsilon : \epsilon \geqslant 0.9, 0.9 > \epsilon > 0.6, \epsilon \leqslant 0.6$.

Taking into account statistical and possible systematic errors, a comparison of the fits for different ranges of ϵ indicate that longitudinal contributions to the cross sections in the resonance region are in general small. In the range $1.3 < W < 1.5$ GeV for $q^2 > 2$ GeV^2 and above $W = 1.6$ GeV for $q^2 > 0.5$ GeV^2, up to 20% contribution of σ_ℓ is possible.

1. Introduction

The total cross section Σ for the absorption of virtual photons on protons in the region of the main nucleon resonances has been determined in many inelastic electron-proton scattering experiments [1–23] where only the scattered electron is detected. Σ is written as

$$\Sigma(q^2, W, \epsilon) = \frac{1}{\Gamma_t} \frac{d^2\sigma}{d\Omega\, dE'} = \sigma_t(W, q^2) + \epsilon\sigma_\ell(W, q^2) \ , \qquad (1)$$

where, as usual [3], q^2 is the four-momentum transfer squared, W the mass of the outgoing hadronic system, Γ_t the flux of transverse polarized virtual photons, ϵ the degree of polarization of the virtual photons, and σ_t and σ_ℓ are the absorption cross section for transverse and longitudinal polarized virtual photons, respectively.

A parametrization of the q^2 dependence of the total cross section by a fitting procedure applied to all existing data allows one to study its behaviour with the best available statistics. As the total cross section Σ consists of two parts σ_t and σ_ℓ a parametrization of the measured cross sections Σ has to take into account their vari-

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ation with ϵ. We have divided therefore all data with respect to different regions of ϵ and have applied fits to these different sets of cross sections. As a result one gets information of the contributions of σ_ℓ to the total cross section.

First fits of the type presented in this report have been shown earlier [24] *. Now some more inelastic ep scattering data in the resonance region have become available [9–12, 16–18]. Also, more data [9,10,12,18,20] are now available between $q^2 = 0$ and $q^2 = 0.5$ GeV^2, a region in which, earlier, there was generally a lack of data as only a small portion of data from refs. [2,6,7] belonged to this range. Furthermore, photoproduction data [25] can now be used for $q^2 = 0$ in place of the SLAC photon-proton cross sections (used earlier in I) obtained from inelastic electron-proton scattering cross sections by extrapolation to the limit of zero four-momentum transfer [8]. We have, therefore, done the fitting of the entire electroproduction data available in the resonance region and present the results obtained therefrom in this report. It must be noted however that data for small values of ϵ are still very scarce and that there is a need for more measurements.

2. The procedure of fitting

The procedure of fitting is the same as used in (I), which is given below.

The cross section Σ from early results shows the following behaviour [2–6]:

$$\Sigma = G_D^2(q^2) \circ A(W) \cdot |\boldsymbol{q}|^{b(W)} , \qquad (2)$$

where $G_D^2(q^2)$ = dipole form factor of the nucleon, $\boldsymbol{q}$ = three-momentum transfer to the hadronic system in the lab frame and $A(W)$ and $b(W)$ are parameters dependent only on W.

For the validity of eq. (2), $|\boldsymbol{q}|$ must be small as for large values of $|\boldsymbol{q}|$, Σ has values smaller than those obtained from the above equation. If the outgoing hadronic system gets a definite total angular momentum, eq. (2) represents the threshold behaviour of Σ for $|\boldsymbol{q}| \to 0$. A plot of $\log (\Sigma/G_D^2)$ *versus* $\log |\boldsymbol{q}|$ gives a straight line; quadratic or higher powers of $\log |\boldsymbol{q}|$ may be introduced to account for deviations from this behaviour at high momentum transfers. One additional term is found to be sufficient (I) and, therefore, the following equation is used to fit the data:

$$\log(\Sigma/G_D^2) = a(W) + b(W) \cdot \log(|\boldsymbol{q}|/|\boldsymbol{q}_0|) + c(W) \cdot |\log(|\boldsymbol{q}|/|\boldsymbol{q}_0|)|^{d(W)} , \qquad (3)$$

where $|\boldsymbol{q}_0|$ is the value of $|\boldsymbol{q}|$ at $q^2 = 0$ for the same W.

The fits done in this work enable us to study the dependence of Σ on ϵ. The data with $\epsilon \geq 0.9$ are taken from refs. [6–12,17–19] while those with $\epsilon \leq 0.6$ from refs. [3–5,8,9,11,19,20]. For the fit in the range $0.6 < \epsilon < 0.9$, the relevant data from the above mentioned references as well as from ref. [2] are used. In W the fits were

* This references will henceforth be referred to as (I).

restricted to the range $1.11 \leqslant W \leqslant 1.99$ GeV. The Daresbury photoproduction cross sections [25], having W in the relevant range, have been included in all ϵ ranges as σ_ℓ is zero at $q^2 = 0$. We have used two bin sizes for W, one, $\Delta W = 0.015$ GeV, across the bumps of the resonances up to $W = 1.755$ GeV, and the other, $\Delta W = 0.020$ GeV, for $1.770 \leqslant W \leqslant 1.990$.

The errors as given in the various references have been increased to include possible systematic errors, since most of the papers give only statistical errors. Furthermore, the final bin size in W introduces an error which is not negligible around the resonances where the cross section changes rapidly with W. Finally, there may be differences in the absolute normalization of the measurements from different experimental arrangements. Therefore, to the errors of the cross sections in the range $\epsilon \geqslant 0.9$ with $W > 1.755$ GeV a 5% error has been added quadratically, to all others a 10% error. Using the above errors the cross sections resulting from the fits were practically not different from those, where only the original errors were used.

3. Results

Most of the additional data [9–12,17,18,20] available after the completion of (I) belong to the range $\epsilon \geqslant 0.9$, and therefore the results of this work for the range $\epsilon \leqslant 0.6$ are not very different from the corresponding results given in (I).

3.1. The parameter d

For the range $\epsilon \leqslant 0.6$ the amount of data is not sufficient to determine the four parameters in eq. (3) for each W bin. Further, b and c cannot be separated if d is around 1. This happens for some W bins in the case $\epsilon \leqslant 0.6$, when cross sections for high values of q^2 are missing or when the errors are too large. Therefore in a first step d was left free for $\epsilon \geqslant 0.9$ only. From these fits an average value of $d = 3.0$ for all bins of W and all ranges of ϵ was taken for the final fits. The choice of d turned out not to be critical for the cross sections resulting from the fits.

3.2. Coefficients a, b and c

The values of the coefficients a, b and c as well as their errors and correlations as determined from the least squares fits of eq. (3) are given in tables 1, 2 and 3 for the three ranges of ϵ. Also given are the weighted average values of ϵ for each W bin, the χ^2 per degree of freedom and the number of degrees of freedom.

3.3. Behaviour of the fits as a function of $|\boldsymbol{q}|^2$

In figs. 1–6 the measured values of Σ/G_D^2 are shown as a function of $|\boldsymbol{q}|^2$ separately for the two extreme ranges of ϵ for six values of W, i.e., for $W = 1.230$, 1.380,

Table 1
Results of the fits for $\epsilon \geqslant 0.9$

W	ϵ	a	$(\Delta a)^2 \cdot 10^4$	b	$(\Delta b)^2 \cdot 10^4$	c	$(\Delta c)^2 \cdot 10^4$	$C_{ab} \cdot 10^4$	$C_{ac} \cdot 10^4$	$C_{bc} \cdot 10^4$	χ^2 / NF	
1.110	0.963	5.045	251.7	0.798	217.3	0.043	2.9	228.2	23.5	23.8	2.3	67
1.125	0.962	5.126	168.5	1.052	166.4	0.024	2.8	163.5	18.8	20.3	2.3	70
1.140	0.963	5.390	109.8	1.213	116.6	0.000	2.3	110.1	13.6	15.4	2.2	71
1.155	0.956	5.621	83.1	1.334	95.9	−0.013	2.0	86.9	11.1	13.1	2.4	71
1.170	0.959	5.913	42.5	1.397	51.8	−0.023	1.4	44.6	5.9	7.8	1.8	69
1.185	0.957	5.955	45.3	1.727	58.1	−0.069	1.5	49.6	6.8	8.7	1.1	83
1.200	0.960	6.139	31.8	1.750	46.0	−0.060	1.5	36.4	5.4	7.6	1.0	75
1.215	0.956	6.178	26.7	1.878	39.7	−0.080	1.4	31.0	4.7	6.7	0.9	83
1.230	0.961	6.125	27.3	1.887	43.9	−0.065	1.8	32.6	5.2	7.9	0.7	65
1.245	0.960	5.999	23.1	1.927	37.9	−0.056	1.5	28.0	4.4	6.8	0.8	80
1.260	0.959	5.769	25.5	2.041	46.7	−0.065	2.3	32.8	5.9	9.3	1.3	83
1.275	0.962	5.622	21.5	2.089	43.0	−0.056	2.4	28.6	5.3	9.0	1.1	80
1.290	0.960	5.431	22.0	2.148	42.0	−0.043	2.4	28.5	5.1	8.7	1.5	76
1.305	0.956	5.288	19.6	2.205	42.0	−0.034	2.8	26.8	5.3	9.5	1.6	86
1.320	0.957	5.175	17.1	2.344	42.1	−0.054	3.2	25.0	5.3	10.2	1.3	85
1.335	0.958	5.131	15.7	2.324	38.6	−0.018	3.2	22.8	4.9	9.6	1.4	90
1.350	0.957	5.003	21.9	2.535	58.3	−0.046	5.6	33.5	7.9	15.8	1.6	76
1.365	0.959	5.065	16.0	2.464	44.3	−0.015	4.4	24.6	5.8	12.1	1.5	86
1.380	0.957	5.045	15.7	2.564	43.5	−0.029	4.2	24.0	5.4	11.5	1.3	77
1.395	0.957	5.078	13.9	2.610	40.1	−0.048	3.4	21.5	4.5	9.7	1.3	71
1.410	0.960	5.145	16.0	2.609	50.1	−0.032	5.9	26.0	6.6	14.8	0.8	75
1.425	0.957	5.156	14.8	2.678	47.2	−0.046	5.4	24.3	6.0	13.4	1.0	79
1.440	0.957	5.234	16.6	2.771	54.1	−0.084	6.2	27.6	6.8	15.4	0.9	67
1.455	0.959	5.298	15.4	2.890	51.5	−0.115	7.1	26.0	7.0	16.1	0.8	75
1.470	0.958	5.371	15.0	2.982	54.3	−0.105	7.6	26.1	7.1	17.2	0.8	67
1.485	0.956	5.457	15.3	3.157	51.6	−0.159	5.4	26.0	6.2	14.1	0.8	69
1.500	0.956	5.543	15.4	3.188	57.1	−0.164	8.5	27.3	7.6	18.3	0.5	65
1.515	0.957	5.519	14.7	3.315	54.7	−0.181	7.5	26.1	7.1	17.1	0.3	65
1.530	0.956	5.465	16.6	3.375	58.1	−0.203	7.4	28.7	7.6	17.6	0.5	65

Table 1 (continued)

1.545	0.957	5.384	16.7	3.450	70.1	−0.220	12.9	31.3	9.6	24.9	0.6	59
1.560	0.955	5.341	14.6	3.477	58.8	−0.245	8.4	26.8	7.3	18.4	0.5	65
1.575	0.954	5.328	15.6	3.471	75.0	−0.264	16.9	31.1	10.2	29.1	0.5	57
1.590	0.958	5.275	17.0	3.554	77.2	−0.239	17.0	32.5	10.6	30.1	0.5	51
1.605	0.955	5.296	16.9	3.633	75.4	−0.302	14.4	32.2	9.8	27.1	0.6	51
1.620	0.954	5.330	15.7	3.695	81.6	−0.299	19.9	32.0	11.0	33.1	0.7	50
1.635	0.956	5.375	15.3	3.804	72.8	−0.318	14.4	30.0	9.3	26.6	0.4	52
1.650	0.955	5.428	14.8	3.900	84.3	−0.388	22.1	31.6	11.2	35.1	0.4	51
1.665	0.954	5.478	14.3	4.047	72.5	−0.393	14.9	28.5	9.0	27.1	0.4	50
1.680	0.955	5.443	14.2	4.290	80.8	−0.466	21.8	29.9	10.7	34.7	0.4	48
1.695	0.953	5.390	13.7	4.519	80.1	−0.588	23.0	29.1	10.6	34.9	0.4	49
1.710	0.953	5.333	13.8	4.709	82.0	−0.622	19.0	29.5	10.1	33.0	0.5	46
1.725	0.955	5.296	14.2	4.757	81.6	−0.568	21.2	29.8	10.7	34.8	0.4	48
1.740	0.953	5.223	13.6	4.840	93.4	−0.574	24.8	30.7	11.3	40.6	0.3	44
1.755	0.960	5.159	5.4	5.017	42.4	−0.727	23.8	13.0	6.2	25.0	0.7	46
1.770	0.959	5.146	4.9	5.015	37.3	−0.665	16.8	11.5	4.9	19.5	0.7	45
1.790	0.960	5.143	4.0	5.129	32.2	−0.704	17.7	9.8	4.6	18.8	0.8	60
1.810	0.957	5.125	3.8	5.285	33.4	−0.856	21.0	9.5	4.7	20.6	1.0	62
1.830	0.954	5.158	3.6	5.322	29.0	−0.798	11.2	8.7	3.6	14.4	1.1	65
1.850	0.959	5.159	3.8	5.546	38.2	−1.048	40.2	10.2	6.1	30.0	0.9	59
1.870	0.958	5.178	3.7	5.623	37.1	−0.980	29.0	10.0	5.3	24.9	1.1	61
1.890	0.961	5.182	3.5	5.775	35.1	−1.021	29.0	9.3	4.9	23.5	1.4	55
1.910	0.955	5.195	3.4	5.894	34.7	−1.092	23.9	9.1	4.7	22.3	1.7	56
1.930	0.956	5.160	3.9	6.138	42.5	−1.313	39.2	10.9	6.5	31.5	1.0	57
1.950	0.956	5.195	3.2	6.151	37.8	−1.341	40.3	9.2	5.7	29.9	1.4	61
1.970	0.954	5.163	3.4	6.301	40.6	−1.266	34.5	9.9	5.8	29.4	1.3	61
1.990	0.953	5.172	4.3	6.542	50.9	−1.473	49.1	12.6	8.1	39.4	1.1	56

Table 2
Results of the fits for $0.9 > \epsilon > 0.6$

W	ϵ	a	$(\Delta a)^2 \cdot 10^4$	b	$(\Delta b)^2 \cdot 10^4$	c	$(\Delta c)^2 \cdot 10^4$	$C_{ab} \cdot 10^4$	$C_{ac} \cdot 10^4$	$C_{bc} \cdot 10^4$	χ^2 / NF	
1.110	0.835	−0.050	16677.8	4.849	9928.3	−0.285	73.8	12807.8	1060.6	838.0	2.3	18
1.125	0.820	1.082	9586.8	4.120	5790.4	−0.212	42.8	7415.9	610.8	486.4	1.7	19
1.140	0.837	4.119	3435.7	2.244	2468.2	−0.100	24.6	2895.1	272.3	238.1	2.5	24
1.155	0.859	3.898	2482.6	2.824	1864.3	−0.155	20.9	2135.3	209.8	189.3	1.5	21
1.170	0.854	5.990	90.1	1.257	90.9	−0.006	3.5	79.5	7.6	13.7	3.0	19
1.185	0.844	6.033	1077.1	1.548	910.2	−0.025	11.5	982.8	101.6	97.3	1.6	26
1.200	0.854	6.160	85.7	1.730	93.6	−0.066	3.5	82.1	8.9	13.9	0.8	22
1.215	0.857	6.219	84.6	1.805	95.1	−0.086	4.0	82.5	9.6	15.1	0.5	25
1.230	0.864	6.117	81.6	1.866	97.5	−0.071	2.5	84.3	10.4	13.7	1.1	28
1.245	0.859	5.959	77.9	1.914	82.8	−0.054	1.5	75.8	7.7	9.5	1.1	24
1.260	0.851	5.451	584.1	2.272	723.0	−0.084	15.8	645.1	89.9	103.2	0.8	31
1.275	0.848	5.675	79.8	1.840	121.0	0.012	4.9	92.4	13.6	20.8	1.6	26
1.290	0.858	5.417	85.8	2.102	127.6	−0.037	5.4	96.8	14.2	22.4	1.2	23
1.305	0.845	5.238	82.2	2.221	134.1	−0.046	7.0	98.4	15.8	25.7	1.8	29
1.320	0.781	5.084	71.9	2.368	105.8	−0.058	2.7	81.7	10.4	15.1	1.7	28
1.335	0.783	4.913	57.5	2.622	76.1	−0.085	1.0	62.3	5.6	7.5	1.9	31
1.350	0.798	4.458	407.6	3.146	695.2	−0.131	25.1	526.1	93.7	127.3	0.9	26
1.365	0.856	5.012	69.0	2.537	148.3	−0.031	10.3	91.7	17.5	34.0	1.1	19
1.380	0.859	5.128	86.1	2.473	172.3	−0.031	9.8	111.5	20.0	36.3	1.4	18
1.395	0.851	5.140	75.9	2.533	158.3	−0.033	13.6	101.1	20.3	38.8	1.7	22
1.410	0.828	5.261	38.2	2.610	67.4	−0.093	2.2	46.2	6.0	10.2	1.7	26
1.425	0.853	5.370	44.7	2.461	121.3	−0.000	13.5	66.1	15.6	34.9	1.3	26
1.440	0.851	5.416	42.3	2.570	114.9	−0.052	11.5	62.3	14.1	31.5	1.2	24
1.455	0.788	5.466	24.5	2.700	77.7	−0.083	8.1	38.5	9.0	21.8	0.7	33
1.470	0.824	5.508	30.2	2.799	97.0	−0.093	10.4	47.7	11.5	27.9	0.9	27
1.485	0.847	5.578	29.6	2.929	102.4	−0.104	13.0	49.2	12.9	31.8	0.9	30
1.500	0.839	5.629	29.4	3.016	111.7	−0.096	19.7	50.7	14.7	39.8	0.9	27
1.515	0.847	5.623	20.4	3.128	71.3	−0.163	7.1	33.1	7.6	19.6	0.9	33
1.530	0.841	5.555	23.2	3.215	103.4	−0.173	22.2	43.3	14.3	41.2	0.5	34

Table 2 (continued)

1.545	0.851	5.503	24.4	3.271	118.3	−0.192	26.0	46.5	15.5	48.1	0.6	26
1.560	0.852	5.471	19.1	3.288	90.7	−0.191	18.3	35.5	11.3	35.5	0.7	32
1.575	0.819	5.419	21.8	3.328	117.0	−0.202	29.0	43.7	15.9	51.2	0.7	30
1.590	0.846	5.390	21.5	3.390	109.6	−0.239	23.3	42.3	14.1	43.5	0.6	30
1.605	0.845	5.423	17.9	3.376	110.9	−0.168	34.2	37.7	14.7	53.2	0.6	31
1.620	0.840	5.396	18.6	3.567	106.6	−0.286	27.1	38.4	14.2	47.2	0.6	33
1.635	0.765	5.451	12.7	3.600	50.5	−0.239	2.6	21.3	3.5	9.7	0.6	36
1.650	0.836	5.400	18.4	3.945	136.6	−0.408	36.3	43.4	17.3	62.8	0.5	31
1.665	0.827	5.446	16.3	4.046	127.4	−0.402	49.4	38.4	16.8	67.9	0.5	30
1.680	0.690	5.448	13.1	4.124	63.5	−0.355	5.9	24.2	5.2	16.2	0.5	34
1.695	0.813	5.421	12.7	4.270	78.7	−0.400	10.9	26.5	7.2	24.8	0.4	35
1.710	0.818	5.308	20.5	4.667	161.0	−0.603	52.2	50.5	22.5	81.9	0.4	32
1.725	0.801	5.248	19.9	4.818	150.7	−0.642	46.5	47.9	21.0	75.7	0.3	32
1.740	0.814	5.193	16.0	4.856	122.9	−0.637	31.1	38.0	14.4	53.9	0.3	33
1.755	0.818	5.099	9.0	5.069	74.0	−0.773	27.9	22.6	10.8	40.7	0.7	29
1.770	0.819	5.076	9.8	5.112	82.0	−0.768	41.7	24.9	13.3	51.4	0.8	29
1.790	0.810	5.054	9.3	5.236	72.7	−0.879	33.8	23.2	12.2	44.2	0.6	35
1.810	0.821	5.064	8.4	5.331	74.5	−0.941	38.3	21.8	11.8	47.3	0.9	31
1.830	0.818	5.028	13.4	5.609	109.5	−1.075	53.0	34.8	19.4	68.7	0.9	27
1.850	0.823	5.080	9.7	5.628	86.7	−1.118	45.9	25.1	14.0	56.8	1.1	24
1.870	0.823	5.076	14.3	5.819	146.4	−1.223	85.6	40.9	25.2	101.5	0.6	20
1.890	0.775	5.173	9.4	5.461	60.4	−0.812	12.5	20.0	6.8	24.4	3.9	20
1.910	0.833	5.011	13.5	6.391	160.0	−1.581	103.7	40.2	25.0	114.4	0.4	15
1.930	0.811	5.136	19.2	5.995	156.6	−1.219	60.2	49.0	25.0	89.8	1.7	15
1.950	0.781	5.055	14.1	6.455	184.0	−1.605	132.1	43.7	29.3	141.6	2.1	14
1.970	0.648	5.035	14.1	6.565	154.2	−1.055	19.7	40.9	12.4	50.3	1.1	15
1.990	0.826	5.059	16.8	7.063	1167.1	−4.060	39509.2	104.7	421.2	6320.8	0.5	15

Table 3
Results of the fits for $\epsilon \leqslant 0.6$

W	ϵ	a	$(\Delta a)^2 \cdot 10^4$	b	$(\Delta b)^2 \cdot 10^4$	c	$(\Delta c)^2 \cdot 10^4$	$C_{ab} \cdot 10^4$	$C_{ac} \cdot 10^4$	$C_{bc} \cdot 10^4$	χ^2 /	NF
1.110	0.488	−3.024	21902.4	6.613	10961.8	−0.350	49.5	15445.2	1014.9	728.0	2.0	8
1.125	0.461	3.796	30067.5	1.935	19307.3	−0.005	153.2	24049.8	2113.8	1707.9	1.5	10
1.140	0.455	6.003	15772.4	0.774	10174.7	0.036	83.0	12622.9	1112.6	907.7	2.2	10
1.155	0.473	6.339	2102.8	0.653	1420.6	0.054	9.7	1714.5	134.7	114.1	2.6	11
1.170	0.499	6.071	92.2	1.017	113.2	0.053	3.9	81.7	8.6	18.3	1.0	14
1.185	0.429	6.834	4390.1	0.709	4387.4	0.083	80.5	4372.0	577.7	586.9	1.4	15
1.200	0.498	6.166	87.5	1.699	97.4	−0.054	2.7	81.6	9.0	14.0	1.2	16
1.215	0.450	6.239	80.6	1.768	87.8	−0.059	2.2	77.7	8.2	11.4	0.4	18
1.230	0.498	6.149	88.0	1.929	120.7	−0.087	4.7	84.4	10.1	20.9	0.2	11
1.245	0.444	5.988	90.5	1.943	156.9	−0.060	11.0	104.8	18.1	36.0	0.7	15
1.260	0.459	5.155	455.3	2.703	537.9	−0.158	11.9	486.6	65.8	76.1	0.6	18
1.275	0.460	5.727	80.2	1.892	128.1	−0.017	6.5	92.4	14.4	24.6	0.8	18
1.290	0.475	5.454	89.9	1.932	176.7	0.064	12.6	112.0	21.3	42.1	0.6	17
1.305	0.439	5.262	77.2	2.244	118.7	−0.050	4.6	88.0	12.6	20.2	1.1	19
1.320	0.458	5.230	87.1	2.040	199.9	0.071	17.8	118.1	25.6	53.2	1.1	18
1.335	0.451	5.153	97.4	1.980	246.2	0.169	32.2	136.7	33.3	77.5	0.8	15
1.350	0.429	4.365	218.7	3.206	362.4	−0.155	14.1	273.3	47.3	67.0	0.7	23
1.365	0.475	5.168	99.7	2.109	320.7	0.159	62.5	145.5	39.8	125.5	0.6	11
1.380	0.494	5.162	93.6	2.587	195.7	−0.106	19.9	113.7	21.6	53.5	1.2	12
1.395	0.462	5.174	83.8	2.479	153.5	−0.046	11.9	100.4	16.6	34.4	1.1	13
1.410	0.578	5.335	78.9	2.493	94.2	−0.071	2.0	76.5	7.2	11.1	0.6	12
1.425	0.471	5.378	89.5	2.437	297.0	0.005	66.7	130.7	36.3	123.1	0.6	11
1.440	0.464	5.396	80.9	2.581	170.0	−0.089	15.8	103.9	20.2	43.3	1.0	14
1.455	0.439	5.436	85.8	2.324	298.2	0.171	66.7	135.8	42.5	125.2	1.0	13
1.470	0.463	5.564	91.3	2.699	235.6	−0.098	43.5	120.8	27.2	83.3	0.6	10
1.485	0.445	5.615	87.1	2.971	185.3	−0.182	20.1	112.6	21.8	48.9	0.8	12
1.500	0.477	5.574	90.6	3.207	249.6	−0.216	52.4	124.1	30.4	95.1	1.6	10
1.515	0.439	5.560	90.9	3.325	208.8	−0.199	31.7	117.5	23.2	63.1	0.2	9
1.530	0.482	5.470	90.6	3.375	282.7	−0.192	68.7	135.3	37.4	114.6	0.5	9

Table 3
Results of the fits for $\epsilon \leqslant 0.6$

W	ϵ	a	$(\Delta a)^2 \cdot 10^4$	b	$(\Delta b)^2 \cdot 10^4$	c	$(\Delta c)^2 \cdot 10^4$	$C_{ab} \cdot 10^4$	$C_{ac} \cdot 10^4$	$C_{bc} \cdot 10^4$	χ^2 / NF
1.110	0.488	−3.024	21902.4	6.613	10961.8	−0.350	49.5	15445.2	1014.9	728.0	2.0 8
1.125	0.461	3.796	30067.5	1.935	19307.3	−0.005	153.2	24049.8	2113.8	1707.9	1.5 10
1.140	0.455	6.003	15772.4	0.774	10174.7	0.036	83.0	12622.9	1112.6	907.7	2.2 10
1.155	0.473	6.339	2102.8	0.653	1420.6	0.054	9.7	1714.5	134.7	114.1	2.6 11
1.170	0.499	6.071	92.2	1.017	113.2	0.053	3.9	81.7	8.6	18.3	1.0 14
1.185	0.429	6.834	4390.1	0.709	4387.4	0.083	80.5	4372.0	577.7	586.9	1.4 15
1.200	0.498	6.166	87.5	1.699	97.4	−0.054	2.7	81.6	9.0	14.0	1.2 16
1.215	0.450	6.239	80.6	1.768	87.8	−0.059	2.2	77.7	8.2	11.4	0.4 18
1.230	0.498	6.149	88.0	1.929	120.7	−0.087	4.7	84.4	10.1	20.9	0.2 11
1.245	0.444	5.988	90.5	1.943	156.9	−0.060	11.0	104.8	18.1	36.0	0.7 15
1.260	0.459	5.155	455.3	2.703	537.9	−0.158	11.9	486.6	65.8	76.1	0.6 18
1.275	0.460	5.727	80.2	1.892	128.1	−0.017	6.5	92.4	14.4	24.6	0.8 18
1.290	0.475	5.454	89.9	1.932	176.7	0.064	12.6	112.0	21.3	42.1	0.6 17
1.305	0.439	5.262	77.2	2.244	118.7	−0.050	4.6	88.0	12.6	20.2	1.1 19
1.320	0.458	5.230	87.1	2.040	199.9	0.071	17.8	118.1	25.6	53.2	1.1 18
1.335	0.451	5.153	97.4	1.980	246.2	0.169	32.2	136.7	33.3	77.5	0.8 15
1.350	0.429	4.365	218.7	3.206	362.4	−0.155	14.1	273.3	47.3	67.0	0.7 23
1.365	0.475	5.168	99.7	2.109	320.7	0.159	62.5	145.5	39.8	125.5	0.6 11
1.380	0.494	5.162	93.6	2.587	195.7	−0.106	19.9	113.7	21.6	53.5	1.2 12
1.395	0.462	5.174	83.8	2.479	153.5	−0.046	11.9	100.4	16.6	34.4	1.1 13
1.410	0.578	5.335	78.9	2.493	94.2	−0.071	2.0	76.5	7.2	11.1	0.6 12
1.425	0.471	5.378	89.5	2.437	297.0	0.005	66.7	130.7	36.3	123.1	0.6 11
1.440	0.464	5.396	80.9	2.581	170.0	−0.089	15.8	103.9	20.2	43.3	1.0 14
1.455	0.439	5.436	85.8	2.324	298.2	0.171	66.7	135.8	42.5	125.2	1.0 13
1.470	0.463	5.564	91.3	2.699	235.6	−0.098	43.5	120.8	27.2	83.3	0.6 10
1.485	0.445	5.615	87.1	2.971	185.3	−0.182	20.1	112.6	21.8	48.9	0.8 12
1.500	0.477	5.574	90.6	3.207	249.6	−0.216	52.4	124.1	30.4	95.1	1.6 10
1.515	0.439	5.560	90.9	3.325	208.8	−0.199	31.7	117.5	23.2	63.1	0.2 9
1.530	0.482	5.470	90.6	3.375	282.7	−0.192	68.7	135.3	37.4	114.6	0.5 9

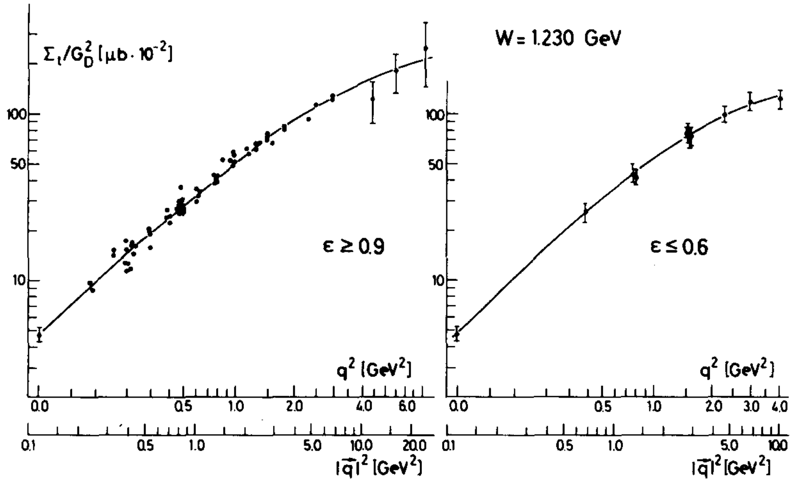

Fig. 1. Σ/G_D^2 as a function of $|q|^2$ in a double logarithmic plot for W = 1.230 GeV. Shown is also the scale of q^2. The curves are our fit with eq. (3) and the parameters in tables 1 and 3, and d = 3.0. For $\epsilon \geqslant 0.9$, error bars are given for extreme positions while for $\epsilon \leqslant 0.6$, they are given for each data point.

1.530, 1.605, 1.695, and 1.890 GeV. Also shown for each W value is the scale of q^2. As mentioned in sect. 1, there are now available some experimental data for the range $\epsilon \geqslant 0.9$, between q^2 = 0 and q^2 = 0.5 GeV^2, as can be seen from figs. 1–6.

Besides the experimental cross sections we have also shown the fit for the two ranges of ϵ for each value of W in figs. 1–6. The fits at W = 1.230, 1.530 and 1.605

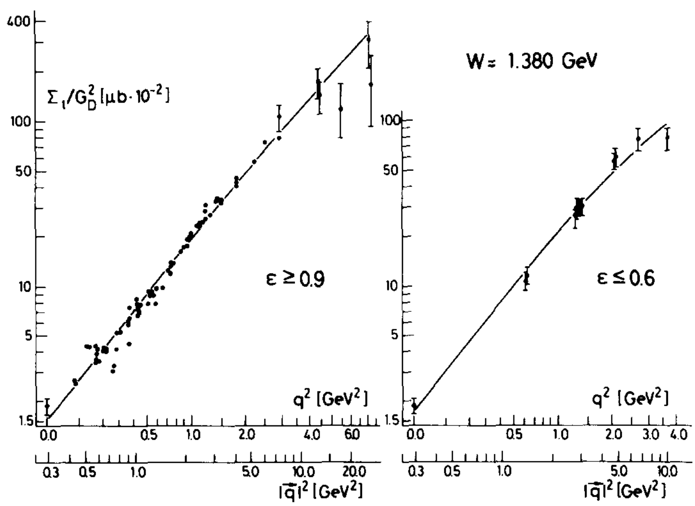

Fig. 2. As for fig. 1, for W = 1.380 GeV.

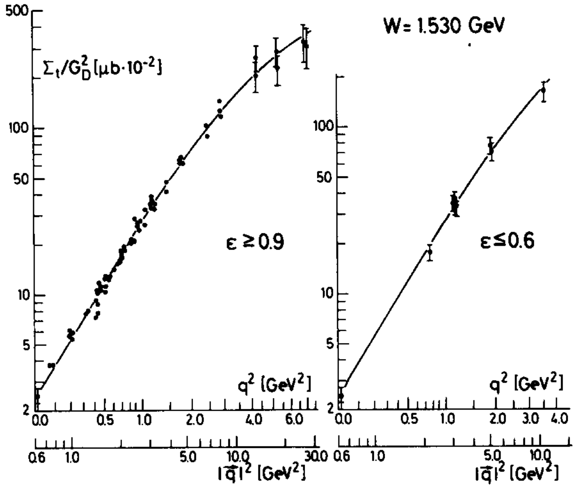

Fig. 3. As for fig. 1, for $W = 1.530$ GeV.

for both ranges of ϵ almost pass through the photoproduction experimental points (figs. 1, 3 and 4). The fits at $W = 1.380$ GeV pass below, while those at $W = 1.695$ and 1.890 GeV pass above the corresponding experimental points but generally touching or passing through the error bars. It should, however, be pointed out that for $\epsilon \leqslant 0.6$ there are no experimental data below $q^2 = 0.5$ to guide the fit while for $\epsilon \geqslant 0.9$ the expression used to obtain the fit is somewhat inadequate to account for a σ_ℓ contribution restricted to small values of q^2.

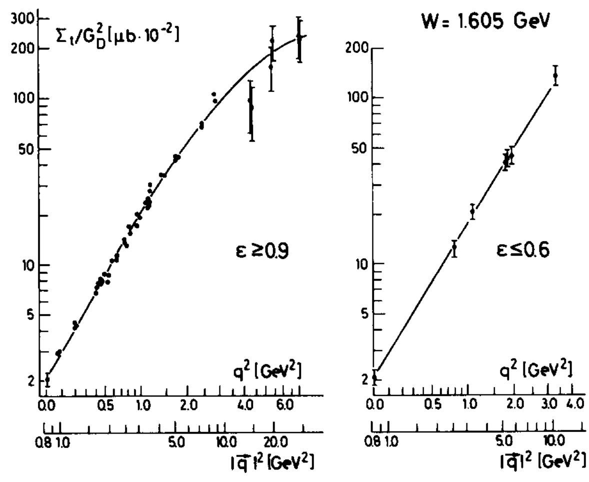

Fig. 4. As for fig. 1, for $W = 1.605$ GeV.

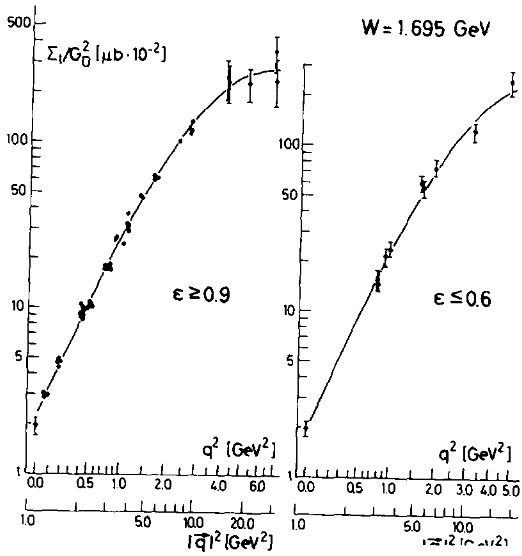

Fig. 5. As for fig. 1, for $W = 1.695$ GeV.

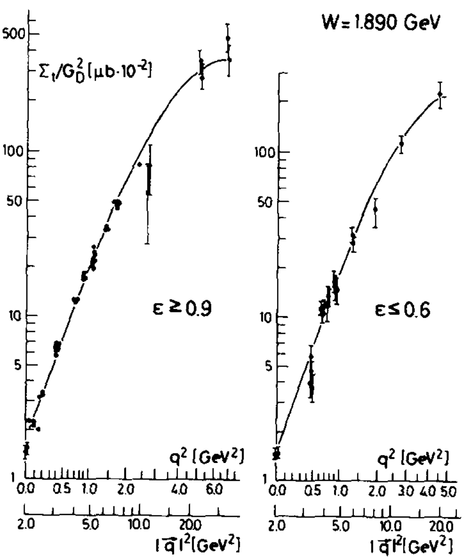

Fig. 6. As for fig. 1, for $W = 1.890$ GeV.

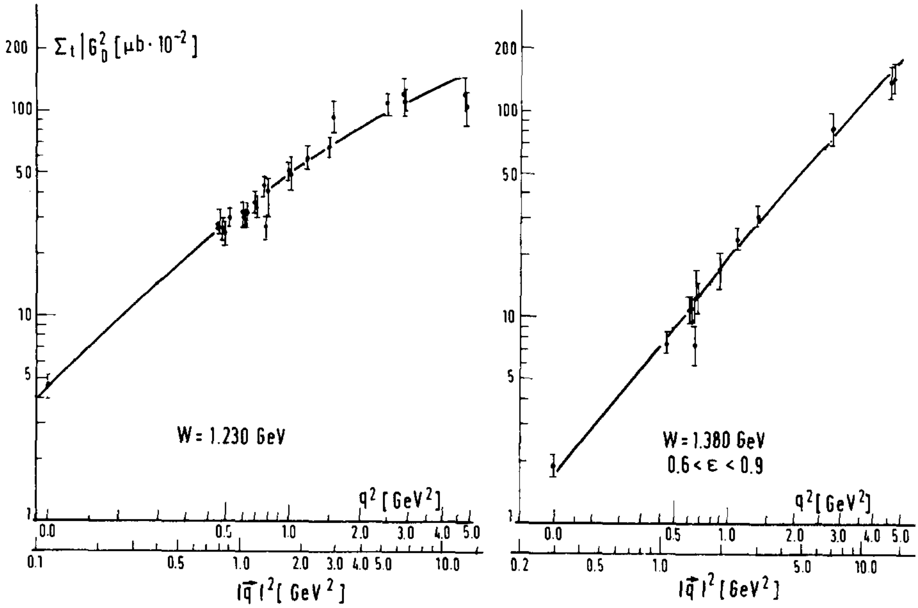

Fig. 7. As for fig. 1, for $0.6 < \epsilon < 0.9$, $W = 1.230$ and 1.380 GeV.

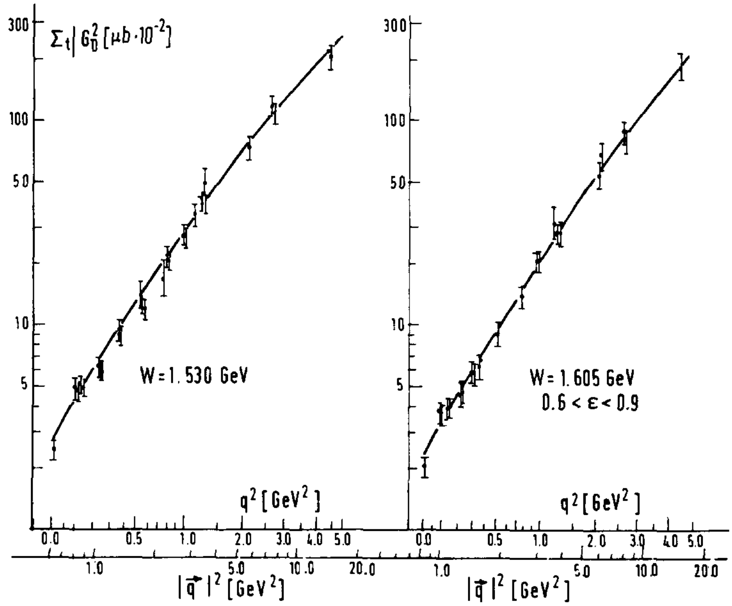

Fig. 8. As for fig. 7, for $W = 1.530$ and 1.605 GeV.

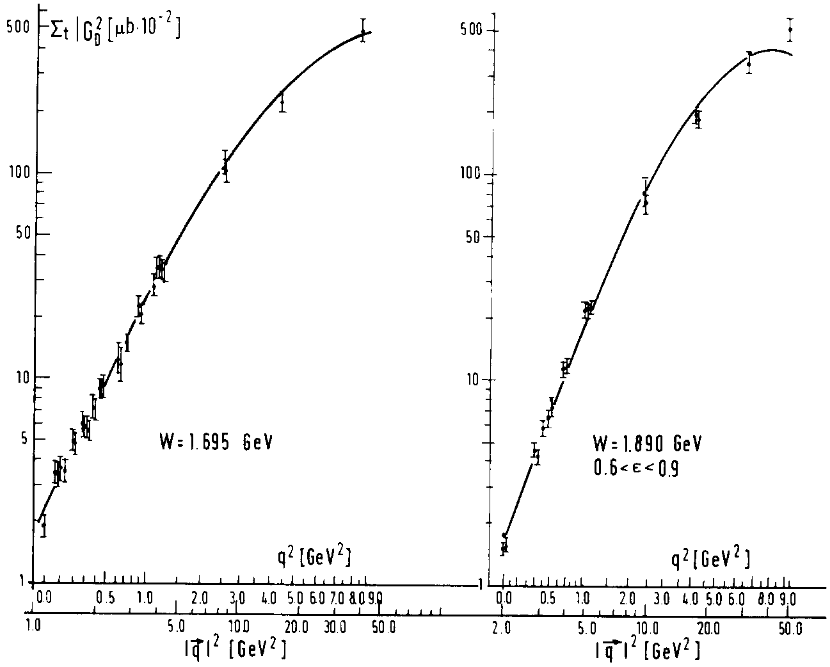

Fig. 9. As for fig. 7, for W = 1.695 and 1.890 GeV.

The corresponding results for $0.6 < \epsilon < 0.9$ are shown in figs. 7 to 9. The behaviour of the fits for the three ranges of ϵ is practically the same. Also, the inclusion or non-inclusion of the photoproduction data does not affect the behaviour of the fits in any appreciable manner.

3.4. Dependence of Σ on W and q^2

In figs. 10–18 we show the cross section Σ for fixed values of q^2, i.e., $q^2 = 0.1$, 0.3, 0.5, 1.0, 2.0, 3.0, 4.0, 5.0 and 6.0 GeV^2 across the range $1.110 \leqslant W \leqslant 1.990$ GeV as computed from out fits. Whereas for $q^2 < 0.5$ GeV^2 we only show results for $\epsilon \geqslant 0.9$, in the figures for $q^2 \geqslant 0.5$ GeV^2 the values of Σ are compared for the two extreme ranges of ϵ. The errors are calculated with the complete error matrix of the coefficients a, b and c according to the following expression:

$$(\Delta\Sigma)^2 = (\Delta a)^2 \cdot \left(\frac{\delta\Sigma}{\delta a}\right)^2 + (\Delta b)^2 \cdot \left(\frac{\delta\Sigma}{\delta b}\right)^2 + (\Delta c)^2 \cdot \left(\frac{\delta\Sigma}{\delta c}\right)^2 - 2C_{ab}$$

$$\times \frac{\delta\Sigma}{\delta a} \cdot \frac{\delta\Sigma}{\delta b} + 2\,C_{ac} \cdot \frac{\delta\Sigma}{\delta a}\,\frac{\delta\Sigma}{\delta c} - 2\,C_{bc}\,\frac{\delta\Sigma}{\delta b} \cdot \frac{\delta\Sigma}{\delta c}\,,$$

where Δa, Δb, Δc, C_{ab}, C_{ac} and C_{bc} are the coefficients of the error matrix as given

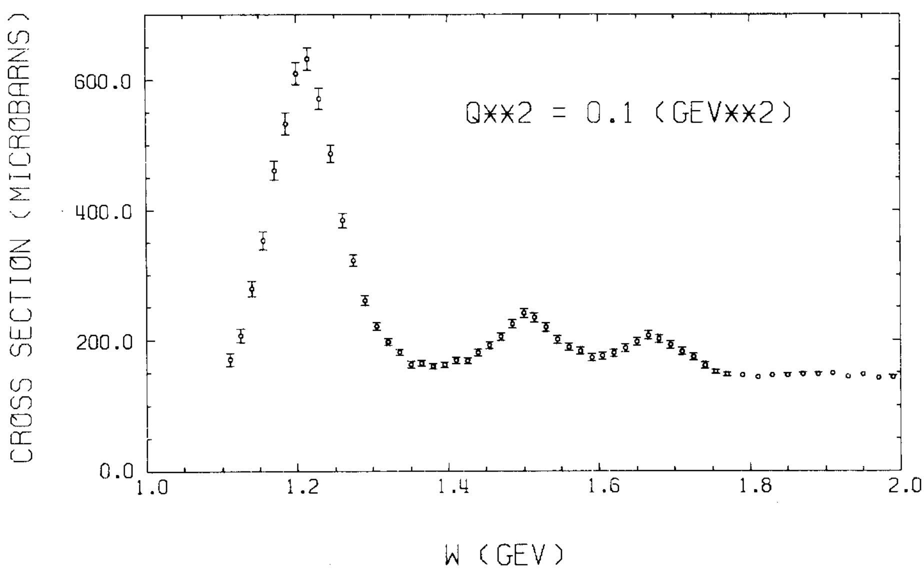

Fig. 10. Σ calculated from our fits for $\epsilon \geqslant 0.9$ as a function of W for $q^2 = 0.1\ \mathrm{GeV}^2$.

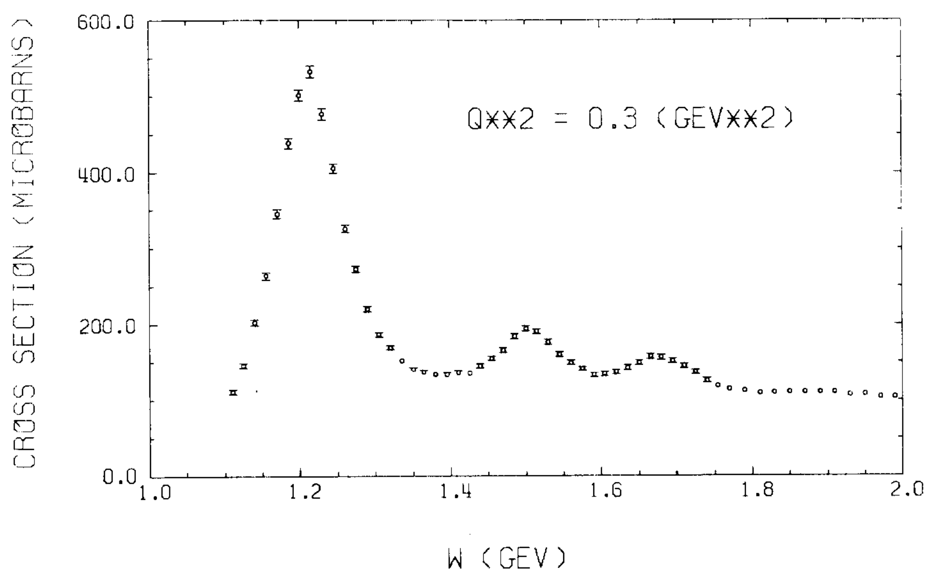

Fig. 11. As for fig. 10, for $q^2 = 0.3\ \mathrm{GeV}^2$.

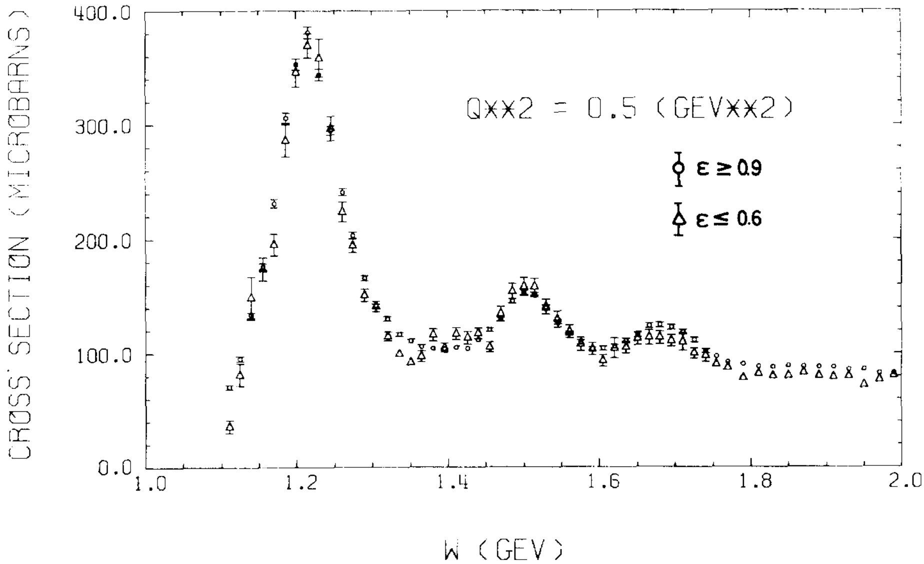

Fig. 12. Σ calculated from our fits for $\epsilon \geqslant 0.9$ and $\epsilon \leqslant 0.6$, as a function of W for $q^2 = 0.5\ \mathrm{GeV}^2$.

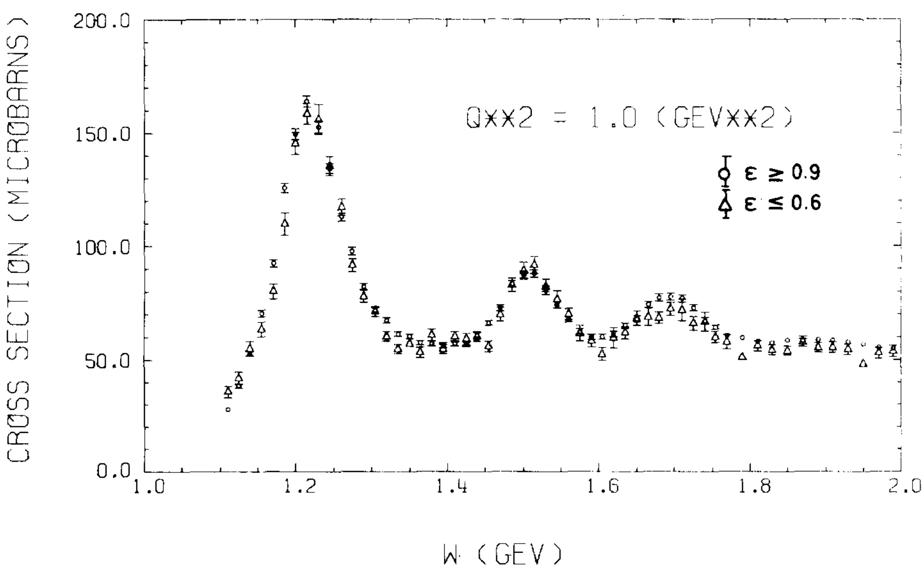

Fig. 13. As for fig. 12, for $q^2 = 1.0\ \mathrm{GeV}^2$.

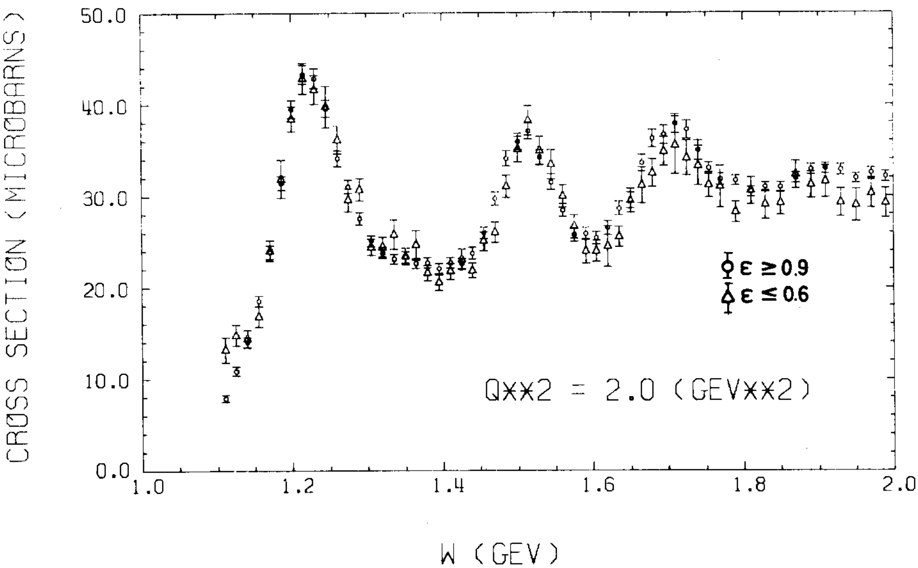

Fig. 14. As for fig. 12, for $q^2 = 2.0\ \mathrm{GeV}^2$.

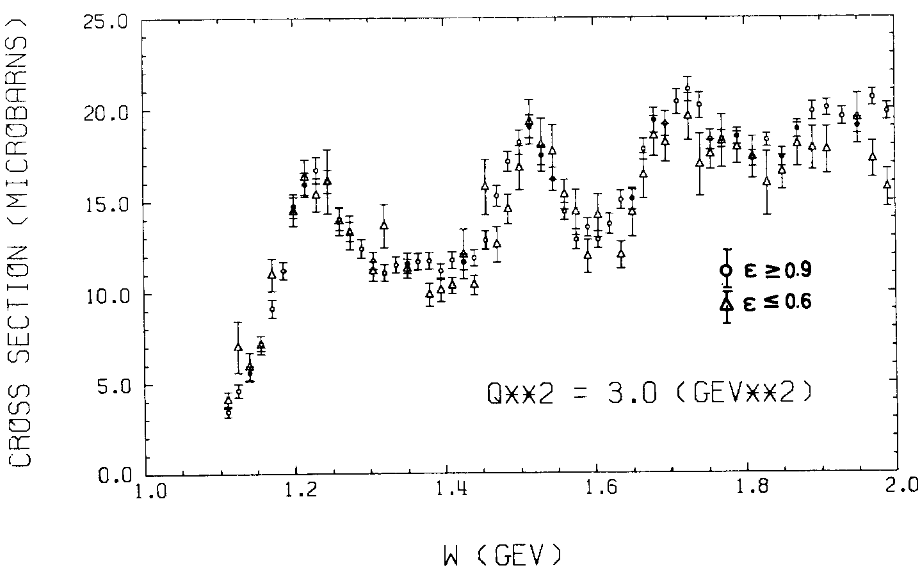

Fig. 15. As for fig. 12, for $q^2 = 3.0\ \mathrm{GeV}^2$.

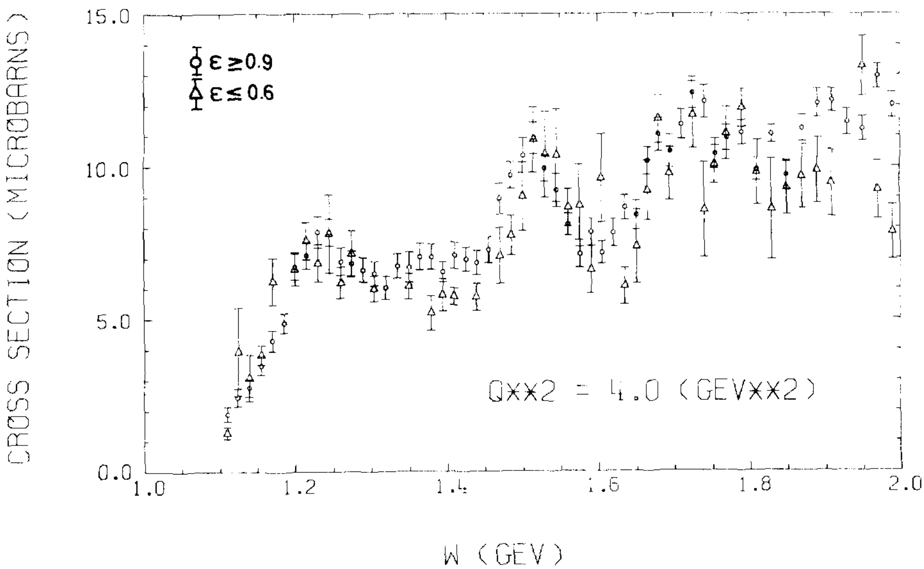

Fig. 16. As for fig. 12, for q^2 = 4.0 GeV^2.

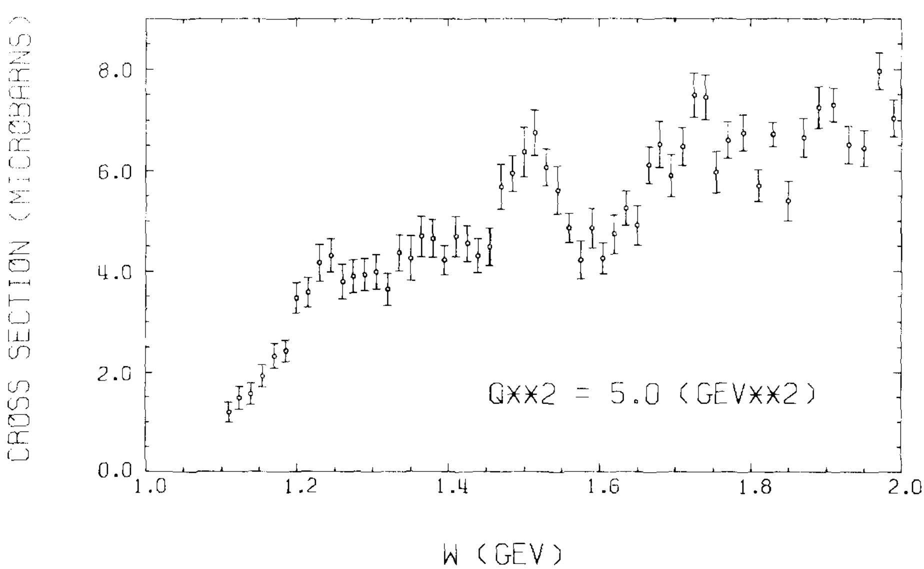

Fig. 17. Σ calculated from our fits for $\epsilon \geq 0.9$ as a function of W for q^2 = 5.0 GeV^2.

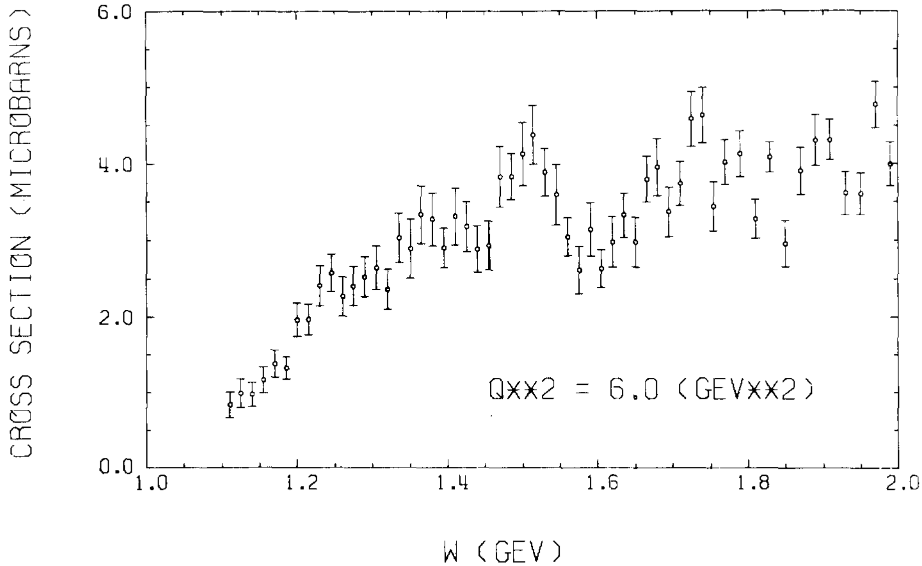

Fig. 18. As for fig. 17, for q^2 = 6.0 GeV^2.

in tables 1–3. We think that the fits are not sufficient to calculate cross sections beyond $q^2 = 4.0$ GeV^2 for $\epsilon \leqslant 0.6$ and beyond $q^2 = 6.0$ GeV^2 for $\epsilon \geqslant 0.9$.

The results show the faster decrease of the first resonance with increasing q^2 compared to the non-resonant background around this resonance and compared to the two other bumps of resonances around 1.5 and 1.67 GeV. The second bump on the contrary increases slightly in relation to the background at the same energy with increasing q^2 whereas the third one stays practically constant. The position of the third bump moves to higher energies with increasing q^2 as has been noticed earlier [18].

3.5. The ratio $R = \sigma_\ell/\sigma_t$

The difference between $\Sigma_{\epsilon \leqslant 0.9}$ and $\Sigma_{\epsilon \geqslant 0.6}$ is a direct measure of σ_ℓ and is very close to $\frac{1}{2}\sigma_\ell$ (eq. (1)), since the average values of ϵ in the two cases are close to 1.0 and 0.5 (tables 1 and 3). Therefore, the comparison of Σ for the two ranges of ϵ in the figs. 12–15 allows one to determine σ_ℓ. There are systematic differences outside the error bars between the two sets of cross sections. For $q^2 = 0.5$ GeV^2 with $W < 1.5$ GeV the difference $\Sigma_{\geqslant 0.9} - \Sigma_{\leqslant 0.6}$ has unphysical values. This is most likely due to the uncertainty of the fit for $\epsilon < 0.6$ for small values of q^2 because of lack of data. It also may show the problem of estimating systematic errors for the different set of measurements, measured with different experimental arrangements. The same conclusion could be made for $W > 1.6$ GeV and small q^2 values, where $\Sigma_{\epsilon \leqslant 0.6}$

is systematically smaller than $\Sigma_{\epsilon \geq 0.9}$. However there the difference stays on to larger values of q^2 up to 3 GeV^2, indicating a possible longitudinal contribution of about 10–20%. This is consistent with a direct determination of R at $q^2 = 1$ GeV^2 [11].

For $W < 1.6$ GeV and $0.5 < q^2 < 2$ GeV^2 no difference outside the error bars between the two sets of measurement is visible except for a small effect on the sides of the first resonance, which might be due to uncertainties in the energy W. A 10% contribution of σ_ℓ at the first resonance around $q^2 = 0.5$ GeV^2 is indicated by direct determinations [20,21]. Above $q^2 = 2$ GeV^2 and for $1.3 < W < 1.5$ GeV a longitudinal contribution of about 20% is not excluded.

References

[1] H.L. Lynch, J.W. Allaby and D.M. Ritson, HEPL-494 B (1967).

[2] A.A. Cone, K.W. Chen, J.R. Dunning, G. Hartwig, N.F. Ramsey, J.K. Walker and R. Wilson, Phys. Rev. 156 (1967) 1490.

[3] F.W. Brasse, J. Engler, E. Ganssauge and M. Schweitzer, Nuovo Cimento 55A (1968) 679, DESY 67/34 (1967).

[4] W. Albrecht, F.W. Brasse, H. Dorner, W. Flauger, K.-H. Frank, J. Gayler, H. Hultschig, J. May and E. Ganssauge, Phys. Letters 28B (1968) 225; DESY 68/48 (1968).

[5] W. Albrecht, F.W. Brasse, H. Dorner, W. Flauger, K.-H. Frank, J. Gayler, H. Hultschig, J. May and E. Ganssauge, Nucl. Phys. B13 (1969) 1; DESY 69/7.

[6] W. Bartel, B. Dudelzak, H. Krehbiel, J. McElroy, U. Meyer-Berkhout, W. Schmidt, V. Walther and G. Weber, Phys. Letters 28B (1968) 148;
W. Bartel, thesis, DESY internal report F22-69/3 (1969).

[7] E.D. Bloom, G. Buschhorn, R.L. Cottrell, D.H. Coward, H. De Staebler, J. Drees, C.L. Jordon, G. Miller, L. Mo, H. Piel, R.E. Taylor, M. Breidenbach, W.R. Ditzler, J.I. Friedman, G.C. Hartmann, H.W. Kendall and J.S. Pucher, SLAC-PUB-795 (1970);
M. Breidenbach, thesis;
R.E. Taylor, private communication.

[8] E.D. Bloom, R.L. Cottrell, D.H. Coward, H. DeStaebler, Jr., J. Drees, G. Miller, L.W. Mo, R.E. Taylor, J.I. Friedman, G.C. Hartmann and H.W. Kendall, SLAC-PUB 653 (1969).

[9] J. Moritz, K.H. Schmidt, D. Wegener, J. Bleckwenn and E. Engles, Jr., DESY 71/61; Nucl. Phys. B41 (1972) 336.

[10] J. Bleckwenn, thesis, DESY internal report F23-71/2 (1971).

[11] J.-C. Alder, F.W. Brasse, E. Chazelas, W. Fehrenbach, W. Flauger, K.-H. Frank, E. Ganssauge, J. Gayler, W. Krechlok, V. Korbel, J. May, M. Merkwitz and P.D. Zimmerman, Nucl. Phys. B48 (1972) 487; DESY 72/38 (1972).

[12] M. Köbberling, J. Moritz, K.H. Schmidt, D. Wegener, D. Zeller, J. Bleckwenn and F.H. Heimlich, Karlsruhe KFK 1822; Nucl. Phys. B82 (1974) 201.

[13] E.D. Bloom, D.H. Coward, H. DeStaebler, J. Drees, G. Miller, L.W. Mo, R.E. Taylor, M. Breidenbach, J.I. Friedman, G.C. Hartmann and H.W. Kendall, Phys. Rev. Letters 23 (1969) 930.

[14] M. Breidenbach. J.I. Friedman, H.W. Kendall, E.D. Bloom, D.H. Coward, H. DeStaebler, J. Drees, L.W. Mo and R.E. Taylor, Phys. Rev. Letters 23 (1969) 935.

[15] G. Miller, E.D. Bloom, G. Buschhorn, D.H. Coward, H. DeStaebler, J. Drees, C.L. Jordan, L.W. Mo, R.E. Taylor, J.I. Friedman, G.C. Hartman, H.W. Kendall and R. Verdier, Phys. Rev. D5 (1972) 528.

[16] J.S. Poucher, M. Breidenbach, R. Ditzler, J.I. Friedman, H.W. Kendall, E.D. Bloom, R.L.A. Cottrell, D.H. Coward, H. DeStaebler, C.L. Jordan, H. Piel and R.E. Taylor, SLAC PUB 1309 (1973); Phys. Rev. Letters 32 (1974) 118.
[17] Arie Bodek, Ph. D. thesis, MIT (1972);
Edward M. Riordan, Ph.D. thesis, MIT (1973) and private communication.
[18] S. Stein, W.B. Atwood, E.D. Bloom, R.L.A. Cottrell, H. DeStaebler, C.L. Jordan, H.G. Piel, C.Y. Prescott, R. Siemann and R.E. Taylor, SLAC-PUB-1528 (1975).
[19] W. Bartel, B. Dudelzak, H. Krehbiel, J. McElroy, U. Meyer-Berkhout, W. Schmidt, V. Walter and G. Weber, Phys. Letters 27B (1968) 660.
[20] W. Bartel, F.W. Büsser, W.R. Dix, R. Felst, D. Harms, H. Krehbiel, P.E. Kuhlmann, J. McElroy, J. Meyer and G. Weber, Phys. Letters 35B (1971) 181;
W.R. Dix, Thesis, Universität Hamburg (1971).
[21] K. Bätzner, U. Beck, K.H. Becks, C. Berger, J. Drees, G. Knop, M. Leenen, K. Moser, C. Nietzel, E. Schlösser and H.E. Stier, Phys. Letters 39B (1972) 575.
[22] W. Albrecht, F.W. Brasse, H. Dorner, W. Flauger, K.-H. Frank, J. Gayler, H. Hultschig, V. Korbel and J. May, DESY 69/46 (1969).
[23] E.M. Riordan, A. Bodek, M. Breidenbach, D.L. Dubin, J.E. Elias, J.I. Friedman, H.W. Kendall, J.S. Poucher, M.R. Sogard and D.H. Coward, SLAC-PUB 1417 (1974).
[24] F.W. Brasse, W. Fehrenbach, W. Flauger, K.-H. Frank, J. Gayler, V. Korbel, J. May, P.D. Zimmerman and E. Ganssauge, DESY 71/2 (1971).
[25] T.A. Armstrong, W.R. Hogg, G.M. Lewis, A.W. Robertson, G.R. Brookes, A.S. Clough, J.H. Freeland, W. Galbraith, A.F. King, W.R. Rawlinson, N.R.S. Tait, J.C. Thompson and D.W.L. Tolfree, Daresbury DNPL/P 88 (1971).