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Mass Breaking in a Relativistic Quark Model for Mesons

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We study symmetry breaking via quark mass differences in a relativistic quark model where mesons are built from heavy ($m > 3$ GeV) spin $\frac{1}{2}$ quarks and antiquarks. The meson (squared-)mass differences are linearly related to the number of strange, charmed, etc. quarks in the mesons. We show that the previously assumed SU_n symmetry of the mesonic couplings holds, i.e., quark mass differences only show up in the masses of the external particles, not in the three meson vertex itself.

1. INTRODUCTION

A central problem of elementary particle physics is to understand the breaking of strong interaction symmetries. In the quark model [1] of hadrons the simplest way to break symmetry is to ascribe different masses to the (n, p), λ, c-quarks. This introduces splittings within the SU_n multiplets in the nonrelativistic quark model [1]. Quark mass differences also successfully account for symmetry breaking effects in the static properties of elementary particles like magnetic moments [2].

In this paper we discuss the consequences of quark mass differences combined with a symmetric quark–antiquark ($q\bar{q}$) interaction in a relativistic dynamical quark model for measons with heavy Fermi quarks [3–6]. In Section 2 we briefly review the main features of this model, derive the dynamical equation, the spectrum, and the wavefunctions of the mesons. We compare our spectrum with the experimental masses of the ordinary mesons [7] and their excitations, and from the J/ψ [8] mass we derive limits on the masses of the expected charmed vector mesons D^* and F^* [2, 9].

Since the quark mass differences ($\Delta_i := m_i - m$; $i =$ quark flavor, $\Delta_c > \Delta_\lambda > 0$, $\Delta_p = \Delta_n = 0$) appear in the wavefunctions only in third order terms in $1/m$ we have no symmetry breaking in simple current matrix elements like the vector meson photon coupling except phase space factors due to the masses of the external particles. In this model, however, the hadronic couplings depend on the small components (order m^{-3}) of the wavefunctions as a consequence of the spin saturation of the superstrong $q\bar{q}$ binding forces [5, 6]. In Section 3 we therefore investigate whether the symmetry of the mesonic couplings and, of course, the spin saturation of the $q\bar{q}$ forces survive in the presence of quark mass differences. In Section 4 we summarize the results.

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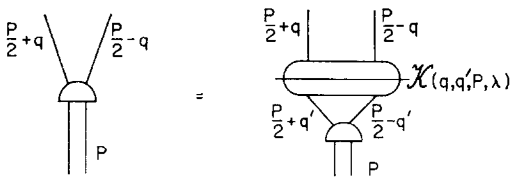
2. The Model, Spectrum, and Wavefunctions

In the framework of a relativistic quark field theory [10] the mesons are described by Bethe Salpeter (BS) amplitudes

$$\chi_{ij}(q, P) = (2\pi)^{3/2} \int dx\, e^{iqx} \langle 0 \mid T(\psi_i(x/2)\, \bar{\psi}_j(-(x/2))) \mid \text{Meson} \rangle \tag{1}$$

where ψ_i, $\bar{\psi}_j$ are the quark and antiquark fields, T the time ordering operator, P the meson momentum, and q the relative momentum of the quarks. The BS amplitudes obey the bound state BS equation

$$S_i^{-1}\left(\frac{P}{2} + q\right) \chi_{ij}(q, P)\, \overline{S_j^{-1}}\left(\frac{P}{2} - q\right) = i \int dq'\, \mathscr{K}(q, q', P, \lambda)\, \chi_{ij}(q', P) \tag{2}$$



with the inverse propagators S_i^{-1} of the quark i, $\overline{S_j^{-1}}$ of the antiquark j and the interaction operator kernel $\mathscr{K}$ with the coupling λ. Böhm, Joos, and Krammer [3, 6] (hereafter referred to as BJK) now made the following assumptions.

(i) Quarks are very heavy ($m > 3$ GeV), this leads to a practical confinement [11];

(ii) Superstrong $q\bar{q}$ binding forces have to compensate the large quark mass. (These superstrong forces have to be saturated in hadronic vertices);

and for simplicity:

(iii) The quark propagators can be approximated by the free quark propagators

$$S_i^{-1}(p) := \not{p} - m_i, \qquad \overline{S_i^{-1}}(p) := \not{p} + m_i;$$

(iv) The interaction kernel shall be of convolution type without derivative couplings.

Now the BS equation reads

$$\left(\frac{\not{P}}{2} + \not{q} - m_i\right) \chi_{ij}(q, P) \left(\frac{\not{P}}{2} - \not{q} + m_j\right) = i\mathscr{K}(q, P, \lambda) * \chi_{ij}(q, P) \tag{2'}$$

while the normalization [5, 12] of the BS amplitude is

$$(2\pi)^{-4}\, i\, \mathrm{Tr}\left[\int dq\, \overline{\chi_{ij}^{r}}(q, P) \left(\frac{\not{P}}{2} + \not{q} - m_i\right) \chi_{ij}^{r'}(q, P) \left(\frac{\not{P}}{2} - \not{q} + m_j\right)\right] = \lambda \frac{dP^2}{d\lambda}\, \delta^{rr'}. \tag{3}$$

For mathematical reasons one performs the Wick [13] rotation, an analytical continuation in the relative energy plane

$$q = (q_0, \mathbf{q}) \to (iq_0, \mathbf{q}) =: (q_4, \mathbf{q}) =: q_E, \\ (\gamma_0, \boldsymbol{\gamma}) \to (\gamma_0, -i\boldsymbol{\gamma}) =: (\gamma_4, \boldsymbol{\gamma}_E), \tag{4}$$

after which we have Euclidean metric, here indicated by the index E. In this Euclidean form the interaction operator $\mathscr{K}$ is the Fourier transform of a four-dim "potential" $V(R)$. BJK [3] have shown that this potential has to have the form of a smooth well in order to yield linear Regge trajectories for the mesons. This potential might be approximated by a harmonic oscillator $V(R) = \alpha + \beta R^2$, $-\alpha \approx m^2$, valid at least for the low excitations [4]. The required quark spin singlet–triplet structure of the BS amplitudes restricts the spin dependence of the interaction operator $\mathscr{K}$. It is uniquely fixed to

$$\mathscr{K}(q, P, \lambda) * \chi(q, P) := -\gamma_5(K(q, P, \lambda) * \chi(q, P))\gamma_5, \\ K(q, P, \lambda) * \chi(q, P) = \widetilde{V(R)} * \chi(q, P) = (\alpha - \beta\Box)\chi(q, P), \tag{5}$$

by the second requirement, that the superstrong $q\bar{q}$ forces are saturated in mesonic vertices [5, 6].

We can now write down the BS Eq. (2) with the quark flavor symmetric interaction (5) but different quark masses. After transforming into the meson rest frame $P = (M, 0)$ we have (now in Euclidean space)

$$\left(\not{q} - im_i + i\frac{M}{2}\gamma_4\right)\chi_{ij}(q)\left(\not{q} - im_j - i\frac{M}{2}\gamma_4\right) = -\gamma_5(\alpha - \beta\Box)\chi_{ij}(q)\gamma_5. \tag{6}$$

When we expand χ according to its Dirac structure

$$\chi = \sum_{\mathscr{S}} \chi^{\mathscr{S}} = S + V + T + A + P = s\mathbb{1} + v^\mu\gamma_\mu + t^{\mu\nu}\sigma_{\mu\nu} + a^\mu\gamma_5\gamma_\mu + p\gamma_5 \tag{7}$$

we see that the spin part of the interaction leaves the vector (V) and axialvector (A) components unchanged while it gives a sign to the scalar (S), tensor (T), and pseudoscalar (P) components of χ. Small perturbations of the ideal spin structure $-\gamma_5 \cdots \gamma_5$ can now be taken into account by modifying the potential parameters according to the Dirac components of χ: $\alpha, \beta \to \alpha^{\mathscr{S}}, \beta^{\mathscr{S}}$ [5, 6, 14]. Equation (6) then explicitly reads

$$\left\{(\not{q} \times \not{q}) + \begin{bmatrix} -2m_im_j - \beta^S\Box - \delta_{ij}^S & & & & \bigcirc \\ & \beta^V\Box + \delta_{ij}^V & & & \\ & & -2m_im_j - \beta^T\Box - \delta_{ij}^T & & \\ & & & \beta^A\Box + \delta_{ij}^A & \\ \bigcirc & & & & -2m_im_j - \beta^P\Box - \delta_{ij}^P \end{bmatrix} + \frac{M^2}{4}(\gamma_4 \times \gamma_4)\right\}\begin{pmatrix} S \\ V \\ T \\ A \\ P \end{pmatrix}$$

$$= \{\ \} \begin{pmatrix} V \\ S \\ A \\ T \\ 0 \end{pmatrix} - [\ ] \begin{pmatrix} 0 \\ T \\ V \\ P \\ A \end{pmatrix} + (\ ) \begin{pmatrix} T \\ A \\ S+P \\ V \\ T \end{pmatrix}$$

$$(\not q \times \not q) Y := \not q Y \not q \qquad \{\ \} Y := \Big\{ i m_{ij} \not q + \Delta_{ij} \frac{M}{2} \gamma_4 , Y \Big\}$$

$$(\gamma_4 \times \gamma_4) Y := \gamma_4 Y \gamma_4 \qquad [\ ] Y := \Big[m_{ij} \frac{M}{2} \gamma_4 + i \Delta_{ij} \not q , Y \Big] \tag{8}$$

$$(\ ) Y := i \frac{M}{2} (\not q Y \gamma_4 - \gamma_4 Y \not q)$$

$$\delta_{ij}^{\mathscr{D}} := -(\alpha^{\mathscr{D}} + m_i m_j) \qquad m_{ij} := \tfrac{1}{2}(m_i + m_j) \qquad \Delta_{ij} := \tfrac{1}{2}(m_i - m_j).$$

The quark mass is not compensated for the components S, T, P. These are therefore small and we can solve the lines for these components by a Taylor expansion up to the order m^{-3} [14]

$$\begin{aligned} S &= \frac{-1}{2m_i m_j} \left(1 + \frac{1}{2m_i m_j} \left(q^2 - \beta^S \Box - \delta_{ij}^S + \frac{M^2}{4}\right)\right) (\{\ \} V + (\ )_S T), \\ T &= \frac{-1}{2m_i m_j} \left(1 + \frac{1}{2m_i m_j} \left(\not q \times \not q - \beta^T \Box - \delta_{ij}^T + \frac{M^2}{4} \gamma_4 \times \gamma_4\right)\right) \\ &\quad \times (\{\ \} A - [\ ] V + (\ )(S + P)), \\ P &= \frac{-1}{2m_i m_j} \left(1 + \frac{-1}{2m_i m_j} \left(q^2 + \beta^P \Box + \delta_{ij}^P + \frac{M^2}{4}\right)\right) (-[\ ] A + (\ )_P T). \end{aligned} \tag{9}$$

By reinserting Eqs. (9) into the lines of the *large* components V, A of Eq. (8) we get the dynamical equation for V and A up to the order m^{-3} [14]. To calculate the spectrum and the V, A wavefunctions it suffices to consider this equation up to the first order in m^{-1}, which gives

$$(\beta^{V,A} \Box - q^2 + (M^2/4) + \delta_{ij}^{V,A})(V, A) = 0. \tag{10}$$

Solutions of Eq. (10) are the quark mass independent wavefunctions $V = v^\mu \gamma_\mu$, $A = \alpha^\mu \gamma_5 \gamma_\mu$ with the eigenfunctions of the four-dimensional harmonic oscillator v^μ, α^μ

$$(v^\mu, \alpha^\mu)(q^2, \Omega^4) = \mathscr{R}_{n,r}^{V,A}(q^2)\, \mathscr{Y}_{l,m}^{n}(\Omega^4);$$

$$\mathscr{R}_{n,r}(q^2) = \left(\frac{2r!}{\beta(n+r+1)!}\right)^{1/2} \left(\frac{q^2}{\beta^{1/2}}\right)^{n/2} e^{-(q^2/2\beta^{1/2})} L_r^{n+1} \left(\frac{q^2}{\beta^{1/2}}\right),$$

while the eigenvalue condition gives the spectrum

$$M_{V,A}^2 = 8(N+2)(\beta^{V,A})^{1/2} - 4\delta_{ij}^{V,A} \simeq M_{0_{V,A}}^2 + 8N(\beta^{V,A})^{1/2} + 4m(\Delta_i + \Delta_j), \tag{11}$$

$$N = 2\nu + n, \qquad n \geqslant l \geqslant 0, \qquad \begin{aligned} r &= \text{radial quantum number}, \\ n &= O_4 \text{ quantum number}, \\ l &= \text{three-dimensional orbital angular momentum}. \end{aligned}$$

Here all terms of order m^{-1} or smaller are neglected. The wavefunctions $v^{\mu}\gamma_{\mu}$, $a^{\mu}\gamma_5\gamma_{\mu}$ represent the spin singlet ($a^4\gamma_5\gamma_4$) and triplet ($v^k\gamma_k$, $k \neq 4$) structure of the nonrelativistic quark model and additionally there are as many pure relativistic states ($a^k\gamma_5\gamma_k$, $v^4\gamma_\nu$). For details see Ref. [4]. The BS equation naturally yields a quadratic mass formula and we have linear Regge trajectories as a consequence of the harmonic interaction.

The spectrum Eq. (11) depends on

(i) the spin. The slopes of the Regge trajectories might differ slightly for quark spin singlet mesons (A component) and quark spin triplet mesons (V component), but they are *quark flavor independent;*

(ii) the quark mass differences. The inner multiplet (squared-)mass differences depend *only* on the quark flavors, but neither on the excitation level nor the spin. They are $4m\Delta_\lambda$ ($4m\Delta_c$) for each strange (charmed) quark.

Result (ii) is identical to a previous one [15] obtained with a scalar model in the same framework. Thus we also get the same "ideally" mixed mesons $(p\bar{p} + n\bar{n})/2^{1/2}$, $\lambda\bar{\lambda}$, $c\bar{c}$, etc. Applying our spectrum Eq. (11) we are therefore forced to omit nonideally mixed mesons like η, η'. We find the equal spacing rules

$$\begin{aligned} M_\phi^2 - M_{K^*}^2 &= M_{K^*}^2 - M_\rho^2 = 4m\Delta_\lambda\,, \\ M_{J/\psi}^2 - M_{D^*}^2 &= M_{D^*}^2 - M_\rho^2 = 4m\Delta_c\,, \\ M_{J/\psi}^2 - M_{F^*}^2 &= M_{F^*}^2 - M_\phi^2 = 4m(\Delta_c - \Delta_\lambda). \end{aligned} \tag{12}$$

These equal spacings should be independent of the excitation level as well as of the spin e.g. $M_k^2 - M_\pi^2 = M_{K^*}^2 - M_\rho^2 = M_{K^*1420}^2 - M_{A2}^2$. This can partly be tested up to the second orbital excitation. Table I shows the experimentally known mass differences.

We find reasonable agreement with our result (ii) only within each excitation level, and in contrast to our model the squared-mass differences seem to increase with excitation. This contradicts our result (i), the Regge trajectories are not parallel for different quark flavors, instead their inverse slope increases with the number of λ-quarks involved.

An indication that the inverse slope also increases with the number of c-quarks

TABLE I

Squared-Mass Differences from Refs. [7, 16][a]

N	Mesons	Experimental mass differences $4m\Delta$ [GeV^2]	
0	$M^2_{K^+} - M^2_{\pi^+}$	$0.496^2 - 0.138^2$	$=0.226$
	$M^2_{K^0} - M^2_{\pi^0}$	$0.498^2 - 0.135^2$	$=0.230$
	$M^2_{K^{*0}} - M^2_{\rho^0}$	$0.89^2 - 0.77^2$	$=0.210$
	$M_\phi{}^2 - M^2_{K^{*0}}$	$1.02^2 - 0.89^2$	$=0.238$
1	$M^2_{K^*_{1420}} - M^2_{A_2}$	$1.42^2 - 1.31^2$	$=0.31$
	$M^2_{f'} - M^2_{K^*_{1420}}$	$1.52^2 - 1.42^2$	$=0.28$
2	$M_L{}^2 - M^2_{A_3}$	$1.77^2 - 1.64^2$	≈ 0.43
	$M^2_{K^*_{1800}} - M_g{}^2$	$1.8^2 - 1.686^2$	≈ 0.40

[a] Our model predicts equal numbers in the right-hand column.

involved is given by the level spacing of the new mesons [8, 17] (all quark spin triplet). Here we have 1^3P wave levels at 3.4 and 3.5 GeV while the 1^3S wave level is $M(J/\psi) = 3.1$ GeV. This gives an inverse slope of the $j = l + 1$ trajectory of 1.95, 2.64 GeV^2 or even more depending on which state is assumed to be the $J = 2$ state, that at 3.4 or 3.5 GeV or even a so long undetected higher ^3P wave state [18]. On the other hand the inverse slope of the old $j = l + 1$ quark spin triplet mesons is $M^2_{A_2} - M_\rho{}^2 = 1.12$ GeV^2.

These different inverse slopes of old and new mesons weaken our inner multiplet equal spacing rules (12). We have to add a negative term $-8((\beta_1)^{1/4} - (\beta_2)^{1/4})^2$ (11), which is of no effect on the $\rho - K^* - \phi$ system but gives corrections of the order of 100–200 MeV to the D^*, F^* masses

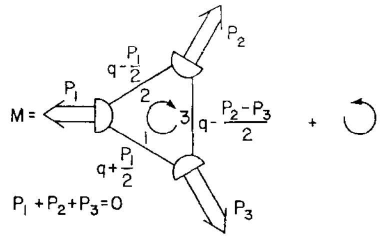
$$M^2_{D^*} = (M^2_{J/\psi} + M_\rho{}^2)/2 - 8((\beta_{J/\psi})^{1/4} - (\beta_\rho)^{1/4})^2 = (2.26\ \text{GeV})^2 - \mathcal{O}(100\ \text{MeV}),$$

$$M^2_{F^*} = (M^2_{J/\psi} + M_\phi{}^2)/2 - 8((\beta_{J/\psi})^{1/4} - (\beta_\phi)^{1/4})^2 = (2.3\ \text{GeV})^2 - \mathcal{O}(100\ \text{MeV}). \qquad (13)$$

For a further discussion of the excitation dependence of the meson mass differences of Table I we would like to refer the reader to Refs. [14], [19].

3. Saturation and the Proof of Symmetric Couplings

In the heavy quark model of BJK the three meson vertex can be calculated from the one leg amputated BS amplitudes $S_i^{-1}(q + (P/2)\,\chi_{ij}(q, P)$ [5, 6],



$$= \operatorname{Tr}[SU_n](2\pi)^{-9/2}\, i \int dq \operatorname{Tr}\Big[S_1^{-1}\Big(q + \frac{P_1}{2}\Big)\chi_{12}(q, P_1) \times S_2^{-1}\Big(q - \frac{P_1}{2}\Big)\chi_{23}\Big(q + \frac{P_3}{2}, P_2\Big) \times S_3^{-1}\Big(q - \frac{P_2 - P_3}{2}\Big)\chi_{31}\Big(q - \frac{P_2}{2}, P_3\Big)\Big]. \tag{14}$$

For this calculation we need the full BS amplitude χ_{ij} which can be obtained in terms of the large components V, A by inserting the expressions (9) into Eq. (7) [14]. After transforming back to Minkowski space we get for the full BS amplitude ($\chi^{(0)}$ denotes any linear combination of V and A) up to the order m^{-3}

$$\begin{aligned}
\chi_{ij} &= \chi_{ij}^{(1)} + \chi_{ij}^{(2)} + \chi_{ij}^{(3)},\\
\chi_{ij}^{(1)}(q, P) &= \frac{1}{2m_i}\overline{S_i^{-1}}\Big(\frac{P}{2} + q\Big)\chi_{ij}^{(0)}(q, P) - \frac{1}{2m_j}\chi_{ij}^{(0)}(q, P)\, S_j^{-1}\Big(\frac{P}{2} - q\Big),\\
\chi_{ij}^{(2)}(q, P) &= \frac{P\cdot q}{4m^3}\Big(\overline{S^{-1}}\Big(\frac{P}{2} + q\Big)\chi_{ij}^{(0)}(q, P) + \chi_{ij}^{(0)}(q, P)\, S^{-1}\Big(\frac{P}{2} - q\Big)\Big),\\
\chi_{ij}^{(3)}(q, P) &= \frac{1}{4m^3}\Big(\{\not{q}, (\delta_{ij}^V - \delta_{ij}^S)\, V_{ij}(q, P) + (\delta_{ij}^A - \delta_{ij}^T)\, A_{ij}(q, P)\}\\
&\quad + \Big[\frac{\not{P}}{2}, (\delta_{ij}^V - \delta_{ij}^T)\, V_{ij}(q, P) + (\delta_{ij}^A - \delta_{ij}^P)\, A_{ij}(q, P)\Big]\\
&\quad + 2\beta\, \partial^\mu\{\gamma_\mu, \chi_{ij}^{(0)}(q, P)\}\Big).
\end{aligned} \tag{15}$$

Quark mass differences appear twice in the mesonic vertex M

(i) In the quark propagators,

$$S_i^{-1}(p) = \not{p} - m_i =: S^{-1}(p) - \frac{m\Delta_i}{m}, \qquad \overline{S_i^{-1}}(p) =: \overline{S^{-1}}(p) + \frac{m\Delta_i}{m}.$$

(ii) In the BS amplitude χ_{ij}, Eq. (15). Here they are only present (in relevant order) in the first line $\chi_{ij}^{(1)}$ and they may be isolated,

$$\chi_{ij}^{(1)} = \frac{1}{2m}\left(\overline{S^{-1}}\left(\frac{P}{2} + q\right)\chi_{ij}^{(0)} - \chi_{ij}^{(0)} S^{-1}\left(\frac{P}{2} - q\right)\right) + \frac{1}{2m^3}\left(m\Delta_j \chi_{ij}^{(0)} \cdot \left(\frac{\not{P}}{2} - \not{q}\right) - m\Delta_i\left(\frac{\not{P}}{2} + \not{q}\right)\chi_{ij}^{(0)}\right),$$

where $m\Delta$ is of the order m^0 as we can see from the mass formula Eq. (11), e.g. $4m\Delta_\lambda = M_K{}^2 - M_\pi{}^2$.

We now proceed calculating the vertex M in the same way as BJK [6] did in the SU_n symmetric case. In particular we calculate the trace over the Dirac components of the BS amplitudes separately for the three different types of combinations

$\mathrm{Tr}^{(1)}$: all BS amplitudes only in terms $\chi^{(1)}$,

$\mathrm{Tr}^{(2)}$: one BS amplitude in term $\chi^{(2)}$, the others $\chi^{(1)}$,

$\mathrm{Tr}^{(3)}$: one BS amplitude in term $\chi^{(3)}$, the others $\chi^{(1)}$.

$\mathrm{Tr}^{(1)}$ might in principle be of the order m^3, $\mathrm{Tr}^{(2,3)}$ of the order m^0, all other contributions to the vertex are of the order m^{-3} or smaller. Note that in $\mathrm{Tr}^{(2,3)}$ even the mass breaking terms are negligible, since they at most contribute to the order m^{-1}. Therefore in $\mathrm{Tr}^{(2,3)}$ we expect no change compared to the SU_n symmetric case and the most interesting part of the vertex M is $\mathrm{Tr}^{(1)}$

$$\begin{aligned}\mathrm{Tr}^{(1)} = \mathrm{Tr}\Big[& S_1^{-1}\left(\frac{P_1}{2} + q\right)\left(\frac{1}{2m_1}\overline{S_1^{-1}}\left(\frac{P}{2} + q\right)\chi_{12}^{(0)}(q, P_1) - \frac{1}{2m_2}\chi_{12}^{(0)}(q, P_1)\, S_2^{-1}\left(\frac{P_1}{2} - q\right)\right) \\ & \times S_2^{-1}\left(\frac{-P_1}{2} + q\right)\left(\frac{1}{2m_2}\overline{S_2^{-1}}\left(\frac{-P_1}{2} + q\right)\chi_{23}^{(0)}\left(q + \frac{P_3}{2}, P_2\right)\right. \\ & \left. - \frac{1}{2m_3}\chi_{23}^{(0)}\left(q + \frac{P_3}{2}, P_2\right) S_3^{-1}\left(\frac{P_2 - P_3}{2} - q\right)\right) \\ & \times S_3^{-1}\left(\frac{P_3 - P_2}{2} + q\right)\left(\frac{1}{2m_3}\overline{S_3^{-1}}\left(\frac{P_3 - P_2}{2} + q\right)\chi_{31}^{(0)}\left(q - \frac{P_2}{2}, P_3\right)\right. \\ & \left. - \frac{1}{2m_1}\chi_{31}^{(0)}\left(q - \frac{P_2}{2}, P_3\right) S_1^{-1}\left(\frac{-P_1}{2} - q\right)\right)\Big]. \end{aligned} \tag{16}$$

In the following we write $\mathscr{D}_i^{-1}(p) := S_i^{-1}(p) \cdot \overline{S_i^{-1}}(p) = p^2 - m_i{}^2$ and omit the arguments of χ, $\mathscr{D}^{-1}$, S^{-1}

$$\begin{aligned}\mathrm{Tr}^{(1)} = \frac{1}{8m_1 m_2 m_3}\mathrm{Tr}\Big[& \chi_{12}^{(0)}\chi_{23}^{(0)}\chi_{31}^{(0)}\mathscr{D}_1^{-1}\mathscr{D}_2^{-1}\mathscr{D}_3^{-1} + \frac{m_3}{m_1}\chi_{12}^{(0)}\chi_{23}^{(0)}S_3^{-1}\chi_{31}^{(0)}\overline{S_1^{-1}}\mathscr{D}_1^{-1}\mathscr{D}_2^{-1} \\ & + \frac{m_2}{m_3}\chi_{12}^{(0)}S_2^{-1}\chi_{23}^{(0)}\overline{S_3^{-1}}\chi_{31}^{(0)}\mathscr{D}_1^{-1}\mathscr{D}_3^{-1} + \frac{m_2}{m_1}\chi_{12}^{(0)}S_2^{-1}\chi_{23}^{(0)}\chi_{31}^{(0)}\overline{S_1^{-1}}\mathscr{D}_1^{-1}\mathscr{D}_3^{-1} \\ & + \frac{m_1}{m_2}S_1^{-1}\chi_{12}^{(0)}\overline{S_2^{-1}}\chi_{23}^{(0)}\chi_{31}^{(0)}\mathscr{D}_2^{-1}\mathscr{D}_3^{-1} + \frac{m_3}{m_2}\chi_{12}^{(0)}\overline{S_2^{-1}}\chi_{23}^{(0)}S_3^{-1}\chi_{31}^{(0)}\mathscr{D}_1^{-1}\mathscr{D}_2^{-1} \\ & + \frac{m_1}{m_3}S_1^{-1}\chi_{12}^{(0)}\chi_{23}^{(0)}\overline{S_3^{-1}}\chi_{31}^{(0)}\mathscr{D}_2^{-1}\mathscr{D}_3^{-1} + \chi_{12}^{(0)}\chi_{23}^{(0)}\chi_{31}^{0()}\mathscr{D}_1^{-1}\mathscr{D}_2^{-1}\mathscr{D}_3^{-1}\Big]. \end{aligned} \tag{17}$$

Because of the odd number of Dirac matrices we have

$$\mathrm{Tr}[\chi_{12}^{(0)}\chi_{23}^{(0)}\chi_{31}^{(0)}] = 0$$

and

$$\mathrm{Tr}[\chi_{ij}^{(0)}S_i^{-1}\chi_{jk}^{(0)}\overline{S_j^{-1}}\chi_{ki}^{(0)}] = \mathrm{Tr}\left[\chi_{ij}^{(0)}\left(\left(\not{q} + \frac{\not{P}_i}{2}\right)\chi_{jk}^{(0)}m_j\chi_{ki}^{(0)} - m_i\chi_{jk}^{(0)}\left(\not{q} + \frac{\not{P}_j}{2}\right)\chi_{ki}^{(0)}\right)\right].$$

With this we get

$$\mathrm{Tr}^{(1)} = \frac{1}{8m_1m_2m_3}\mathrm{Tr}\left[\chi_{12}^{(0)}\chi_{23}^{(0)}\left(\not{q} - \frac{\not{P}_2 - \not{P}_3}{2}\right)\chi_{31}^{(0)}\cdot C_{312} + \chi_{12}^{(0)}\chi_{23}^{(0)}\chi_{31}^{(0)}\left(\not{q} + \frac{\not{P}_1}{2}\right)\cdot C_{123} + \chi_{12}^{(0)}\left(\not{q} - \frac{\not{P}_1}{2}\right)\chi_{23}^{(0)}\chi_{31}^{(0)}C_{231}\right]. \tag{18}$$

Here

$$C_{ijk} := 2m_i\mathscr{D}_j^{-1}\mathscr{D}_k^{-1} - \frac{1}{m_i}\mathscr{D}_i^{-1}(m_j{}^2\mathscr{D}_k^{-1} + m_k{}^2\mathscr{D}_j^{-1})$$
$$= m_im_jm_k(2p_i{}^2 - p_j{}^2 - p_k{}^2 + \mathscr{O}(m^{-2}))$$

where p_i, p_j, p_k are the momenta of $\mathscr{D}_i^{-1}$, $\mathscr{D}_j^{-1}$, $\mathscr{D}_k^{-1}$, respectively. With the explicit expressions for the C_{ijk} we have

$$\begin{aligned}\mathrm{Tr}^{(1)} = &-\frac{1}{4}(P_2P_3 - q(P_2 - P_3))\,\mathrm{Tr}\left[\chi_{12}^{(0)}\chi_{23}^{(0)}\left(\not{q} - \frac{\not{P}_2 - \not{P}_3}{2}\right)\chi_{31}^{(0)}\right)\\ &+ \frac{1}{8}(P_2P_3 + q(3P_1 + P_2 - P_3))\,\mathrm{Tr}\left[\chi_{12}^{(0)}\chi_{23}^{(0)}\chi_{31}^{(0)}\left(\not{q} + \frac{\not{P}_1}{2}\right)\right]\\ &+ \frac{1}{8}(P_2P_3 - q(3P_1 - P_2 + P_3))\,\mathrm{Tr}\left[\chi_{12}^{(0)}\left(\not{q} - \frac{\not{P}_1}{2}\right)\chi_{23}^{(0)}\chi_{31}^{(0)}\right] + \mathscr{O}(m^{-2}).\end{aligned} \tag{19}$$

This is the *same* expression as the corresponding term of the SU_n symmetric treatment, Eq. (40) of Ref. [6]. We can already state here, that

(i) the quark mass breaking does not affect saturation, since Eq. (19) still is independent of the quark mass;

(ii) the vertex M and therefore the mesonic couplings will be SU_n symmetric (except phase space factors). As we already mentioned there is no further SU_n breaking term which contributes to the vertex in the order m^0.

To complete our calculation we will write down $\mathrm{Tr}^{(2)}$ and $\mathrm{Tr}^{(3)}$ which are identical with those of the SU_n symmetric calculation of BJK [6].

$$\mathrm{Tr}^{(2)} = \mathrm{Tr}[S_1^{-1}\chi_{12}^{(2)}S_2^{-1}\chi_{23}^{(1)}S_3^{-1}\chi_{31}^{(1)} + S_1^{-1}\chi_{12}^{(1)}S_2^{-1}\chi_{23}^{(2)}S_3^{-1}\chi_{31}^{(1)} + S_1^{-1}\chi_{12}^{(1)}S_2^{-1}\chi_{23}^{(1)}S_3^{-1}\chi_{31}^{(2)}].$$

$$\begin{aligned}\mathrm{Tr}^{(2)} = &-\frac{1}{4}\mathrm{Tr}\left[P_1\cdot q\left(\overline{S^{-1}}\left(\frac{P_1}{2} + q\right)\chi_{12}^{(0)} + \chi_{12}^{(0)}S^{-1}\left(\frac{P_1}{2} - q\right)\right)\chi_{23}^{(0)}\chi_{31}^{(0)}\right.\\ &+ \chi_{12}^{(0)}P_2\cdot\left(q + \frac{P_3}{2}\right)\left(\overline{S^{-1}}\left(q - \frac{P_1}{2}\right)\chi_{23}^{(0)} + \chi_{23}^{(0)}S^{-1}\left(\frac{P_2 - P_3}{2} - q\right)\right)\chi_{31}^{(0)}\\ &\left.+ \chi_{12}^{(0)}\chi_{23}^{(0)}P_3\cdot\left(q - \frac{P_2}{2}\right)\left(\overline{S^{-1}}\left(q - \frac{P_2 - P_3}{2}\right)\chi_{31}^{(0)} + \chi_{31}^{(0)}S^{-1}\left(\frac{-P_1}{2} - q\right)\right)\right]\\ &+ \mathscr{O}(m^{-1}).\end{aligned} \tag{20}$$

We easily find $\mathrm{Tr}^{(1)} = -\mathrm{Tr}^{(2)}$. Therefore the mesonic couplings are given by $\mathrm{Tr}^{(3)}$ only

$$\begin{aligned} \mathrm{Tr}^{(3)} &= \mathrm{Tr}[S_1^{-1}\chi_{12}^{(3)}S_2^{-1}\chi_{23}^{(1)}S_3^{-1}\chi_{31}^{(1)} + \cdots], \\ \mathrm{Tr}^{(3)} &= m^3\,\mathrm{Tr}[\chi_{12}^{(3)}\chi_{23}^{(0)}\chi_{31}^{(0)} + \chi_{12}^{(0)}\chi_{23}^{(3)}\chi_{31}^{(0)} + \chi_{12}^{(0)}\chi_{23}^{(0)}\chi_{31}^{(3)}] + \mathcal{O}(m^{-1}). \end{aligned} \tag{21}$$

Since $\chi^{(3)} \sim m^{-3}$, $\mathrm{Tr}^{(3)}$ is of the order m^0 and we find for the mesonic vertex in leading order the quark mass independent expression

$$M = \mathrm{Tr}[SU_n](2\pi)^{-9/2}\, i \int dq\, \mathrm{Tr}^{(3)}. \tag{22}$$

4. Summary

Our starting point was the relativistic dynamical quark model for mesons by BJK [6]. In this model very heavy ($m > 3$ GeV) Fermi quarks and their antiquarks form bound states obeying the Bethe–Salpeter equation. The model, SU_n symmetric so far, shows spin saturation of the three meson vertex, i.e., the spin structure of the $q\bar{q}$ interaction leads to the quark mass independence of mesonic couplings. We then introduced a conventional quark mass breaking, $m_{p,n} < m_\lambda < m_c$, etc.

Concerning the spectrum we found that the meson squared-mass differences are strictly proportional to the number of λ or c quarks in the mesons. This result is identical to a previous one from the same but scalar model [15, 20]. Experiment seems to verify this result within each $q\bar{q}$-excitation level only, while it further indicates an increase of these squared-mass differences with the main quantum number. A further indication for this flavor dependence of the Regge slope is the level spacing of the new mesons. This implies a modification on the equal spacing rules which is neglegible for $\rho - K^* - \phi$ but lowers the masses of D^* and F^*. From the ρ, ϕ, J/ψ masses we are only able to give an upper limit on the masses of D^* and F^*, $M_{D^*} < 2.26$ GeV, $M_{F^*} < 2.3$ GeV, while the correct values may lie 100–200 MeV below.

Concerning the mesonic couplings of the model we found that the spin saturation mechanism described above survives symmetry breaking and that even symmetry breaking terms have no effect on the matrix element in leading order. We could therefore verify the assumption of symmetric mesonic couplings in spite of quark mass breaking, on which the calculation of meson decay rates in Ref. [6] was based.

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