NEUTRAL CURRENT CONSTRAINTS IN THE SU(2) ⊗ U(1) MODELS

T.C. YANG
Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

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We present a systematic analysis of hadronic neutral currents in low energy neutrino reactions applicable to SU(2) ⊗ U(1) models with any number of quarks. We point out a relation to determine the Weinberg angle independent of the models for the heavy quarks. We discuss the constraints of atomic neutral current experiments on the SU(2) ⊗ U(1) gauge models.

New data on various neutrino reactions induced by neutral currents are recently reported [1]. While the structure of neutral currents is not yet established, the data can be used to set limits on the various gauge models. Comparison of several SU(2) ⊗ U(1) models for the hadronic neutral currents (i.e. the Weinberg–Salam (W–S) [2], the six quark vector (V) [3], the Gürsey–Sikivie (G–S) [4], and the five-quark model due to Achiman, Koller and Walsh [5], with the low energy inclusive neutrino cross section data and the elastic νp scattering can be found in the recent work of Albright et al. [6] assuming $z = m_Z^2 \cos^2\theta_W / m_W^2 = 1$ ‡1.

We present here, first, an analysis similar to that in ref. [6] but *general* enough to be applicable to any SU(2) ⊗ U(1) model with an arbitrary number of quarks (in which the up and down quarks appear in doublet and/or singlet representations). We confine ourselves also to the discussion of the hadronic neutral currents below heavy quark thresholds with the same assumptions as in ref. [6]. We find a relation which can be used to determine the Weinberg angle in terms of the measurable quantities independent of our knowledge about any additional heavy quarks that might exist. Secondly, we discuss the atomic neutral current experiments which also place a very useful constraint on our analysis. Putting all the experimental constraints together, we can systematically narrow down the classes of models that are consistent with the data. The atomic neutral current data can distinguish the W–S model from other models with more than 4 quarks and/or 4 leptons. Among the latter class of models, if we assume the Weinberg–Salam model for the known leptons and the Higgs scalars ($z = 1$), we conclude that at least seven quarks are needed in order to be consistent with the data. An eight quark vectorlike (but non-vector) model is presented.

Since the analysis is limited to low energy region where no new particle is excited, on can parametrize the neutral currents of the up and down quarks in a very general way. For the left-handed up and down quarks we take the GIM scheme [7] which is fixed by the Cabibbo angle. For the right-handed up and down quarks in the SU(2) ⊗ U(1) model, only the pieces belonging to the doublets (but not the singlets) contribute to the $I = 1$ neutral currents. We introduce two mixing angles θ and ϕ such that $u_R \cos\theta + \ldots$ and $d_R \cos\phi + \ldots$ belong to different doublets (further mixing of these states with other heavy quarks does not change the diagonal netural currents). The orthogonal components $u_R \sin\theta + \ldots$ and $d_R \sin\phi + \ldots$ belong to singlets. The values of $\cos\theta$ and $\cos\phi$ in several models are given in table 1. The neutral currents can be written in the following general form,

Table 1
Mixing angles for the four models of interest.

Model	$\cos\theta$	$\cos\phi$
W–S	0	0
Vector	1	1
AKW	$\cos\alpha$	0
G–S(B)	1	0

‡1 The experimental evidence supports $Z \approx 1$ [1].

$$J_\mu^{\rm neut} = J_\mu^3 - X_{\rm w} J_\mu^{\rm em}$$
$$= \tfrac{1}{2}\{\bar u_L \gamma_\mu u_L + \cos^2\theta\, \bar u_R \gamma_\mu u_R - \bar d_L \gamma_\mu d_L \quad (1)$$
$$- \cos^2\phi\, \bar d_R \gamma_\mu d_R - 2X_{\rm w}(\tfrac{2}{3}\,\bar u \gamma_\mu u - \tfrac{1}{3}\,\bar d \gamma_\mu d)\} + \ldots$$

where $X_{\rm w} = \sin^2\theta_{\rm w}$, $\theta_{\rm w}$ is the Weinberg angle, and $L, R = (1 \pm \gamma_5)/2$. In terms of isospin indices,

$$J_\mu^{\rm neut} = \alpha V_\mu^3 + \beta A_\mu^3 + \tfrac{1}{3}\gamma V_\mu^{\rm o} + \delta A_\mu^{\rm o} \quad (2)$$

one has

$$\alpha = 1 - 2X_{\rm w} + \tfrac{1}{2}(\cos^2\theta + \cos^2\phi),$$
$$\beta = 1 - \tfrac{1}{2}(\cos^2\theta + \cos^2\phi),$$
$$\gamma = \tfrac{3}{2}(\cos^2\theta - \cos^2\phi) - 2X_{\rm w}, \quad (3)$$
$$\delta = \tfrac{1}{2}(\cos^2\phi - \cos^2\theta).$$

Using the parton model with eq. (1), one can calculate the ratio of the neutral-current and charged-current cross sections for the neutrino and anti-neutrino beams on an isoscalar target,

$$R^\nu = \frac{\sigma_{\rm nc}^\nu}{\sigma_{\rm cc}^\nu} = \frac{1}{4z^2}\left\{A + \frac{B}{3}\right\} + {\rm O},$$
$$R^{\bar\nu} = \frac{\sigma_{\rm nc}^{\bar\nu}}{\sigma_{\rm cc}^{\bar\nu}} = \frac{1}{4z^2}\{A + 3B\} + {\rm O}, \quad (4)$$

where

$$z = \frac{m_{\rm Z}^2 \cos^2\theta_{\rm w}}{m_{\rm w}^2}$$
$$A = (1 - \tfrac{4}{3}X_{\rm w})^2 + (1 - \tfrac{2}{3}X_{\rm w})^2 \quad (5)$$
$$B = (\cos^2\theta - \tfrac{4}{3}X_{\rm w})^2 + (\cos^2\phi - \tfrac{2}{3}X_{\rm w})^2$$

and O denotes the corrections due to neglecting the $\sin^2\theta_{\rm c}$ and the strange sea contribution which are (experimentally) estimated to be less than 10% for low neutrino reactions [1].

We find from eqs. (4) a general relation

$$9R^\nu - R^{\bar\nu} = z^{-2}\{4(1 - X_{\rm w})^2 + \tfrac{4}{9}X_{\rm w}^2\} + {\rm O} \quad (6)$$

which is independent of the structure of the heavy quarks and can be used to determine the Weinberg angle. The ratios R^ν, $R^{\bar\nu}$ are plotted in fig. 1 for $z = 1$ as functions of $X_{\rm w} = \sin^2\theta_{\rm w}$ for the W–S, vector, AKW models and one variant of the G–S model. Different model predictions for the same value of $X_{\rm w}$ are seen to lie approximately on a straight line according to eq. (6). We also show the data point corresponding to the world average values [8].

$$R^\nu = 0.26 \pm 0.04, \quad R^{\bar\nu} = 0.37 \pm 0.10\,. \quad (7)$$

From fig. 1, one immediately reads off the corresponding value of $X_{\rm w}$ for eq. (7),

$$X_{\rm w} = \sin^2\theta_{\rm w} = 0.37 \pm 0.06\,. \quad (8)$$

One notes that the value of $X_{\rm w}$ is more sensitive to the error of R^ν than $R^{\bar\nu}$.

Parity violation of neutral currents in atoms could shed significant information on the structure of weak neutral currents [e.g. 9]. The experiments in heavy atoms measure the interference term of the axial-vector electron current and the vector hadronic current where the contribution adds up coherently and get multiplied by the atomic numbers. For the W–S model with the known lepton neutral currents, the relevant quantity is the hadronic weak charge which is a function of $X_{\rm w} = \sin^2\theta_{\rm w}$

$$Q_{\rm w} = Z\langle {\rm p}|V_{\rm o}^{\rm neut}|{\rm p}\rangle + N\langle {\rm n}|V_{\rm o}^{\rm neut}|{\rm n}\rangle$$
$$= Z(1 - 4X_{\rm w}) - N \quad (9)$$

where Z and N are the numbers of the protons and neutrons in the nucleus. The hadronic weak charge for the general class of SU(2) ⊗ U(1) models is, using eq. (1), given by

$$Q_{\rm w} = Z\{1 + 2\cos^2\theta - \cos^2\phi - 4X_{\rm w}\}$$
$$+ N\{-1 + \cos^2\theta - 2\cos^2\phi\}\,. \quad (10)$$

The experiments are done by measuring the optical rotation of laser light tuned to frequences close to the resonant one [10]. Two experiments, one at Seattle and one at Oxford, are reporting perhaps the parity violation effects in atomic $^{209}_{83}$Bi. Using the atomic physics calculation [11], the W–S model predicts an angle of rotation $\Delta\phi \sim 3 \times 10^{-7}$ rad. for $X_w \sim 0.4$. The preliminary data from both experiments indicate an effect of the order 10^{-8} rad. [12]. We are aware that the data are by no means final and the atomic calculation could be modified. The point, however, is that if we put together the neutrino data of neutral currents with the atomic neutral current experiments, there exist severe constraints on the gauge models [9] which can be analyzed systematically. The W–S model predication for the atomic neutral current experiments need not be further commented on. If the parity violation effect in atoms is absent or small, the reasons can be either that the electron neutral current is a pure vector up to a small mixing angle or the hadronic neutral currents are different from the W–S model since there might exist more than 4 quarks. Modification of the W–S model of the leptons [9, 13] have unique predictions for the (anti-) neutrino-electron elastic scattering ‡2, and will not be covered here. We proceed below with our general analysis of the hadronic neutral currents assuming the W–S model of leptons and Higgs scalars ($z = 1$) ‡3.

We present in fig. 2 the allowed region of the mixing angles from the data. We assumed $X_w = 0.37$ from (8) as discussed above. We see that there exists a solution with non-zero cos θ and cos ϕ; the specific values depend on the atomic neutral current data. This solution corresponds to assuming more than 4 quarks ‡4 – heavy quarks have been suggested in the literature to explain the rising anti-neutrino charged-current cross section and the y-anomaly [14]. We note that the assignments of the right-handed quarks are severely restricted: one cannot allow u_R and d_R (or s_R) in the

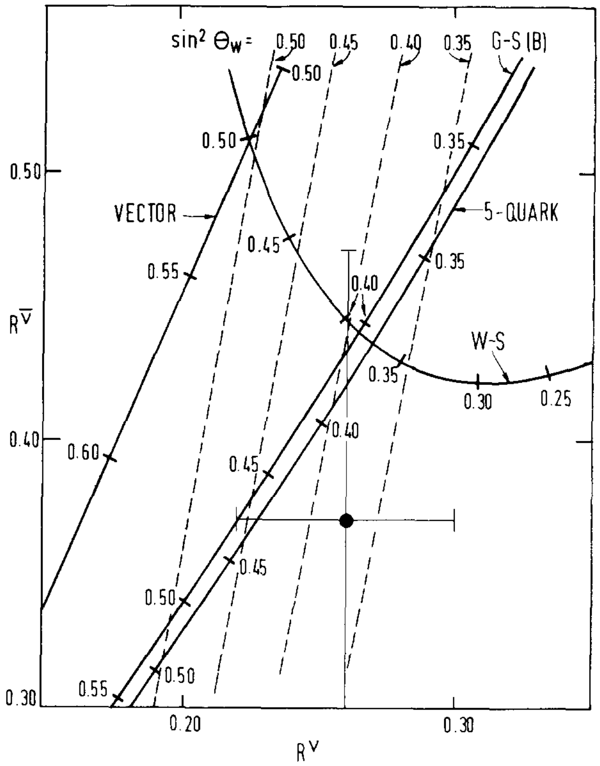

Fig. 1. The ratios R^{ν} and $R^{\bar{\nu}}$ as functions of $\sin^2\theta_W$ for the Weinberg–Salam (W–S) vector, Gürsey–Sivikie (G–S) model (B), and the five-quark model of Achiman et al. The curves are for E below heavy quark thresholds (see Albright et al., ref. [7]). The tick marks on each curve denote the value of $\sin^2\theta_W$. The dashed lines are given by eq. (6) as compared with the model predictions. The data point corresponds to the world average values given in eq. (7). See ref. [11] for details.

‡2 If the electron neutral current is a pure vector, then $\sigma^{\mathrm{elast}}(\nu_\mu e) = \sigma^{\mathrm{elast}}(\bar{\nu}_\mu e)$.

‡3 The analysis can be done for other values of Z as well. See also Achiman and Walsh, ref. [6].

‡4 Non-zero cos θ and cos ϕ means that neutral currents could excite heavy quarks in neutrino reactions above the heavy quark thresholds.

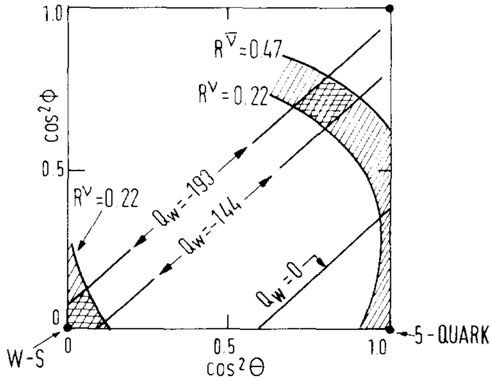

Fig. 2. Constraints on the mixing angles from the data. The cross hatched area denote the bounds $0.22 \leqslant R^{\nu} \leqslant 0.30$, $0.37 \leqslant R^{\bar{\nu}} \leqslant 0.47$. The Weinberg–Salam model predicts $Q_W = -144 \sim -193$ for $X_W = 0.3 \sim 0.4$. The corresponding limits are shown by the crossed area. Small parity violation effect in atoms would limit the solution to the intersection near $Q_W = 0$.

same doublet since low energy meson and hyperon decay via V–A currents. One must also avoid right-handed strangeness-changing neutral currents in lowest order. With in addition the constraint on $\cos\theta$ and $\cos\phi$, one is forced to have a minimum of seven quarks using only doublet and/or singlet representations. We suggest below an eight-quark vector-like (but non-vector) model with lepton-quark symmetry. The quarks and leptons are assigned in the following doublets ‡5:

$$\left(\begin{array}{c|c} u & \nu_e \\ d(\theta_c) & e^- \\ \hline s(\theta_c) & \mu^- \\ c & \nu_\mu \end{array}\right)_L \quad \left(\begin{array}{c|c} t(\theta) & \nu_L \\ b & L \\ \hline b'(\phi) & M^- \\ t' & \nu_M \end{array}\right)_R$$

and singlets: $e^-_R, \mu^-_R, L^-_L, M^-_L; t_L, b_L, b'_L, t'_L; u(\theta)_R, d(\phi)_R, s_R, c_R$, where L^-, M^- are two sequential heavy leptons (one of them may be responsible for the anomalous μe events [15], and ν_L, ν_M are two massless neutrinos. t, b, t′, b′ are four heavy quarks (the b quark may be responsible for the rising anti-neutrino charged-current cross section and the y-anomaly). The notations used above are denoted by: $d(\theta_c) = d\cos\theta_c + u\sin\theta_c$, $s(\theta_c) = -d\sin\theta_c + s\cos\theta_c$, $t(\theta) = t\sin\theta + u\cos\theta$, $u(\theta) = t\cos\theta - u\sin\theta$, $b'(\phi) = b'\sin\phi + d\cos\phi$, $d(\phi) = b'\cos\phi - d\sin\phi$, where θ_c is the Cabibbo angle, θ, ϕ the two mixing angles discussed above. The difference $1-\cos\theta$ and $1-\cos\phi$ reflect the amount of parity violation in the hadronic neutral currents. Why the mixing angles are not 0 or π? For the vectorlike theories, the mass eigenstates are determined by spontaneous symmetry breaking. The mixing angles can only be determined by experiments.

For the (anti-)neutrino proton elastic scattering, we refer the reader to Albright et al. [6] who concluded that the five quark model [5] and the Gürsey–Sikivie [4] model satisfactorily account for the observed data in shape and magnitude, and the Weinberg–Salam model predicts a cross section smaller than observed. We repeat a similar calculation using the decomposition (2). For $X_W = 0.37$, and $Q_W \sim -167$ ($\cos^2\theta \sim 0.8$, $\cos^2\phi \sim 0.7$) we find a cross section (and q^2 dependence) for νp scattering in between the W–S model and the G–S(B) model. For $Q_W = 0$,(i.e. no parity violation in atoms) the prediction is closer to the five quark model and the G–S(B) model. Noting the theoretical assumptions in the analysis and possible experimental corrections, we conclude that the νp elastic data are compatible with the other data and could in the future place additional restrictions in a more quantitative analysis.

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‡5 One can also assign (t_L, b_L) and (c_R, s_R) in doublets. I thank K. Fukikawa for the remarks.

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