e^+e^- annihilation into baryon-antibaryon pairs

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Using generalized-vector-dominance-model–type form factors we calculate the e^+-e^- production cross sections for the reactions $e^+e^- \to B_{1/2^+}\bar{B}_{1/2^\pm}$, $B_{1/2^+}\bar{B}_{3/2^\pm}$, $B_{1/2^+}\bar{B}_{5/2^+}$, and $B_{3/2^+}\bar{B}_{3/2^+}$ including all prominent baryon resonances at energies of present and planned e^+-e^- storage-ring machines. The calculated cross sections indicate that the interference of direct and one-photon decay of the ψ particles into baryon pairs is small.

I. INTRODUCTION

In the next few years one anticipates, in addition to total and inclusive e^+-e^- cross-section measurements, data on exclusive two-body and quasi-two-body production cross sections which are expected to provide important information on the electromagnetic structure of hadrons in the timelike region. Obviously the rate for the production of baryon-antibaryon pairs will be much suppressed compared to the production rate of meson pairs and will constitute only a minute fraction of the total production cross section at any given energy. The question then arises whether present and/or future machine luminosities and detection efficiencies are sufficient to collect enough data on baryon pair production to enable one to study this potentially rich area of physics. In order to answer this question we present some cross-section estimates of two-body and quasi-two-body baryon pair production based on present knowledge of the form-factor behavior of elastic and inelastic baryon production in the spacelike region (for previous estimates see Refs. 1 and 2).

A principal problem in the calculation of annihilation cross sections into two-body and quasi-two-body states involving particles with spins is the appearance of kinematic q^2 powers through momentum factors which have to be damped by some dynamical mechanism in order not to violate general theoretical principles as, e.g., unitarity bounds. In addition, if form factors have a dynamical power behavior in q^2, one expects a relation between this power and the spins of the produced particles[3-6] from simple dimensional arguments.

In the framework of the generalized vector-dominance model (GVDM) one would attempt to obtain a representation of such dynamically well-behaved form factors by writing down a set of superconvergence relations and saturate these through a finite number of vector mesons.[7] The resulting vector-meson pole residues are such that the vector-meson contributions can be summed into product forms as in Eq. (1). Such GVDM-type form factors appear explicitly in the dual current model.[5] The form factors consist of a product of poles in the timelike region with pole positions located at the vector-meson poles and their Veneziano recurrences. The product terminates at the appropriate recurrence to ensure the correct asymptotic q^2 behavior (see Ref. 8).[9] Such form factors were shown in Ref. 10 to give a good fit in the spacelike transition-form-factor analysis of Devenish and Lyth[11] (for a comparison with elastic-form-factor data see Ref. 12).

In this paper we extend the analysis of Ref. 10 to the timelike region. We calculate the production cross sections for all members of the ground-state octet and decuplet with $J^P=\frac{1}{2}^+$ and $\frac{3}{2}^+$ in the processes $e^+e^- \to B_{1/2^+}\bar{B}_{1/2^+}$, $B_{3/2^+}\bar{B}_{3/2^+}$, $B_{1/2^+}\bar{B}_{3/2^+}$ and resonance production in $e^+e^- \to N\bar{N}^*$, where we consider the resonances $N^* = D_{13}(1520)$, $F_{15}(1688)$, $P_{11}(1470)$, and $S_{11}(1535)$ with $J^P=\frac{3}{2}^-$, $\frac{5}{2}^+$, $\frac{1}{2}^+$, and $\frac{1}{2}^-$, respectively.

Our main results may be summarized as follows:

(1) The cross sections rise steeply from threshold to their peak value and then decrease faster than what corresponds to a naive $(q^2)^{-5}$ power law in an intermediate q^2 region. The faster-than-$(q^2)^{-5}$ decrease is due to the presence of the vector-meson mass singularities in the timelike region, and, for the "elastic" production reactions, is due to destructive interference effects between the various contributing form factors. The cross sections are in general very small. For example, the peak cross section for $\Omega^-\bar{\Omega}^-$ production is predicted to be a mere 6×10^{-6} nb.

(2) The one-photon decay mode of the ψ particle into baryon pairs is small and the interference with the direct decay is likely to occur only at a level of a few percent.

II. FORM FACTORS

We shall be working with the constraint-free form factors that have been introduced in Ref. 10. It is then straightforward to obtain the relations between these and the c.m. helicity amplitudes which are given in the Appendix. Since the constraint-free form factors are independent of each other for all values of q^2 the corresponding helicity amplitudes exhibit the correct kinematical threshold and pseudothreshold constraint structure (see Refs. 10 and 13).

As in Ref. 10 the q^2 dependence of the constraint-free form factors is assumed to arise from the coupling of many vector mesons in the form of a product of poles. Thus we have for a form factor

$$F(q^2)=\sum_{i=\rho,\omega,\phi} C_i \prod_{n=0}^{N(J,c)-1}\left(1-\frac{q^2}{m_i{}^2+n\alpha_i'^{-1}}\right)^{-1}. \tag{1}$$

The product in Eq. (1) extends over as many poles $N(J,c)$ as is necessary to obtain the correct asymptotic q^2 dependence. In terms of the Drell-Yan threshold relation one has, for the contribution of a given resonance with spin J to e.g. νW_2, $\nu W_2 \propto (\omega-1)^{2c-1}$ in the limit $q^2 \to -\infty$, where c determines the threshold power in νW_2 and is set to $c=2$ for canonical dipole behavior. Also, we shall assume asymptotic suppression of longitudinal resonance contributions. In the timelike region these assumptions lead to $\sigma_T \propto (1/q^2)^{2c+1}$ and $\sigma_L/\sigma_T \propto (1/q^2)$ for the annihilation cross sections defined in the Appendix. A common power-law behavior for two-body baryon-antibaryon production has also been advocated in Ref. 14.

The sum in Eq. (1) extends over the different vector mesons ρ, ω, and ϕ and their recurrences that couple to the photon. In terms of quark language we decompose the photon according to the contributions $(2)^{-1/2}(u\bar{u}-d\bar{d})$, $(2)^{-1/2}(u\bar{u}+d\bar{d})$, and $\lambda\bar{\lambda}$. We shall ignore the small deviations from ideal mixing for ω and ϕ and assume the complete validity of the Okubo-Zweig-Iizuka (OZI) rule.[15,16] The resulting couplings are given in Tables I–IV.

For the Regge slopes α_i' appearing in Eq. (1) we shall use $\alpha_i' \approx 0.9$–1.0 GeV^{-2} and for the masses of the lowest-lying vector mesons we use $m_\rho{}^2 = 0.593$ GeV^2, $m_\omega{}^2=0.614$ GeV^2, and $m_\phi{}^2=1.040$ GeV^2.

As has been already stressed in the Introduction, form factors of the GVMD type as Eq. (1) have been shown in Ref. 10 to account quite well for the spacelike behavior of some of the resonance excitation data. In general the $q^2=0$ values of the form factors will be determined from data if possible. In some cases some additional theoretical input has to be used, which will be discussed in Sec. III.

Finally we remark that we shall always be working in the narrow-resonance approximation for the vector mesons. This is well justified since the production thresholds of all the cases treated here are at least 1 GeV^2 above the highest-lying vector-meson pole coupling to the respective production amplitude.

TABLE I. Coupling strengths C_1^V of vector mesons ρ, ω, and ϕ to the $B_{1/2^+}\bar{B}_{1/2^+}$ system at $q^2=0$.

V	$p\bar{p}$	$n\bar{n}$	$\Sigma^+\bar{\Sigma}^+$	$\Sigma^0\bar{\Sigma}^0$	$\Sigma^-\bar{\Sigma}^-$	$\Xi^0\bar{\Xi}^0$	$\Xi^-\bar{\Xi}^-$	$\Lambda\bar{\Lambda}$
ρ	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0
ω	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
ϕ	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$

III. CROSS SECTIONS

The kinematic quantities used in this section are all defined in the Appendix. We shall not in every case be referring to the relevant equations in the Appendix.

A. $B_{1/2^+}\bar{B}_{1/2^+}$ production

The canonical dipole behavior corresponds to the choice of two poles for $F_1(q^2)$ and three or more poles for $F_2(q^2)$. We shall represent $F_2(q^2)$ by three poles, i.e., the minimum number of poles required to satisfy the Drell-Yan relation.[17] The F_1 are normalized to the charge at $q^2=0$ and the $F_2(0)$ to the anomalous magnetic moment. For p, n, Λ, Σ^+, and Ξ^- we use the experimental magnetic moments as input and for the remaining members of the octet including the $\Sigma^0\bar{\Lambda}$ transition moment SU(6) values. The tabulated values of relative ρ, ω, and ϕ couplings in Tables I and II correspond to pure F coupling for $F_1(0)$ and the SU(6) F/D ratio $F/D=\frac{2}{3}$ for $G_M(0)$.

In Figs. 1 and 2 we compare our form factors with the experimental proton and neutron form factors from the analysis of Felst[18] and Atwood.[19] In the case of the magnetic form factor of the proton G_M^p better agreement with the data is obtained for $\alpha'=1$, whereas there is not much difference between $\alpha'=0.9$ and $\alpha'=1$ for the other three cases G_E^p, G_M^n, and G_E^n. The gross features of the form-factor behavior are reproduced quite well and the

TABLE II. Coupling strengths $C_1^V+C_2^V$ of the vector mesons ρ, ω, and ϕ to the $B_{1/2^+}\bar{B}_{1/2^+}$ system at $q^2=0$. The values are in units of the magnetic moment of each particle and the total magnetic moment μ is in units of the proton magnetic moment μ_p.

V	$p\bar{p}$	$n\bar{n}$	$\Sigma^+\bar{\Sigma}^+$	$\Sigma^0\bar{\Sigma}^0$	$\Sigma^-\bar{\Sigma}^-$	$\Xi^0\bar{\Xi}^0$	$\Xi^-\bar{\Xi}^-$	$\Lambda\bar{\Lambda}$	$\Lambda\bar{\Sigma}^0$
ρ	$\frac{5}{6}$	$\frac{5}{4}$	$\frac{2}{3}$	0	2	$\frac{1}{4}$	$-\frac{1}{2}$	0	1
ω	$\frac{1}{6}$	$-\frac{1}{4}$	$\frac{2}{9}$	$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{1}{12}$	$\frac{1}{6}$	0	0
ϕ	0	0	$\frac{1}{9}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	1	0
μ	1	$-\frac{2}{3}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$(\frac{1}{3})^{1/2}$

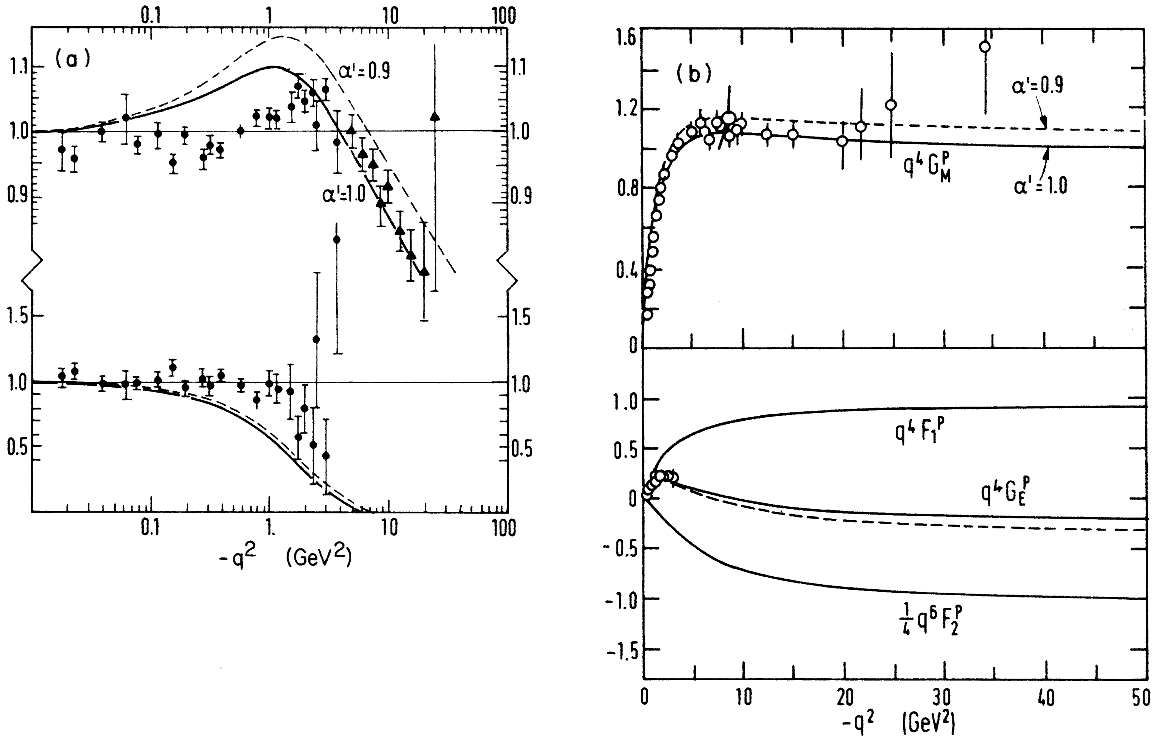

FIG. 1. Electromagnetic form factors of the proton in the spacelike region. Data are from Ref. 18 and Ref. 19. In (b) $q^4F_1^p$ and $q^6F_2^p$ are also included in order to show the large-q^2 behavior of F_1^p and F_2^p.

deviations from the experimental values are within approximately 10%.

In Fig. 3 we show our predictions for the production cross sections. For $p\bar{p}$ production the calculated cross section close to threshold is quite sensitive to the choice $\alpha' = 0.9$ or $\alpha' = 1.0$ owing to destructive interference between the two-pole ansatz F_1 and the three-pole ansatz of F_2. For $\alpha' = 0.9$ and $\alpha' = 1.0$ our predicted cross sections are within 1 and 3 standard deviations, respectibely, of the Frascati measurement at $q^2 = 4.41$ GeV^2.[20,21] For higher values of q^2 the predicted $p\bar{p}$ cross sections for the two different values of α' approach one another and lie within ~15% for $q^2 = \infty$. Whereas the spacelike data seem to favor a slope of $\alpha' = 1$, the value $\alpha' = 0.9$ is more favorable for a fit through the Frascati point. In the case of $n\bar{n}$ production the interference between F_1 and F_2 is negligible [$F_1^n(q^2) \equiv 0$ for $m_\omega{}^2 = m_\rho{}^2$] resulting in a larger cross section in the intermediate-q^2 region. For the same reason the difference between choosing $\alpha' = 1.0$ or $\alpha' = 0.9$ is small in this case. Since it is at present rather difficult to commit oneself to a definite value for the spacing of the vector mesons and their recurrences, one has to accept our results modulo this uncer-

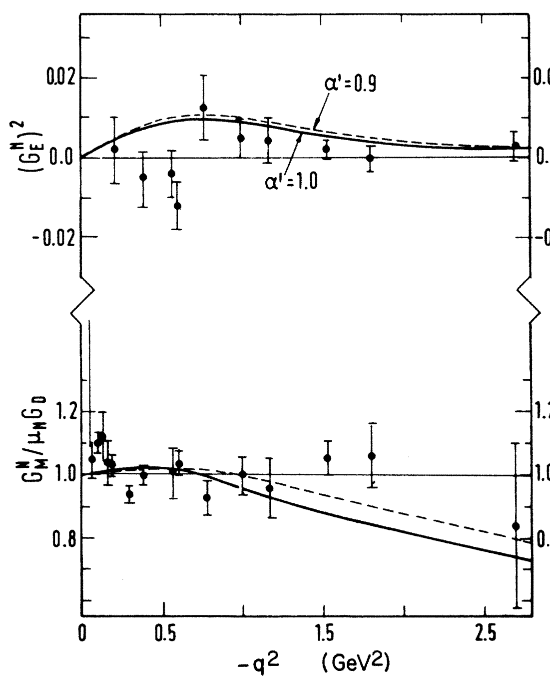

FIG. 2. Electromagnetic form factors of the neutron in the spacelike region. Data are from Ref. 18.

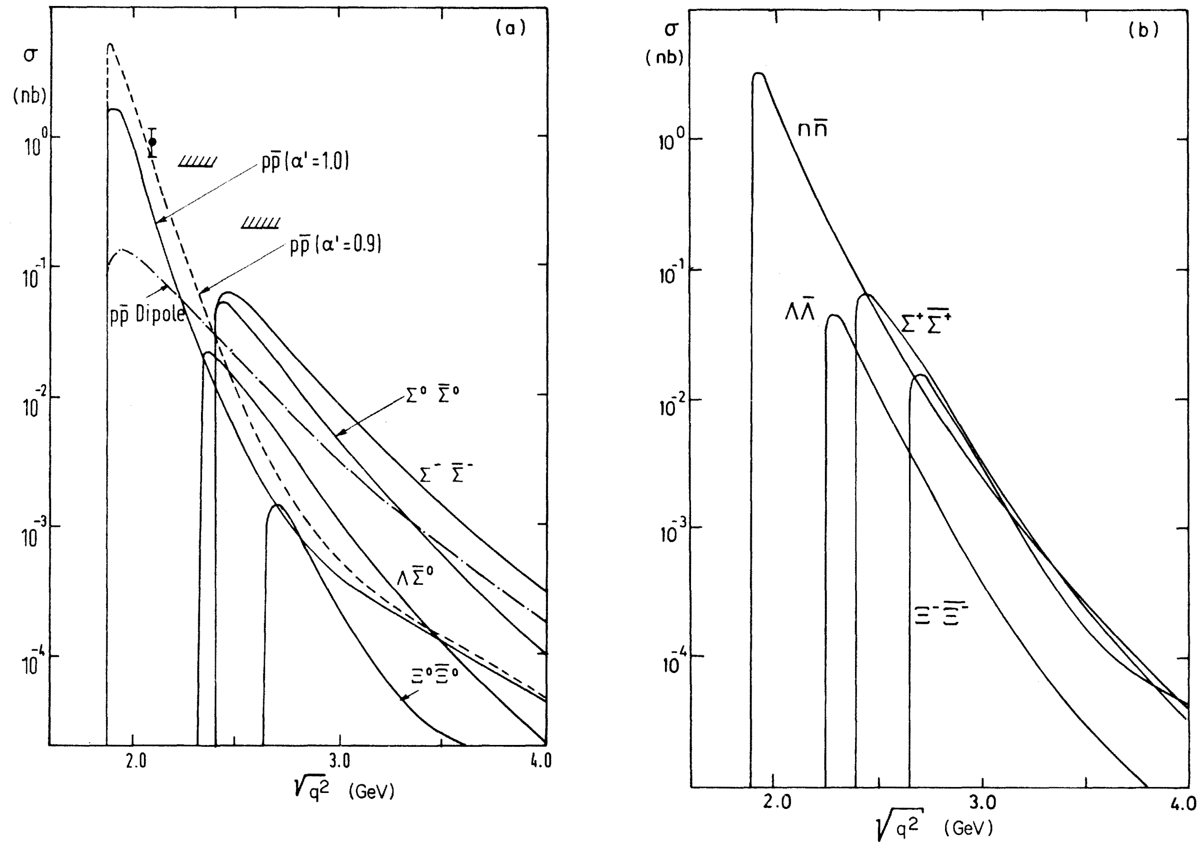

FIG. 3. Cross sections $\sigma(e^+e^- \to B_{1/2^+}\bar{B}_{1/2^+})$. Dashed line: $\alpha' = 0.9$, solid lines: $\alpha' = 1$; dashed-dotted line: cross section according to simple dipole law with dipole mass $m_D = 0.71$ GeV. Upper bounds are from Refs. 23 and 24.

tainty in the slope value. At any rate, except for a region close to threshold our results are not so extremely sensitive to the value of the Regge slope. The remaining figures are drawn for $\alpha' = 1$.

In Ref. 22 a detailed analysis of elastic proton and neutron form-factor data was performed using a parametrization in terms of vector mesons. Apart from the usual $q^2 = 0$ constraints most of the masses and residues of the vector-meson recurrences were treated as free parameters. In our treatment the only free parameter is the Regge slope α' which is constrained to vary in a limited range only. It is clear that we can therefore only expect to reproduce the gross features of the form-factor behavior in the spacelike region. Expanding our product-type form factors in terms of sums of vector-meson contributions, one obtains residues which show the alternating sign pattern characteristic of all form-factor fits. However, the values of the residues are in general different from those in Ref. 22. Our residues reflect the asymptotic constraints which we imposed from the outset, whereas in the treatment of Ref. 22 no such constraints are used.

It is well known that different satisfactory fits to the elastic spacelike data can lead to widely varying predictions in the timelike region. For example, Renard maintains that finite-width effects

TABLE III. Coupling strengths C_1^V and C_3^V of vector mesons ρ, ω, and ϕ to the $B_{3/2^+}\bar{B}_{3/2^+}$ system at $q^2 = 0$. C_2^V and C_4^V are obtained by multiplying C_1^V by the proton anomalous moment $\kappa_p = 1.79$.

V	$\Delta^{++}\bar{\Delta}^{++}$	$\Delta^{+}\bar{\Delta}^{+}$	$\Delta^{-}\bar{\Delta}^{-}$	$\Sigma^{*+}\bar{\Sigma}^{*+}$	$\Sigma^{*-}\bar{\Sigma}^{*-}$	$\Xi^{*-}\bar{\Xi}^{*-}$	$\Omega^{-}\bar{\Omega}^{-}$
ρ	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	1	-1	$-\frac{1}{2}$	0
ω	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	0
ϕ	0	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	-1

and inelasticity corrections to the vector-meson propagators are important, and he arrives at a satisfactory representation of the spacelike data.[2] He presents $p\bar{p}$-production cross sections that are rather large. Independent of the details of his model the large cross sections can be traced to an *ad hoc* assumption about how F_1 and F_2 are related to each other. This assumption produces a strong constructive interference in the timelike region between the contributions of the two form factors which leads to a large cross section.

In Fig. 3(a) we have also included upper bounds from Refs. 23 and 24. A preliminary upper bound of 6×10^{-2} nb at $q^2=9$ GeV^2 due to a tentative $p\bar{p}$ event is reported from SPEAR.[25]

The cross sections for the other reactions such as $e^+e^-\to\Sigma^+\bar{\Sigma}^+$, $\Lambda\bar{\Lambda}$, etc. are small and are therefore expected to be measurable only near the threshold where the cross sections peak at 10^{-2}–10^{-1} nb.

We close this subsection by remarking that the asymptotic constraints we impose on the q^2 behavior of F_1 and F_2 are also compatible with F_2 having four poles. The fit to the spacelike data would not be affected so much by such a choice. However, in the timelike region F_1 and F_2 would now constructively interfere. We have checked that such a choice would not be compatible with the Frascati point and the upper bounds provided by Refs. 23 and 24. For example, at the Frascati point the $p\bar{p}$ cross section would be more than 20 times the experimental value.

B. $B_{3/2^+}\bar{B}_{3/2^+}$ production

The canonical choice for F_1, F_2, F_3, and F_4 is three, four, four, and five poles. From SU(6) (see Ref. 26) or from the quark model including effects due to the difference of current and constituent quarks,[27] one has for the multipoles

$$\begin{aligned} G_E(0)&=F_1(0)=Q,\\ G_M(0)&=F_1(0)+F_2(0)=Q\mu_p,\\ G_Q(0)&=F_1(0)-F_3(0)=0,\\ \sqrt{6}\,G_O(0)&=F_1(0)+F_2(0)-F_3(0)-F_4(0)\\ &=0, \end{aligned} \qquad (2)$$

where Q is the charge and μ_p is the magnetic moment of the proton. The relative ρ, ω, and ϕ couplings of the various SU(3) production channels are given in Table III. We show the results for $e^+e^-\to B_{3/2^+}\bar{B}_{3/2^+}$ in Fig. 4. The Δ^{++}-pair production has the largest cross section in the decuplet which is due to its double charge and its lightness. Although the $\Omega^-\bar{\Omega}^-$ production cross section is enhanced by the ϕ-meson dominance of its form factor, its cross section is small (~10^{-5} nb at

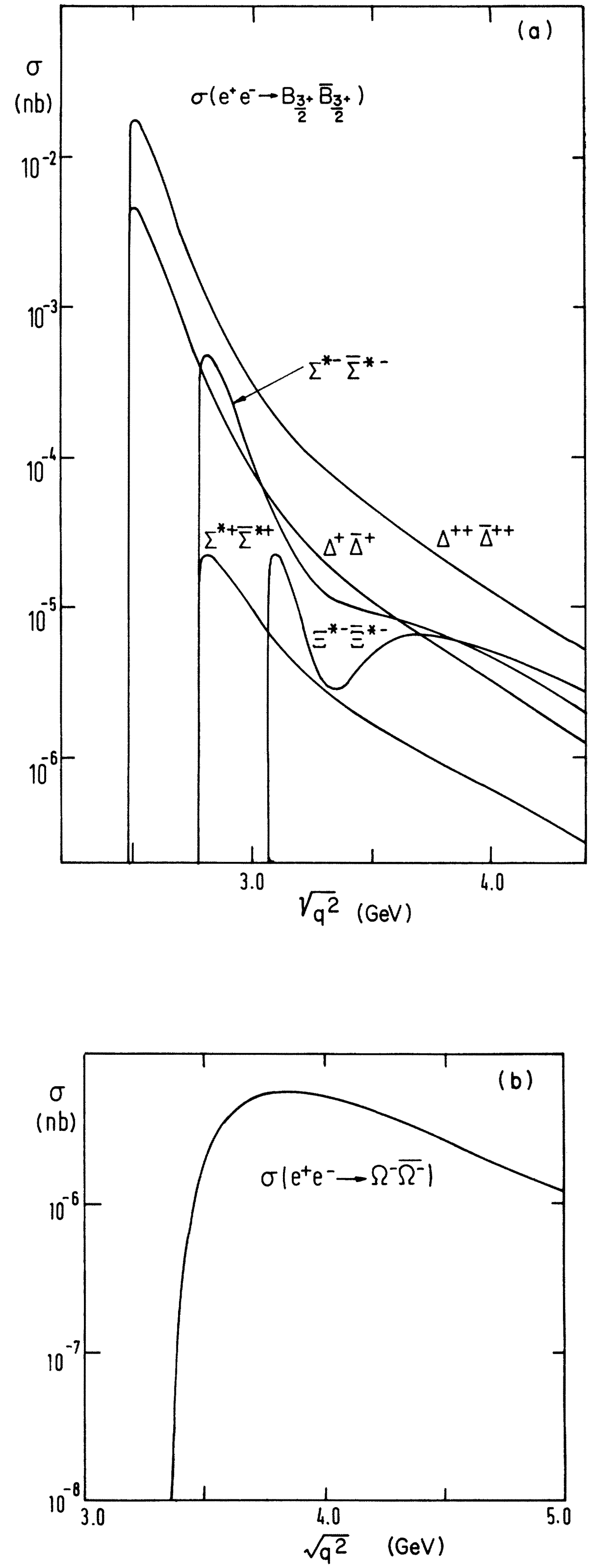

FIG 4. Cross sections $\sigma(e^+e^-\to B_{3/2^+}\bar{B}_{3/2^+})$.

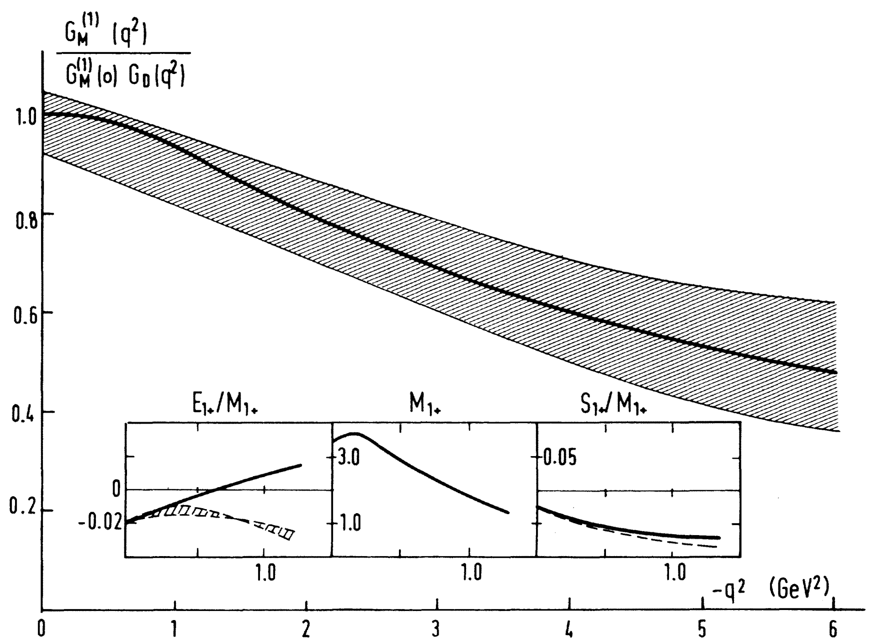

FIG. 5. $B_{1/2^+}B_{3/2^+}$ transition form factors in the spacelike region. Inset: Dashed lines give the result of the analysis of Ref. 11; solid lines give our fit with $c=3$, $\alpha'=1$ and coupling values Eq. (3). For M_{1+} our fit is not discernible from results of Ref. 11. Main figure: Magnetic transition form factor normalized to dipole form factor ($m_D=0.71$ GeV) (for definitions see Ref. 10). Shaded area: experimental band taken from Ref. 29; solid line: our fit with the above parameter values.

peak value) and thus the direct electromagnetic production of Ω^- pairs will be extremely hard to observe. After reaching its peak value the $\Omega^-\bar{\Omega}^-$ production decreases at a slower rate than the $\Delta\bar{\Delta}$ cross section, since $\Omega^-\bar{\Omega}^-$ production occurs at higher q^2 values.

C. $B_{1/2^+}\bar{B}_{3/2^+}$ production

The canonical choice for the three form factors G_1, G_2, and G_3 is three, three, and four poles. However, in Ref. 10 it was shown that the form-factor data require a one-below-canonical form-factor behavior, which is in nice accord with theoretical ideas related to the attainment of the value $\frac{1}{4}$ for the ratio $\nu W_2^n/\nu W_2^p$ in the threshold region $\omega \to 1$.[28] A good fit to the spacelike data[11,29] was obtained for $\gamma_v p \to \Delta^+$ using

$$G_1(0)/M = 1.77\ \text{GeV}^{-2},\quad G_2(0) = -1.31\ \text{GeV}^{-2},\quad G_3(0) = 0, \qquad (3)$$

and using four poles for G_1 and G_2 (Fig. 5) (see also Ref. 10). A choice of $G_1(0)/MG_2(0) = -1$ in Eq. (3) would correspond to a pure $M1$ transition as predicted by SU(6) (see e.g., Ref. 26) or the quark model (see e.g., Ref. 27). The deviation from this value indicated in Eq. (3) allows for the small $E2$ and Coulombic contributions that are observed in the data (see Fig. 5). We give the relative ρ, ω, and ϕ couplings to the various SU(3) channels in Table IV.

TABLE IV. Relative dimensionless coupling strengths C_1^V and C_2^V of the vector mesons ρ, ω, and ϕ to the $B_{1/2^+}\bar{B}_{3/2^+}$. For details see the main text.

V	$p\bar{\Delta}^+$	$n\bar{\Delta}^0$	$\Sigma^+\bar{\Sigma}^{*+}$	$\Sigma^0\bar{\Sigma}^{*0}$	$\Sigma^-\bar{\Sigma}^{*-}$	$\Xi^0\bar{\Xi}^{*0}$	$\Xi^-\bar{\Xi}^{*-}$	$\Lambda\bar{\Sigma}^{*0}$
ρ	1	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\sqrt{3}/2$
ω	0	0	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{6}$	0
ϕ	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	0

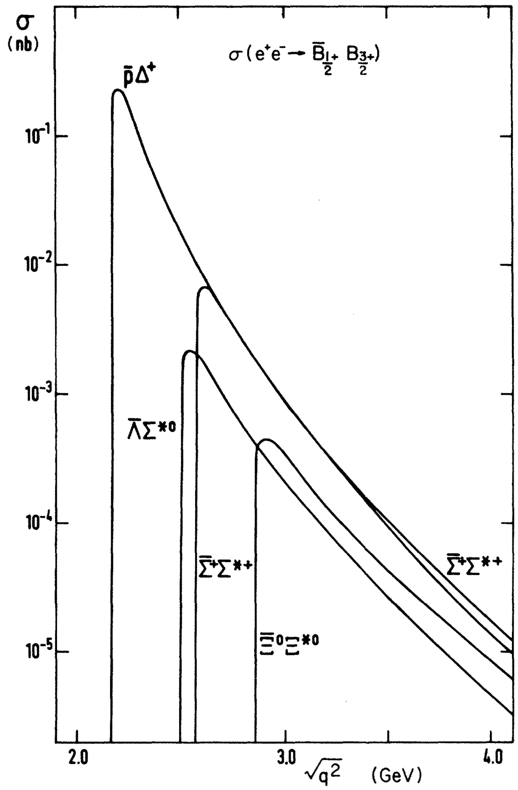

FIG. 6. Cross sections $\sigma(e^+e^- \rightarrow B_{1/2^+}\bar{B}_{3/2^+})$.

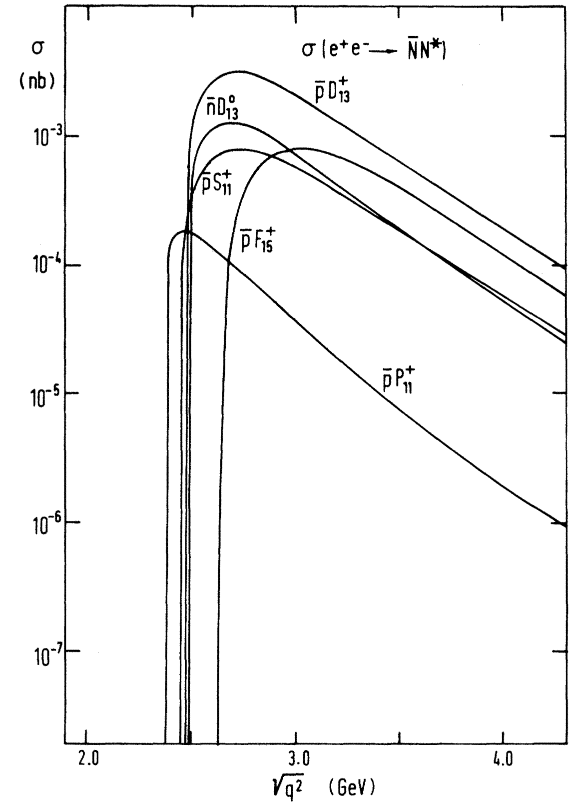

FIG. 7. Cross sections $\sigma(e^+e^- \rightarrow \bar{N}N^*)$, where $N^* = P_{11}(1470)$, $S_{11}(1535)$, $D_{13}(1520)$, and $F_{15}(1688)$.

Our results are shown in Fig. 6. Since G_1 and G_2 have dimensions of -1 and -2, we have multiplied these by M and M^2 in order to apply SU(3) to dimensionless couplings. The resulting predictions for the various cross sections are of course subject to the well-known ambiguities connected with the choice of mass factors that are used to obtain the dimensionless couplings.

D. $B_{1/2^+}\bar{B}_{3/2^-}$ production $[D_{13}(1520)]$

In Ref. 10 a reasonable fit to the results of the analysis of Devenish and Lyth[11] was found for the canonical choice G_1' and G_2' with three poles each. For the mass we take $M = 1.514$ GeV.[11]

The coupling values at $q^2 = 0$ are

$$G_1'^{+}(0)/M = 1.73\ \text{GeV}^{-2},\quad G_1'^{0}(0)/M = -0.008\ \text{GeV}^{-2},$$
$$G_2'^{+}(0) = 1.38\ \text{GeV}^{-2},\quad G_2'^{0}(0) = 0.671\ \text{GeV}^{-2},\qquad (4)$$
$$G_3'^{+}(0) = 0,\quad G_3'^{0}(0) = 0$$

for the charge (+) and (0) members of the isodoublet. The resulting prediction is shown in Fig. 7.

E. $B_{1/2^+}\bar{B}_{5/2^+}$ production $[F_{15}(1688)]$

One has ($M = 1.682$ GeV[11])

$$G_1'^{+}(0)/M = 2.10\ \text{GeV}^{-3},$$
$$G_2'^{+}(0) = 1.14\ \text{GeV}^{-3},\qquad (5)$$
$$G_3'^{+}(0) = 0,$$

and the canonical choice of four poles. The coupling of the neutral isobar state is small (see e.g., Ref. 30). Results are shown in Fig. 7.

F. $B_{1/2^+}\bar{B}_{1/2^+}[P_{11}(1470)]$ and $B_{1/2^+}\bar{B}_{1/2^-}[S_{11}(1535)]$ production

At present a reliable prediction of the production cross sections of these two baryon resonances is rather difficult for the following reasons. Experimentally the q^2 dependences of the respective multipoles in the spacelike region are not determined in a very reliable way. The multipoles of the $P_{11}(1470)$ as determined from the fit procedure of Ref. 11 are quite sensitive to input assumptions and tend to be unstable.[31] With the $S_{11}(1535)$ there are still uncertainties connected with the separation of transverse and longitudinal contributions. Theoretically their form factors cannot be expected to have as simple a form as the transition form factors of the leading baryon resonances since S_{11} and P_{11} occur at the daughter level. One would have to reach a better understanding of the q^2 dependence of these multipoles in the spacelike region before one could continue to the timelike region with much confidence.

Nevertheless we take the results of Ref. 10 and attempt a continuation to the timelike region by using an effective form-factor parametrization of the spacelike data. For the $S_{11}^{+}(1535)$ such an *effective* parametrization is given by

$$G_1'^{+}(0) = -1.31\ \text{GeV}^{-2},\qquad G_2'^{+}(0) = 0.25\ \text{GeV}^{-2}, \tag{6}$$

${m_V}_{\text{eff}}{}^2 = 0.35\ \text{GeV}^2$, $\alpha'_{\text{eff}} = 1.3\ \text{GeV}^{-2}$ with (the canonical) three poles each for $G_1'(q^2)$ and $G_2'(q^2)$. We use $M = 1.505$ GeV.[11] Similarly for the $P_{11}^{+}(1470)$ (we use $M = 1.434$ GeV; see Ref. 11)

$$G_1^{+}(0) = 1.57\ \text{GeV}^{-2},\qquad G_2^{+}(0) = 1.33\ \text{GeV}^{-2}, \tag{7}$$

${m_V}_{\text{eff}}{}^2 = 0.08\ \text{GeV}^2$, $\alpha'_{\text{eff}} = 1.5\ \text{GeV}^{-2}$ and (the canonical) three poles each for $G_1(q^2)$ and $G_2(q^2)$. We stress that the vector mesons appearing in the parametrization of the form factors are meant to represent only *effective* pole parametrizations and we do not mean to imply that such particle singularities exist.

Our predictions are shown in Fig. 7. As emphasized before, these predictions must at present be considered tentative at best.

G. Estimate of three-body production rates

The three-body production processes $e^+e^- \to p\bar{p}\pi^0$ and $e^+e^- \to p\bar{p}\eta$ are estimated by assuming that the two-body subsystems $p\pi^0$, $\bar{p}\pi^0$, etc. result from the decay of one of the resonance states P_{11}, S_{11}, P_{33}, D_{33}, and F_{15}. Using the known branching ratios of these resonances into $p\pi^0$ and $p\eta$ we show our estimates for the production of $p\bar{p}\pi^0$ and $p\bar{p}\eta$ in Fig. 8. We do not give any figures for the other three-body charge states because we lack complete information on the neutral quasi-two-body states. It is difficult to estimate the contribution from nonresonant background and from coherence effects. Figure 8 should therefore be interpreted as giving a lower bound on $p\bar{p}\pi^0$ and $p\bar{p}\eta$ production (see also Ref. 1).

Compared to the rate estimates of Ref. 1 our $p\bar{p}\pi^0$ cross section is down by approximately three orders of magnitude. The large production cross sections of Ref. 1 may have resulted from using too large timelike form factors. We would like to mention that a preliminary upper bound for $p\bar{p}\pi^0$ production at $q^2 = 9\ \text{GeV}^2$ is reported from SPEAR at 0.1 nb.[25]

IV. COMMENTS

Recently several authors discussed the possibility of using the interference of one-photon decay and direct decay of the ψ particles into some given final state to study the SU(3) properties of the ψ decays.[32] Using the results of Sec. III we can estimate the strength of such interference effects.

Let us first consider the decay $\psi \to \Lambda\bar{\Sigma}^0$ which occurs only via the one-photon intermediate state $\psi \to \gamma \to \Lambda\bar{\Sigma}^0$, since the ψ is an isosinglet. From factorization one has

$$\frac{\Gamma(\psi\to\Lambda\Sigma^0)}{\Gamma(\psi\to\mu^+\mu^-)} = \frac{\sigma(e^+e^-\to\gamma\to\Lambda\Sigma^0)}{\sigma(e^+e^-\to\gamma\to\mu^+\mu^-)}\bigg|_{q^2=m_\psi{}^2} = 8.8\times10^{-5}, \tag{8}$$

using the results of Sec. III. Experimentally[33] $\Gamma(\psi\to\Lambda\bar{\Sigma}^0)/\Gamma(\psi\to p\bar{p}) < 0.1$ and $\Gamma(\psi\to p\bar{p})/\Gamma(\psi\to \text{all}) = 0.0023 \pm 0.0006$[34] giving for the above ratio an upper bound of 3.3×10^{-3} which is consistent with our prediction.

In a similar way we can calculate the partial decay rates $\psi \to N\bar{\Delta}$ since these decays also proceed

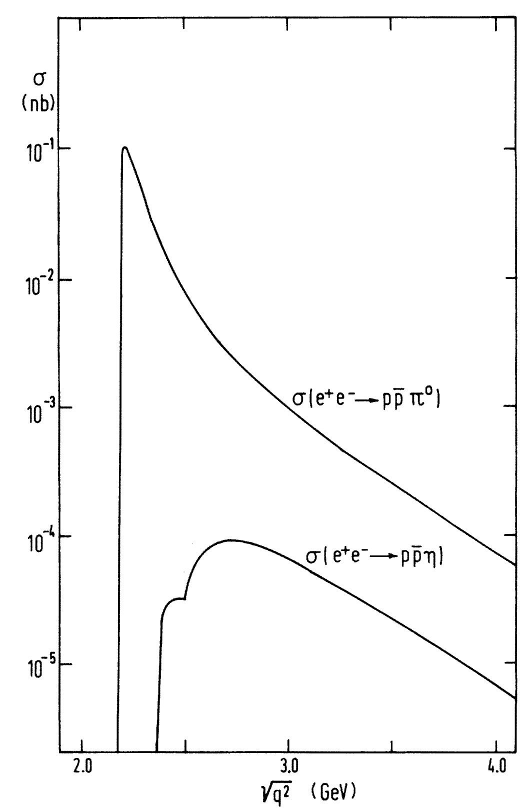

FIG. 8. Three-body cross sections $\sigma(e^+e^- \to p\bar{p}\pi^0, p\bar{p}\eta)$.

via the one-photon intermediate state. Using the results of Sec. III we obtain

$$\Gamma(\psi\to N\bar{\Delta}\to p\bar{p}\pi^0+p\bar{n}\pi^-+n\bar{p}\pi^+)/\Gamma(\psi\to\text{all})=1.2\times10^{-5}.$$

Experimentally one has

$$\Gamma(\psi\to p\bar{p}\pi^0+p\bar{n}\pi^-+n\bar{p}\pi^+)/\Gamma(\psi\to\text{all})=(3.7\pm1.9)\times10^{-3}.$$ [35]

This would imply that a Dalitz-plot analysis of the decay $\psi\to N\bar{N}\pi$ would show a Δ enhancement only at the 0.3% level. In the case $\psi\to p\bar{p}$ both the direct and one-photon decay contribute. One has

$$\frac{\Gamma(\psi\to p\bar{p})}{\Gamma(\psi\to\mu^+\mu^-)}=\frac{\sigma(e^+e^-\to\psi\to\gamma\to p\bar{p})}{\sigma(e^+e^-\to\psi\to\mu^+\mu^-)}+\frac{\sigma(e^+e^-\to\psi\to p\bar{p})}{\sigma(e^+e^-\to\psi\to\mu^+\mu^-)}+\text{interference term.} \quad (9)$$

The first term on the right-hand side of Eq. (9) is evaluated from our $p\bar{p}$ production cross section to be 3.3×10^{-5} ($\alpha'=0.9$), which is negligible compared to the experimental value 0.033 ± 0.010 of the left-hand side.[34] Therefore the direct decay accounts for almost 100% of the $\psi\to p\bar{p}$ rate. Numerically one obtains $\approx3\times10^{-2}$ for the ratio of the one-photon and direct decay amplitudes. This value is so small that it will be rather difficult to establish such an interference effect.

We close by remarking that the measurement of baryon-antibaryon pair production off the ψ resonances is not likely to be an easy experimental task. For example, at $(q^2)^{1/2}\simeq3$ GeV, our calculations indicate that one can only expect production rates of the order of one hundreth of a percent of the total hadronic production rate for any exclusive baryonic channel. Nevertheless, one hopes that enough data on baryon pair production can be collected eventually to enable one to study this potentially rich area of physics.

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APPENDIX

In Refs. 13 and 36 it was shown that in the one-photon approximation the differential cross section for the annihilation into two particles with mass, helicity, and momentum M, λ^*, p^* and m, λ, p, respectively, is given by

$$\frac{d\sigma}{d\cos\theta}=\frac{d\sigma_T}{d\cos\theta}+\frac{d\sigma_L}{d\cos\theta}, \quad (A1)$$

where

$$\frac{d\sigma_T}{d\cos\theta}=2\pi\frac{\alpha^2p_c}{4(q^2)^2(q^2)^{1/2}}\,\tfrac{1}{2}\sum_\lambda\left(|\Gamma^{\lambda+1,\lambda}|^2+|\Gamma^{\lambda-1,\lambda}|^2\right)\times(1+\cos^2\theta),$$
$$\frac{d\sigma_L}{d\cos\theta}=2\pi\frac{\alpha^2p_c}{4(q^2)^2(q^2)^{1/2}}\sum_\lambda|\Gamma^{\lambda,\lambda}|^2\sin^2\theta. \quad (A2)$$

The $\Gamma^{\lambda^*,\lambda}$ are the c.m. helicity amplitudes with helicities λ and λ^*, and p_c is the magnitude of the c.m. momentum. The total cross sections σ_T and σ_L are obtained from (A2) by $\cos\theta$ integration.

1. $B_{1/2^+}\bar{B}_{1/2^+}$; equal-mass case $M=m$

One defines constraint-free gauge-invariant form factors by

$$\langle N(p^*,\lambda^*)\bar{N}(p,\lambda)|J_\mu(0)|0\rangle=e\bar{u}(p^*,\lambda^*)\left(F_1\gamma_\mu+F_2\frac{i\sigma_{\mu\nu}q^\nu}{2m}\right)v(p,\lambda), \quad (A3)$$

where $q=p^*+p$. The helicity amplitudes are given by

$$\Gamma^{1/2,1/2}=-2m(F_1+\eta F_2)=-2mG_E,$$
$$\Gamma^{1/2,-1/2}=(2q^2)^{1/2}(F_1+F_2)=(2q^2)^{1/2}G_M, \quad (A4)$$

where $\eta=q^2/4m^2$. Here and in the following cases we shall be listing only the independent helicity amplitudes. The other helicity amplitudes entering in the cross-section formula Eq. (A2) can be obtained from parity and charge-conjugation invariance (see Refs. 13 and 36). The differential cross section is given by

$$\frac{d\sigma}{d\cos\theta}=\frac{4\pi\alpha^2p_c m^2}{(q^2)^2(q^2)^{1/2}}\left[\sin^2\theta|G_E|^2+\eta(1+\cos^2\theta)|G_M|^2\right]. \quad (A5)$$

2. $B_{1/2^+}\bar{B}_{1/2^+}$; unequal-mass case

One defines constraint-free amplitudes by (see Ref. 10)

$$\langle N^*(p^*,\lambda^*)\bar{N}(p,\lambda)|J_\mu(0)|0\rangle=e\bar{u}(p^*,\lambda^*)\Gamma_\mu v(p,\lambda),$$
$$\Gamma_\mu=F_1'(q^2\gamma_\mu-\not{q}q_\mu)+F_2'(P\cdot q\gamma_\mu-P_\mu\not{q}), \quad (A6)$$

where $P=\frac{1}{2}(p^*-p)$. The transverse and longitudinal helicity amplitudes are

$$\Gamma^{1/2,-1/2}=(\tfrac{1}{2})^{1/2}(Q^-)^{1/2}(2q^2F_1'+\sigma F_2'),$$
$$\Gamma^{1/2,1/2}=-\tfrac{1}{2}(Q^-)^{1/2}(q^2)^{1/2}[2(M+m)F_1'+(M-m)F_2'], \quad (A7)$$

where we have introduced the abbreviations $Q^\pm=q^2-(M\pm m)^2$ and $\sigma=M^2-m^2$. We caution the reader that some care has to be exercised to take the equal-mass limit $M=m$ of Eq. (A7).

3. $B_{1/2}{}^{+}\bar{B}_{1/2}{}^{-}$

One defines corresponding unprimed amplitudes by multiplying Γ_μ by $i\gamma_5$ from the left:

$$\Gamma_\mu = i\gamma_5 F_1(q^2\gamma_\mu - \not{q} q_\mu) + i\gamma_5 F_2(P\cdot q\gamma_\mu - P_\mu \not{q}) . \quad \text{(A8)}$$

The corresponding helicity amplitudes are given by the replacement $F_i \to F_i'$ and $M \leftrightarrow -M$ in Eq. (A7).

4. $B_{3/2}{}^{+}\bar{B}_{3/2}{}^{+}$

Constraint-free gauge-invariant form factors are defined by

$$\langle N_{3/2^+}(p^*, \lambda^*)\bar{N}_{3/2^+}(p, \lambda)|J_\mu(0)|0\rangle = e\bar{u}^\alpha(p^*, \lambda^*)\Gamma_{\alpha\beta\mu} v^\beta(p, \lambda) , \quad \text{(A9)}$$

with

$$\Gamma_{\alpha\beta\mu} = g_{\alpha\beta}\left(F_1\gamma_\mu + F_2\frac{i\sigma_{\mu\nu}q_\nu}{2M}\right) + \frac{q_\alpha q_\beta}{2M^2}\left(F_3\gamma_\mu + F_4\frac{i\sigma_{\mu\nu}q_\nu}{2M}\right) . \quad \text{(A10)}$$

The four independent helicity amplitudes are given by

$$\begin{aligned}
\Gamma^{3/2,1/2} &= (\tfrac{2}{3})^{1/2}(q^2)^{1/2}(F_1+F_2) ,\\
\Gamma^{1/2,-1/2} &= \tfrac{2}{3}\sqrt{2}\,(q^2)^{1/2}[-(1-2\eta)(F_1+F_2) - 2\eta(1-\eta)(F_3+F_4)] ,\\
\Gamma^{3/2,3/2} &= -2M(F_1+\eta F_2)\\
\Gamma^{1/2,1/2} &= (1-\tfrac{4}{3}\eta)2M(F_1+\eta F_2) + \tfrac{4}{3}\eta(1-\eta)2M(F_3+\eta F_4) .
\end{aligned} \quad \text{(A11)}$$

It is often convenient to work with multipole amplitudes[37] which are given by

$$\begin{aligned}
G_E &= \frac{1}{4M}(-\Gamma^{3/2,3/2} + \Gamma^{1/2,1/2}) ,\\
G_M &= \frac{3}{5\sqrt{2}}\frac{1}{(q^2)^{1/2}}(\sqrt{3}\,\Gamma^{3/2,1/2} - \Gamma^{1/2,-1/2}) ,\\
G_Q &= -\frac{3}{2\eta}\frac{1}{4M}(\Gamma^{3/2,3/2} + \Gamma^{1/2,1/2}) ,\\
G_O &= \frac{1}{4\eta}\frac{1}{(q^2)^{1/2}}\left(\Gamma^{3/2,1/2} + \frac{\sqrt{3}}{2}\Gamma^{1/2,-1/2}\right),
\end{aligned} \quad \text{(A12)}$$

and which, at $q^2 = 0$, are normalized to the charge (in units of e), magnetic dipole moment (in units of $e/2M$), electric quadrupole moment (in units of e/M^2), and magnetic octupole moment (in units of $e/2M^3$).

In terms of the multipole amplitudes one has for the differential cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{4\pi\alpha^2 p_c}{(q^2)^2}\frac{M^2}{(q^2)^{1/2}} \times [\tfrac{2}{3}\eta(\tfrac{5}{3}|G_M|^2 + \tfrac{48}{5}\eta^2|G_O|^2)(1+\cos^2\theta) + (2|G_E|^2 + \tfrac{8}{9}\eta^2|G_Q|^2)\sin^2\theta] . \quad \text{(A13)}$$

5. $B_{1/2}{}^{+}\bar{B}_{3/2}{}^{-}$

The constraint-free amplitudes are defined by

$$\langle N^*(p^*, \lambda^*)\bar{N}(p, \lambda)|J_\mu(0)|0\rangle = e\bar{u}^\beta(p^*, \lambda^*)\Gamma_{\beta\mu} v(p, \lambda), \quad \text{(A14)}$$

$$\Gamma_{\beta\mu} = -G_1'(q_\beta\gamma_\mu - \not{q} g_{\beta\mu}) + G_2'(q_\beta p_\mu^* - p^* q g_{\beta\mu}) + G_3'(q_\beta q_\mu - q^2 g_{\beta\mu}) , \quad \text{(A15)}$$

and the helicity amplitudes are given by

$$\Gamma^{1/2,-1/2} = -\frac{p_c(q^2)^{1/2}}{(Q^-)^{1/2}}(\tfrac{1}{3})^{1/2}\left\{-[q^2 + m(M-m)]\frac{2G_1'}{M} + (\sigma+q^2)G_2' + 2q^2G_3'\right\}, \quad \text{(A16a)}$$

$$\Gamma^{3/2,1/2} = \frac{p_c(q^2)^{1/2}}{(Q^-)^{1/2}}\left[-2M(M-m)\frac{G_1'}{M} + (\sigma+q^2)G_2' + 2q^2G_3'\right], \quad \text{(A16b)}$$

$$\Gamma^{1/2,1/2} = -\sqrt{q^2}\,\frac{p_c(q^2)^{1/2}}{\sqrt{Q^-}}(\tfrac{2}{3})^{1/2} \times \left[2M\frac{G_1'}{M} - 2MG_2' - \frac{1}{M}(\sigma+q^2)G_3'\right] . \quad \text{(A16c)}$$

6. $\bar{B}_{1/2}{}^{+}B_{3/2}{}^{+}$

The corresponding unprimed invariants G_1, G_2, and G_3 are obtained by multiplying $\Gamma_{\beta\mu}$ in (A15) by $i\gamma_5$ from the left. The helicity amplitudes are given by the replacement $G_i' \to G_i$ and $M \leftrightarrow -M$ in Eq. (A16).

7. $\bar{B}_{1/2}{}^{+}B_{5/2}{}^{+}$

One has

$$\langle N^*(p^*, \lambda^*)\bar{N}(p, \lambda)|J_\mu(0)|0\rangle = e\bar{u}^{\beta_1\beta}(p^*, \lambda^*)q_{\beta_1}\Gamma_{\beta\mu} v(p, \lambda)$$

and defines three invariants $G_1'^{(5/2)}$, $G_2'^{(5/2)}$, and $G_3'^{(5/2)}$ in analogy to (A15). The helicity amplitudes are obtained from (A16) by multiplying the right-hand side of (A16a) and (A16c) by $(\tfrac{3}{5})^{1/2}p_c(q^2)^{1/2}/M$ and (A16b) by $(\tfrac{2}{5})^{1/2}p_c(q^2)^{1/2}/M$.

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