# ELECTROMAGNETIC MASS DIFFERENCES OF THE OLD AND THE CHARMED MESONS

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We predict for $M_{\rho^+} - M_{\rho^0}$ the values $-3.4 \pm 0.8$ MeV and $-3.8 \pm 1.2$ MeV using the $\rho$-$\omega$ mixing and the quark model, respectively. The extracted parameters indicate the necessity of a relativistic treatment of the old mesons. The problem of extrapolating these parameters to the charmed mesons is discussed. Under conservative assumptions, we predict $1.7 \leqslant M_{D^+} - M_{D^0} \leqslant 2.2$ MeV and $-1.4 \pm 1.1 \leqslant M_{D^{*+}} - M_{D^{*0}} \leqslant 0.0 \pm 0.6$ MeV.

Recent contradictory predictions [1] for $M_{D^+} - M_{D^0}$, $M_{D^{*+}} - M_{D^{*0}}$ have stimulated us to study the electromagnetic mass differences of the old [2] and the new [3] mesons. We first reinvestigated $\pi$, $\rho$, K and K* in order to derive *consistent* parameters, which can then be extrapolated to the charmed mesons. Thereby we found that we can successfully predict $M_{\rho^+} - M_{\rho^0}$ using two completely different methods, one based on SU(3) and $\rho^0$-$\omega$ mixing, and the other based on the quark model. If one believes the errors for $\Delta_\pi$, $\Delta_K$, $\Delta_{K^*}$ ($\Delta_A \equiv M_{A^+} - M_{A^0}$; $\delta_A \equiv M^2_{A^+} - M^2_{A^0}$) and $\Gamma(\omega \to 2\pi)$, as quoted by the Particle Data Group [4], then our predictions for $\Delta\rho$ have much smaller error bars than the experimental value [4] for $\Delta\rho$.

For the $\rho^0$ mass, we have

$$M^2_{\rho^0} = M^2_{\rho^+} + \delta^{\text{ann}}_{\rho^0} - \delta^{\text{n.a.}}_{\rho} + \delta^{\text{mix}}_{\rho^0\omega}\,, \tag{1}$$

where $\delta^{\text{ann}}_{\rho^0}$ is the e.m. mass shift due to the one-photon exchange in the $s$-channel (fig. 1a), $\delta^{\text{n.a.}}_{\rho}$ is due to one photon plus hadrons, the Cottingham contribution (fig. 1b), and $\delta^{\text{mix}}_{\rho^0\omega}$ is due to $\rho^0$-$\omega$ mixing (fig. 1c). $\delta^{\text{ann.}}_{\rho^0}$ is related to the leptonic decay width of the $\rho^0$:

$$\delta^{\text{ann.}}_{\rho^0} = \operatorname{Re}\frac{W^2_{\rho^0\gamma}}{Z_\rho} = \frac{3}{\alpha} M_\rho \Gamma(\rho^0 \to e^+e^-) \\ = 2.07 \pm 0.24 \text{ MeV GeV}\,, \tag{2}$$

with $Z_V \equiv M^2_V - iM_V\Gamma_V$.

The two methods differ in the way they estimate $\delta^{\text{n.a.}}_{\rho}$.

i) SU(3) gives[‡1] $\delta^{\text{n.a.}}_{\rho} = \delta_{K^*} - \operatorname{Re} W^{\text{n.a.}}_{\rho^0\omega}$, where $W^{\text{n.a.}}_{\rho^0\omega}$ is the nonannihilation part of the $\rho^0$-$\omega$ mixing [5][‡2], $W_{\rho^0\omega} \equiv W^{\text{n.a.}}_{\rho^0\omega} + W^{\text{ann.}}_{\rho^0\omega} \approx W^{\text{n.a.}}_{\rho^0\omega} + \delta^{\text{ann.}}_{\rho^0}/3$.

$W_{\rho^0\omega}$ is related to the coupling constant $g_{\omega\pi\pi}$ (fig. 1g).

$$W_{\rho^0\omega} = (z_\omega - z_\rho) g_{\omega\pi\pi}/g_{\rho\pi\pi} \\ = |z_\omega - z_\rho| \exp[i(\phi_Z + \phi)] \sqrt{\Gamma(\omega \to 2\pi)/\Gamma(\rho \to 2\pi)}\,, \tag{3}$$

where $\phi_Z = \operatorname{arctg}(\Gamma_\rho - \Gamma_\omega)/2(M_\omega - M_\rho) \approx 80°$.

Experimentally one finds [6] $\phi = 85° \pm 15°$ so that $W_{\rho^0\omega}$ is mainly real and negative. $\delta^{\text{mix}}_{\rho^0\omega} = \operatorname{Re}(W^2_{\rho\omega}/(z_\rho - z_\omega))$ is estimated to be less than 100 keV and will be neglected. Therefore we obtain

‡1 This relation follows from $W_{\rho^0\rho^0} - W_{\rho^+\rho^+} + W_{K^{*+}K^{*+}} - W_{K^{*0}K^{*0}} = \sqrt{3}\,W_{\rho^0\omega_8} \approx W_{\rho^0\omega} + \sqrt{2}\,W_{\rho^0\phi}$, which holds separately for the annihilation part $W^{\text{ann.}}_{AB}$ and the non-annihilation part $W^{\text{n.a.}}_{AB}$ of $W_{AB}$, and from $W^{\text{n.a.}}_{\rho^0\phi} \approx 0$ (Zweig rule).

‡2 We have found that for a *small* mixing angle, the two-channel mixing formalism can be translated into a very simple diagramatic language indicated in figs. 1f,g with the following rules: i) Although the physical states $|A\rangle$ and $|B\rangle$ are orthogonal, $\langle A|B\rangle = 0$, they should be treated in these diagrams as if they were coupled together, by an "effective coupling constant" $W_{A^0B^0}$, which is equal to the off-diagonal element of the mass matrix $W$ between the original (before mixing) states $|A^0\rangle$ and $|B^0\rangle$. ii) For an intermediate state B, a propagator $1/(s - z_B)$, with $z_B = M^2_B - i\Gamma_B M_B$, should be used, as usual. iii) However, for $s$ one should substitute $z_A$ and not $M^2_A$, for an external particle A. For example, $\delta^{\text{mix}}_{\rho\omega}$ is mainly imaginary, due to the $\rho$-width in $W^2_{\rho\omega}/(z_\rho - z_\omega)$ (fig. 1c). The above equivalence follows, since the mixing angle for $|W_{A^0B^0}| \ll |W_{A^0A^0} - W_{B^0B^0}|$ is given by $\epsilon = \langle B^0|A\rangle \approx W_{A^0B^0}/(W_{A^0A^0} - W_{B^0B^0}) = W_{A^0B^0}/(z_{A^0} - z_{B^0}) \approx W_{A^0B^0}/(z_A - z_B)$.

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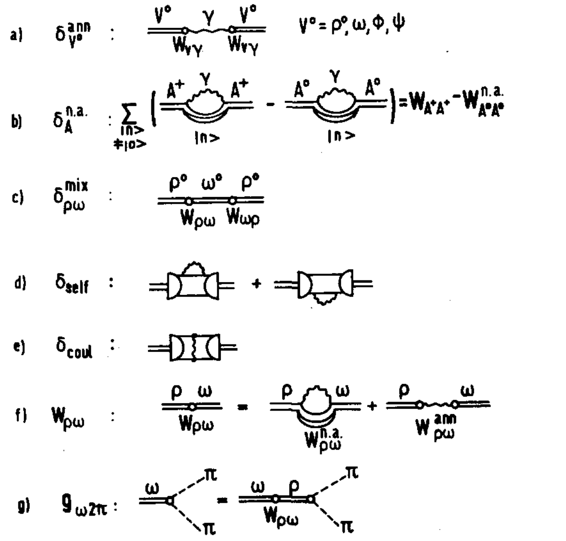


Fig. 1.

$$\delta_\rho \approx \delta_{K^*} - \cos(\phi_z + \phi) M_\rho \sqrt{\Gamma_\rho \Gamma_\omega B(\omega \to \pi\pi)} - \tfrac{2}{3}\delta_{\rho^0}^{\mathrm{ann.}} . \tag{4}$$

Eq. (4) leads to two sets of predictions given in table 1, which follow from using the world average for $\Delta_{K^*}^{\mathrm{av}} \equiv \langle M_{K^{*+}} \rangle - \langle M_{K^{*0}} \rangle$ or $\Delta_{K^*}^{\mathrm{dir}}$ from direct fits [4], i.e. using data from the *same* experiment.

ii) The quark model parametrization of the *quadratic* mass differences

$$\begin{aligned} &\delta_\pi = \tfrac{1}{2}\delta^{\mathrm{P}}, && \delta_{\mathrm{K}} = \tfrac{1}{3}\delta^{\mathrm{P}} + \delta_{\mathrm{ud}}, \\ &\delta_\rho = \tfrac{1}{2}\delta^{\mathrm{V}} - \delta_{\rho^0}^{\mathrm{ann.}}, && \delta_{\mathrm{K}^*} = \tfrac{1}{3}\delta^{\mathrm{V}} + \delta_{\mathrm{ud}}, \end{aligned} \tag{5}$$

where $\delta^{\mathrm{P}}, \delta^{\mathrm{V}}$ correspond to quark-antiquark diagrams (fig. 1e) for the pseudoscalar and vector mesons, and where $\delta_{\mathrm{ud}}$ corresponds to the quark self energy indicated in fig. 1d, gives

$$\delta_\rho = \tfrac{3}{2}(\delta_{\mathrm{K}^*} - \delta_{\mathrm{K}}) + \delta_\pi - \delta_{\rho^0}^{\mathrm{ann.}} . \tag{6}$$

The prediction from this formula (table 1) is in agreement with experiment and with our first prediction.

A parametrization similar to (5), with $\Delta^{\mathrm{P,V}}$, $\Delta_{\mathrm{ud}}$ for the *linear* masss differences $\Delta_{\mathrm{A}}$, would give

$$\Delta_\rho = \tfrac{3}{2}(\Delta_{\mathrm{K}^*} - \Delta_{\mathrm{K}}) + \Delta_\pi - \Delta_{\rho^0}^{\mathrm{ann.}} . \tag{7}$$

This equation leads to $\Delta_\rho = (3.1 \pm 1.0, 0.5 \pm 2.4)$ MeV for $\Delta_{\mathrm{K}^*} = (-4.1 \pm 0.6, -5.8 \pm 1.6)$ MeV, in disagreement with experiment. Eq. (5) gives the values

$$\begin{aligned} &\delta^{\mathrm{P}} = 2\delta_\pi = 2.52\ \mathrm{MeV\,GeV}, \\ &\delta_{\mathrm{ud}} = \delta_{\mathrm{K}} - \tfrac{2}{3}\delta_\pi = -4.79 \pm 0.13\ \mathrm{MeV\,GeV}, \\ &\delta^{\mathrm{V}} = 3(\delta_{\mathrm{K}^*} - \delta_{\mathrm{K}}) + 2\delta_\pi \\ &\quad = \begin{cases} \delta_{\mathrm{I}}^{\mathrm{V}} = -7.6 \pm 3.2\ \mathrm{MeV\,GeV\ from}\ \Delta_{\mathrm{K}^*}^{\mathrm{av}} \\ \delta_{\mathrm{II}}^{\mathrm{V}} = -16.7 \pm 8.6\ \mathrm{MeV\,GeV\ from}\ \Delta_{\mathrm{K}^*}^{\mathrm{dir}} \end{cases}, \\ &\delta^{\mathrm{V}} = 2(\delta_\rho + \delta_{\rho^0}^{\mathrm{ann.}}) \\ &\quad = \delta_{\mathrm{III}}^{\mathrm{V}} = -9.1 \pm 7.4\ \mathrm{MeV\,GeV\ from}\ \Delta_\rho^{\mathrm{av}}, \end{aligned} \tag{8}$$

and leads to the ratio

$$\begin{aligned} &(\delta_{\mathrm{I}}^{\mathrm{V}}/\delta^{\mathrm{P}}, \delta_{\mathrm{II}}^{\mathrm{V}}/\delta^{\mathrm{P}}, \delta_{\mathrm{III}}^{\mathrm{V}}/\delta^{\mathrm{P}}) \\ &\quad = (-3.0 \pm 1.3, -6.6 \pm 3.4, -3.6 \pm 2.9) . \end{aligned} \tag{9}$$

Since all three $\delta^{\mathrm{V}}/\delta^{\mathrm{P}}$ are consistent with each other, it seems very unlikely that the negative large values of the ratio are accidental. This result cannot be understood in the usual non-relativistic theory: in the non-relativistic limit, the linear parameters $\Delta^{\mathrm{P}}$ and $\Delta^{\mathrm{V}}$ are given by $\Delta_{\mathrm{el}} + 3\Delta_{\mathrm{mag}}$ and $\Delta_{\mathrm{el}} - \Delta_{\mathrm{mag}}$ respectively, so that

$$-1/3 \leqslant \Delta_{\mathrm{n.r.}}^{\mathrm{V}}/\Delta_{\mathrm{n.r.}}^{\mathrm{P}} \leqslant 1 , \tag{10}$$

where the lower limit is only achieved in the extreme unlikely case of $\Delta_{\mathrm{el}} = 0$. However, we do not find the ratios (9) disappointing or bad. On the contrary! We believe that the result (9) is very significant: it shows clearly that, either the old mesons cannot be described at all as a bound state of a quark and an antiquark, or that they are truely relativistic $q\bar{q}$ systems, which cannot be approximated by non-relativistic dynamics. This conclusion is also supported by the fact that the quark model is successful for quadratic mass differences but fails for linear mass differences.

Therefore, it is encouraging that there already exists a relativistic model [7] which does give a ratio $\delta^{\mathrm{V}}/\delta^{\mathrm{P}} = -1$ [8]. This model is based on a Bethe–Salpeter description of strongly bound heavy quarks. If the strong interaction is approximated by a four-dimensional harmonic oscillator, one obtains the following meson mass spectrum [7,9]

$$M_{l,n,r}^2(q_i, \bar{q}_j) = M_0^2 + 4m_{\mathrm{u}}(\Delta_i + \Delta_j) + (n + 2r)/\alpha' , \tag{11}$$

where $n, r = 0, 1, 2, \ldots, l = n, n-2, \ldots 1$ or $0$, $m_{\mathrm{u}}$ = mass of the heavy up-quark, $\Delta_i \equiv m_{q_i} - m_{\mathrm{u}}$, $\alpha'$ = Regge slope ($\alpha'_\rho \sim 1.0\ \mathrm{GeV}^{-2}$). The leading term of the Wick rotated

Table 1
Prediction for $M_{\rho^+} - M_{\rho^0}$. These calues are to be compared with the experimental values [4], $\Delta_\rho^{\mathrm{av}} = -4.3 \pm 2.4$ MeV, based on world average, and $\Delta_\rho^{\mathrm{ed.g.}} = -7.1 \pm 3.8$ MeV, based on an educated guess [4] of $773 \pm 3$ MeV for $M_{\rho^0}$.

| Model | $\Delta_\rho^{\mathrm{pred}}$ (MeV) (Input: $\Delta_{\mathrm{K}^*}^{\mathrm{av}} = -4.1 \pm 0.6$ MeV) | $\Delta_\rho^{\mathrm{pred}}$ (MeV) (Input: $\Delta_{\mathrm{K}^*}^{\mathrm{dir}} = -5.8 \pm 1.6$ MeV) |
|---|---|---|
| i) $\rho^0$-$\omega$ mixing | | |
| $0.01 \leqslant B(\omega \to 2\pi) \leqslant 0.04$ | $-3.7 \pm 0.8 \leqslant \Delta\rho \leqslant -1.8 \pm 0.8$ | $-5.7 \pm 1.9 \leqslant \Delta\rho \leqslant -3.7 \pm 1.9$ |
| $B^{\mathrm{world}}_{(\omega\to 2\pi)} = 0.013 \pm 0.003$ | $-3.4 \pm 0.8$ | $-5.4 \pm 1.9$ |
| ii) Quark model | $-3.8 \pm 1.2$ | $-6.8 \pm 2.8$ |

Bethe–Salpeter wave functions of the ground state pseudoscalar and vector mesons is:

$$\chi^{\mathrm{P}}_{0,0,0}(q, P) = 32\pi\alpha'(\gamma_5\gamma^\mu P_\mu/M)\exp(-4\alpha' q^2),$$
$$\chi^{\mathrm{V}}_{0,0,0}(q, P, s_3) = 32\pi\alpha'\gamma^\mu\epsilon_\mu^{s_3}(P)\exp(-4\alpha' q^2). \tag{12}$$

Calculating the diagram fig. 1e with these wave functions gives [10, 8]

$$\delta^{\mathrm{P}} = -\delta^{\mathrm{V}} = \frac{\alpha}{\pi}\alpha'^{-1} \approx 2.3\ \mathrm{MeV\,GeV}. \tag{13}$$

It is tempting to extend our parametrization to the charmed mesons D and D*:

$$\delta_{\mathrm{D}} = \tfrac{2}{3}\delta^{\mathrm{P}}(\mathrm{D}) - \delta_{\mu d}(\mathrm{D});\quad \delta_{\mathrm{D}^*} = \tfrac{2}{3}\delta^{\mathrm{V}}(\mathrm{D}) - \delta_{\mu d}(\mathrm{D}). \tag{14}$$

The problem is now how to extrapolate our parameters from the old mesons to the charmed mesons. Let us consider a subset of possible extrapolations:

$$\delta_{\mu d}(\mathrm{D}) = \lambda_1\delta_{\mu d}(\mathrm{old}),\quad \delta^{\mathrm{P,V}}(\mathrm{D}) = \lambda_2\delta^{\mathrm{P,V}}(\mathrm{old}).$$

a) The simplest assumption would be the universality of all parameters for the old and the new mesons, i.e. $\lambda_1 = \lambda_2 = 1$, so that

$$\delta_{\mathrm{D}} = -\delta_{\mathrm{K}} + 2\delta_\pi,\quad \delta_{\mathrm{D}^*} = 2\delta_{\mathrm{K}^*} - 3\delta_{\mathrm{K}} + 2\delta_\pi. \tag{15a, b}$$

Eq. (15a) was also derived by Fritzsch [1] using strong PCAC.

b) The assumption $\lambda_1 = 1$ and $\lambda_2 > 1$ is suggested by (11) and (13) where $\delta_{\mu d}$ remains unchanged [‡3] and where $\lambda_2$ would correspond to the ratio of the Regge slopes $\lambda_2 = \alpha'_\rho/\alpha'_{\mathrm{D}}$. Since D and D* are made up of a charmed quark and an ordinary quark, one would expect $\alpha'_\rho \geqslant \alpha'_{\mathrm{D}} \geqslant \alpha'_\psi \sim 0.5\,\alpha'_\rho$ (the assumption $\lambda_2 > 1$ also corresponds to the common belief that the charmed mesons have a smaller size), and we obtain

$$1.7\ \mathrm{MeV} \leqslant M_{\mathrm{D}^+} - M_{\mathrm{D}^0} \leqslant 2.2\ \mathrm{MeV},\quad \text{with } M_{\mathrm{D}} = 1.87\ \mathrm{GeV},$$

$$-1.4 \pm 1.1\ \mathrm{MeV} \leqslant M_{\mathrm{D}^{*+}} - M_{\mathrm{D}^{*0}} \leqslant \pm 0.0 \pm 0.6\ \mathrm{MeV},$$

if $M_{\mathrm{D}^*} \sim 2$ GeV and with $\delta_{\mathrm{I}}^{\mathrm{V}}$ where the lower (upper) limit for $\Delta_{\mathrm{D}}$ ($\Delta_{\mathrm{D}^*}$) corresponds to the universality assumption a). This estimate is to be compared with the present experimental value $\Delta_{\mathrm{D}} = 11 \pm 11$ MeV [11].

c) We may have $\lambda_1, \lambda_2 > 1$ in models which are more general.

Naturally there exist many more possibilities. For example, in all the above cases we assumed the universality of the ratio $\delta^{\mathrm{V}}/\delta^{\mathrm{P}}(\mathrm{D}) = \delta^{\mathrm{V}}/\delta^{\mathrm{P}}$ (old). But this ratio would change drastically from $-3$ to $+1$, *if* the charmed mesons were non-relativistic systems. In this case one would get $\Delta_{\mathrm{D}^*} \approx \Delta_{\mathrm{D}}$. But then, there would be no point to use parameters derived from the old mesons: the charmed mesons would be dynamically completely different from the old mesons, which – as we have clearly shown – are truely relativistic systems.

[‡3] If one uses for the baryons (N, Σ, Ξ) a parametrization similar to (5), but based on *nonrelativistic* SU(6) spin wave functions of the baryons, one obtains: $\delta_{\mu d}(\mathrm{B}) = -3.85$ MeV GeV, $\delta^0(\mathrm{B}) = 24.0$ MeV GeV, $\delta^1(\mathrm{B}) = 4.3$ MeV GeV, where $\delta^{0,1}$ denotes the q-q interaction in the singlet and triplet states. It is interesting to note, that $\delta_{\mu d}(\mathrm{B}) \approx \delta_{\mu d}(\mathrm{M})$, but (!) $\delta^{0,1}(\mathrm{B})$ are completely different from $\delta^{\mathrm{P,V}}(\mathrm{M})$.

## References

[1] A. De Rújula, H. Georgi, S.L. Glashow, Harvard preprint 6/76;
K. Lane, E. Eichten, CLNS-335, C00-2220-78 (1976);
H. Fritzsch, Ref. TH. 2191 – CERN (1976).
[2] W. Thirring, Acta Phys. Austr. Suppl. II (1966) 205;
G. Barton, D. Dare, Nuovo Cim. 46A (1966) 433;
H.R. Rubinstein, F. Scheck, R.H. Socolow, Phys. Rev. 154 (1967) 1608;

A. Gal, F. Scheck, Nucl. Phys. B2 (1967) 110;
S. Ishida, K. Konno, P. Roman, H. Shimodaira, Nucl. Phys. Phys. B2 (1967) 307;
J. Körner, Phys. Rev. 170 (1968) 1401.

[3] C. Itoh, T. Minamikawa, K. Miura, T. Watanabe, Progr. Theor. Phys. 54 (1975) 908;
D.B. Lichtenberg, Phys. Rev. D12 (1975) 3760.

[4] Particle Data Group, Rev. Mod. Phys. 48 (1976) No. 2, Part II.

[5] For reviews see, e.g.: M. Gourdin, in Hadronic interactions of electrons and photons, eds. J. Cumming, H. Osborn (Academic Press, London-New York, 1971) p. 395;
F.M. Renard, Springer Tracts in Modern Physics, ed. G. Höhler, 63 (1972) 98.

[6] R. Marshall, in: Meson resonances and related electromagnetic phenomena, eds. R.H. Dalitz, A. Zichichi (Editrice Compositori, Bologna, 1972) p. 101;
J. Daboul, in preparation.

[7] M. Böhm, H. Joos, M. Krammer, in Recent developments in mathematical physics, ed. P. Urban (Springer Verlag, Wien-New York, 1973) p. 3; Ref. TH 1949 – CERN (1974).

[8] W. Bauhoff, Phys. Lett. 62B (1976) 200.

[9] H. Krasemann, DESY 76/25 (1976).

[10] C.O. Escobar, Lett. al Nuovo Cim. 10 (1974) 741.

[11] R. Schwitters, Conference Tbilisi, July 1976.