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ACCELERATION OF ELECTRONS IN THE FOCUS OF A LASER-BEAM

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A new concept in accelerating electrons by a high power laser beam is discussed The acceleration takes place in the field patterns produced by the superposition of four waves in the focus of a lens The matching of the particle velocity is done by a birefringent crystal outside the focus

1. Introduction

Since several years there have been discussions on how to use the high electrical fields of laser beams for accelerating electrons The problem is to find electromagnetic field configurations in which the electron is accelerated Several approaches to that problem were tried. There exist two schools of thinking on how to use the electric field of a laser beam for acceleration

a) Use of electromagnetic fields near the boundary of a dielectric material The ideas were influenced by experiments done by Smith and Purcell in 1953[1]) The authors discovered that electrons travelling along an optical grating emit radiation The wavelength of this radiation depends on the energy of the electrons and the angle between electron trajectory and the light ray A general theory of these effects was given by Di Francia[2]) Di Francia pointed out that waves exist with phase velocities less than c, which he called evanescent waves, near the surface of an optical grating under certain circumstances These waves exist only within a distance of one wavelength from the surface of the grating
Several authors discussed how to use the inversion of the Smith–Purcell effect for acceleration particles[3–5]) Takeda and Matsui[3]) started experiments but failed in demonstrating this acceleration principle[6]) The problems in the Matsui and Takeda suggestion were the matching of phase and particle velocity and the complicated field configurations with both evanescent and travelling waves

b) The second set of suggestions is based on a paper by McMillan in 1950[7]) McMillan discussed the generation of cosmic rays and studied the acceleration of particles in the interstellar electromagnetic fields The idea is very simple: an electron is injected perpendicular to a travelling wave Since the field of the travelling wave is of TEM type, the electric field accelerates the particles and the magnetic field bends the particle towards the propagation direction of the wave Thus the particle is accelerated not only by one half-wave of the electromagnetic field but is riding on the wave gaining energy until it is out of phase When the particle is out of phase, deceleration begins Several papers discuss these proposals[8–14]) Chiao and Feldman[15]) discussed this acceleration mechanism for high power lasers and calculated that under certain circumstances a net energy gain of the electron is possible when the electron passes through the laser focus But it seems that these calculations are too optimistic for real laser beams Beside these two main schools there exists a paper by Shimoda proposing the use of a cavity similar to an rf cavity[17]) Both schools of thinking come to results which are almost impracticable

In this paper a new concept is discussed which seems to be more realistic than previous proposals. The suggestion is based on the use of standing waves instead of travelling waves superposed in the focus of a lens In standing waves the electric and the magnetic fields are separated by a quarter wavelength Matching between waves and particle velocity is done far away from the interaction point by a birefringent crystal, so that problems with dimensions in the order of one wavelength of proposal (a) do not exist anymore

The accelerating of electrons by optical fields is interesting for the following reasons conventional linear accelerarators accelerate particles by using TM-fields in the microwave frequency range The TM-fields are generated in metallic wave guides surrounding the beam The maximum field strength is limited by electrical breakdowns in the vacuum chamber Conventional linacs work between 7 and 15 MV/m In a laser focus, 2 GV/cm is achieved, but it seems to be difficult to accelerate particles in this field. The main interest in this article is in the search for field configurations and not in calculating maximum currents and related topics This article describes the principal direction in which to proceed and not the ultimate solution of this problem

2. Field configuration in the focus of a laser beam

The idea to use the field in the focus of a lens was first conceived by Boivin and Wolf[16]) The authors discussed the field configurations in a focus under several conditions and pointed out that some of these configurations must be able to accelerate charged particles Theoretical research on fields in the focus of a laser beam were done by several authors[18–20]), but the basic idea of Boivin and Wolf of particle acceleration was not discussed anymore

Independently from each other, Csonka[21]) and the author[22]) proposed to use multibeam interference patterns for particle acceleration In this paper both ideas are combined and it is shown that this proposal leads to a practicable technical solution of the problem

The diameter of a focus is given by

$$D = 2f \operatorname{tg}\phi , \tag{1}$$

where ϕ is the beam divergence This formula holds when the beam diameter is much more smaller than the lens diameter and errors of the lens can be neglected With the beam diameter small compared to the diameter of the lens, this condition is always fulfilled In the words of geometrical optics in the focus rays with identical inclination are superposed on one point Assuming two linearly polarized beams [one coming from the right-hand side and the other from the left-hand side (fig. 1)] in the focus, standing waves are produced with electric and magnetic fields given by

$$\begin{aligned} E &= E_0 \cos \omega t \sin K_z , \\ H &= H_0 \sin \omega t \cos K_z , \end{aligned} \tag{2}$$

with $K_z = 2\pi_r/\lambda$, E and H electric and magnetic field Thus electric and magnetic fields are separated by a quarter wavelength Assuming that the diameter of a focus is less than half a wavelength (i.e both the emittance and the focal length of the lens are small), particles injected in the local maximum of the electric field can be accelerated When the decelerating half-wave comes the particle is beyond the region of the focus But nature does not behave that way Usually the focus is extended over several wavelengths. The particle is accelerated by one half-wave, deacelerated by the next half-wave and so on. A net acceleration is not achieved

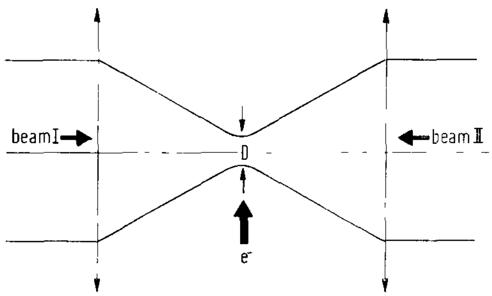

Fig 1 Production of standing waves in the focus of two laser beams coming from opposite directions

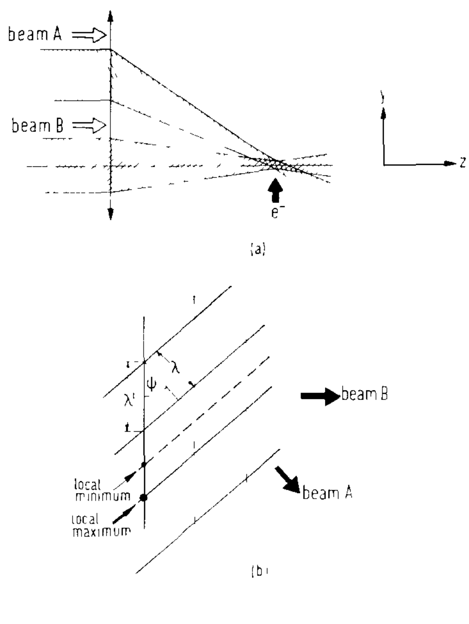

Fig 2 Superposition of two beams in the focus (a) Principal arrangment, (b) production of local minima and maxima in the focus The solid lines are lines of equal phase The distance between two local maxima is given by λ'

3. Accelerating field configurations in the focus of a laser beam

The simplest way to overcome the problem of deacelerating half-waves is to produce local maxima and minima along the particle trajectory The local minima must be at points where the decelerating half-wave can decelerate the particle Such a field configuration can be produced, e g , by splitting the laser beam in to two parallel beams and superposing both beams in the focus (fig 2) The diameter of the focus is not influenced by the superposition Inside the focus the two beams superpose in the following manner (fig 2b) In the interaction region of the two beams local maxima and minima are produced The distance between two maxima is given by

$$\lambda' = \lambda/\cos\psi \tag{3}$$

The angle ψ is defined in fig 2b

Instead of two beams, several beams may be superposed Superposing beams with

$$\frac{\lambda'}{2}, \frac{\lambda'}{3}, \quad , \frac{\lambda'}{n} \tag{4}$$

leads to a sharpening of the local maximum and an extension of a region with low electric field strength The intensity distribution along the y-axis is

$$E_y^2 = E_0^2 \frac{\sin^2 2\pi n y/\lambda}{\sin^2 2\pi y/\lambda}, \tag{5}$$

where n is the number of partial waves It is easy to imagine without calculations that such a superposition must lead to an accelerating structure

In the following considerations the accelerating condition will be derived Assuming a field configuration

$$E_y = E_0 \sin(\omega t+\phi) \cos(2\pi y/\lambda'), \tag{6}$$

with ϕ the phase describing the point of time the particle is injected into the field, the field seen by the particle is given by

$$E_{\text{particle}} = E_0 \sin\left(\frac{\omega y}{v_p} + \phi\right) \cos\left(\frac{2\pi}{\lambda'} y\right), \tag{7}$$

with v_p the particle velocity

The energy change of a particle along the y-axis is proportional to

$$I_2 = \int_{-\infty}^{+\infty} E_{\text{particle}}\, e^{-y^2/\sigma^2}\, dy \tag{8}$$

The term $\exp(-y^2/\sigma^2)$ describes the extension of the focus In this formula the assumption is made that the field distribution in the focus has a Gaussian shape This assumption is valid when both the beam dimensions are small compared with the lens dimensions and the laser cross section has a Gaussian shape σ can be derived from eq. (1) Integrating along the particle trajectory eq (7), eq. (8) yields

$$I_2 = \tfrac{1}{2}\sqrt{\pi}\,\sigma \sin(-\phi) \left\{\exp\left[-\sigma^2\pi^2\left(\frac{1}{\lambda'} - \frac{c}{\lambda v_p}\right)^2\right] + \exp\left[-\sigma^2\pi^2\left(\frac{1}{\lambda'} + \frac{c}{\lambda v_p}\right)^2\right]\right\}, \tag{9}$$

c is the velocity of light

When only one wave of fig 2 interacts with the particle, the integration along the particle trajectory is given by

$$I_1 = \sigma\sqrt{\pi} \sin(-\phi) \exp\left(-\pi^2 \frac{c^2}{v_p^2}\frac{\sigma^2}{\lambda^2}\right) \tag{10}$$

Comparing eqs (9) and (10) one finds that in the case of a two-beam focus [eq (9)] the particle can accumulate

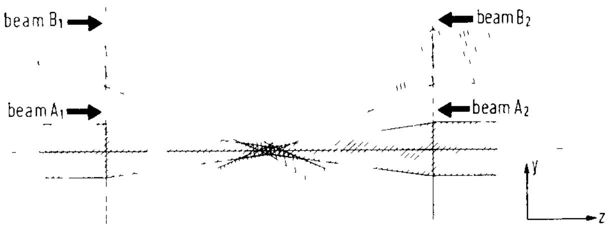

Fig 3 Superposition of four beams in the focus

energy similar to a resonance effect At resonance

$$v_p = c\lambda'/\lambda, \tag{11}$$

the total increase of energy is about

$$I_2 \approx \tfrac{1}{2}\sqrt{\pi}\,\sigma \sin(-\phi) \tag{12}$$

In these calculations the change of velocity with energy was neglected This case will be discussed later

Eq (6) does not exactly describe the field components of fig 2 The electric field component of the beam in the z-direction has been ignored and the magnetic fields of both beams were totally neglected To find fields obeying eq (6) exactly four waves have to interact (fig 3) The electric fields produced by the four waves are

$$E_{\text{total}} = E_y \exp\{i(\omega t - k_z)\} + E_y \exp\{i(\omega t + k_z)\} + (\tilde{E}_y + \tilde{E}_z)\exp\{i(\omega t - k_z \sin\psi - k_y \cos\psi)\} + \\ + (\tilde{E}_y - \tilde{E}_z)\exp\{i(\omega t + k_z \sin\psi - k_y \cos\psi)\} \tag{13}$$

E_y describes the electric field of beams A_1 and A_2, $\tilde{E}_y$ and $\tilde{E}_z$ the field component of the beams B_1 and B_2 (fig 3)

The coordinates are defined in fig 3 The electric field strength in the y-direction is, after some simple calculations,

$$E_{y\,\text{total}} = 2E_y \sin\omega t \cos k_z + 2\tilde{E}_y \cos(k_z \sin\psi)\ [\sin(\omega t - k_y \sin\psi)] \tag{14}$$

For

$$E_y \cos k_z = \tilde{E}_y \cos(k_z \sin\psi) = E_y, \tag{15}$$

eq (14) yields

$$E_{y\,\text{total}} = 4E_y \cos(\tfrac{1}{2}k_y \cos\psi)\sin(\omega t - \tfrac{1}{2}k_y \cos\psi) \tag{16}$$

Now the same conclusions arrived at for eqs (7)–(9) can be repeated Resonance acceleration takes place when

$$\tfrac{1}{2}k_y \cos\psi = \omega t - \tfrac{1}{2}k_y \cos\psi, \tag{17}$$

or

$$v_p = \frac{c}{\lambda}\frac{\lambda}{\cos\psi} = \frac{c\lambda'}{\lambda} \tag{18}$$

Eq (18) is identical with the previously derived eq (11) Similar calculations can be done with the z-components of the electric field of eq (13), yielding

$$E_{z\,\text{total}} = 2\tilde{E}_z \sin(k_z \sin\psi)\cos(\omega t - k_y \cos\psi) \tag{19}$$

The y- and the z-components are separated locally by a quarter wavelength. At resonance the z-component of the electric fields vanishes along the acceleration channel Similar to the E_z fields the H-fields can be calculated The H-fields are

$$H_{x\,\text{total}} = H_x[\exp\{i(\omega t - k_z)\} - \exp\{i(\omega t + k_z)\}] + \tilde{H}_x[\exp\{i(\omega t - k_z \sin\psi - k_y \cos\psi)\} - \\ - \exp\{i(\omega t + k_z \sin\psi - k_y \cos\psi)\}] \tag{20}$$

Using the same arguments given in eqs (13) and (19) and eq (2), it can be demonstrated that the magnetic field vanishes along the acceleration channels Thus eq (16) describes the total field, and resonance acceleration is possible Unfortunately the acceleration condition (18) can not be used for real particles λ' cannot become less than λ, meaning that particles with velocities less than c cannot be accepted In the following section a modification of this system is described allowing the acceleration of particles with velocities less than c

4. Acceleration of particles with velocities less than c

For particles with velocities less than c, λ' must become smaller than λ or $\cos\psi$ must become greater than 1 In the calculation of eq (11) the assumption was made that the lines of equal phases are perpendicular to the direction of propagation in both overlapping beams Assuming that the lines of equal phases have an angle ϕ to the normal of the direction of propogation (fig 4), λ' becomes

$$\lambda' = \frac{\lambda\cos\phi}{\cos(\phi-\psi)} \tag{21}$$

Before discussing how to produce such fields technically it should be demonstrated that the fields fulfill the acceleration condition of a resonance channel accelerator described in the previous section

As mentioned above rays with equal inclination to the optical axis are superposed in the focus Fields with λ' less than λ can be generated by delaying each ray of one beam individually one of the two superposed beams must pass an element with an angle-dependent index of refraction. Now the superposition of the E_z-fields can be written by modifying eq (19)

$$E_{z\,\mathrm{total}} = -2\tilde{E}_z \sin(k_z \sin\omega t)\cos(\omega t+\omega\Delta t-k_y\cos\psi), \tag{22}$$

where Δt denotes the time delay of the individual ray As seen in eq. (22) the term describing the local dependence vanishes along the particle trajectory in the same way it has done before The same argument leads to an extinction of the magnetic field when the particle trajectory is parallel to the y-axis

As mentioned above, matching of the particle velocity can be done by an element with an angle-dependent index of refraction, for instance a birefringent crystal one of the laser beams enters the crystal as the e-beam This is possible since the laser beam must be linearly polarized for the acceleration The energy of the electron being accepted for acceleration by the channel depends both on the angle ψ (fig 3) and the index of refraction of the birefringent modulator For a spheric lens (focal length 2 5 cm) and a beam divergence of 10 mrad the accepted particle energy as function of the length of the modulator is calculated in fig. 5 The modulator medium is calcite The index of refraction varies within the 10 mrad from 1 590719 to 1 591639 As seen from fig 5 the accepted energy is less than 100 keV with the modulator 10 cm long 10 cm seems to be something of a technical limitation. Since 100 keV electrostatic preaccelerators are no technical problem it seems to be possibly to study channel

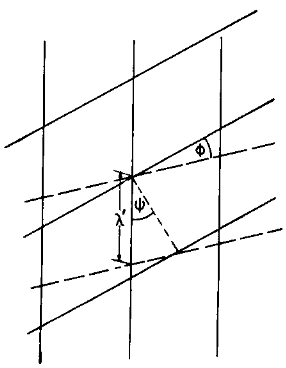

Fig 4 Superposition of two beams similar to fig. 2 but with the difference that the line of equal phase of one beam is inclined by an angle ϕ to the normal of the propagation direction.

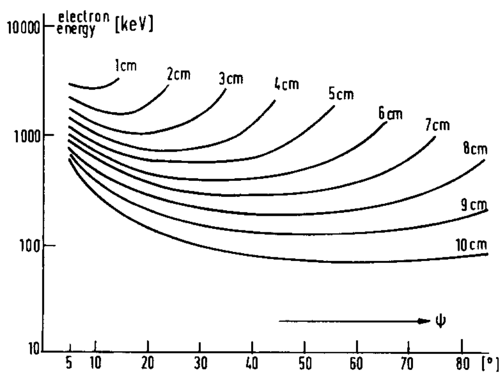

Fig 5 Acceptance energy of a channel accelerator as a function of ψ and modulator length.

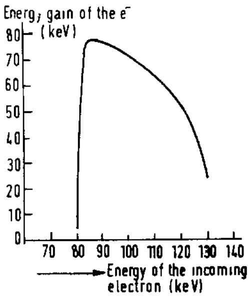

Fig 6 Energy gain of an incoming electron vs starting energy The channel is tuned to 110 keV, $\sigma = 2\ \mu$m, voltage across the channel 100 kV

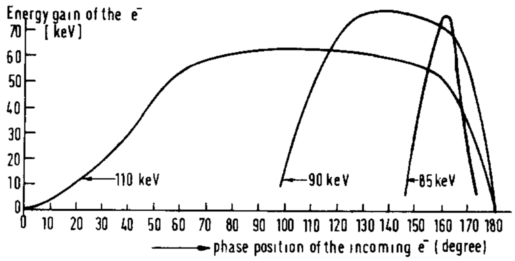

Fig 7 Energy gain vs phase position of an injected electron at different energies Channel parameters are equal to those in fig 6

acceleration experimentally in the laboratory. Injecting 100 keV electrons into the channel the electrons gain energy and change their velocity, thus leading to a mismatch between the channel acceptance and the particle velocity Fig 6 shows the maximum energy gain of a particle travelling through a 100 kV ($\sigma = 2\ \mu$m, $\lambda = 1\ 06\ \mu$m) channel versus the energy of the injected electron The channel accepts between 85 and 120 keV The maximum transferred energy does not exceed 77 keV In fig 7 the energy gain of the particle versus the phase position of the incoming particle is plotted The channel parameters are the same as in fig 6 At 85 keV the accepted electrons are very small in phase, which explains the asymmetry of fig 6 At 110 keV there is a broad phase acceptance The wide range of phase in which the output energy is independent with the phase of the incoming electron is well known at linear accelerators In linear accelerators this is known as the longitudinal focussing effect

This simple example should demonstrate that the particle trajectory is stable under certain circumstances in the longitudinal direction. Now there is only one problem left the stabilisation of the particle trajectory in the transverse direction For transverse stability fields must be produced which are able to focus the particle beam

5. Transverse focussing field configurations

For the production of focussing fields two field components can be used the E_z and/or the H_x components of the field [eqs (19) and (20)] Both fields generate a transverse force on the particle travelling outside the channel and both fields act in the same direction

Generalizing the superposition of four waves in the focus of a laser beam similar to eq (13) but with all four waves having an inclination to the optical axis

$$E_z \exp\{i(\omega t - k_z \sin\psi - k_y \cos\psi)\} - E_z \exp\{i(\omega t + k_z \sin\psi - k_y \cos\psi)\} + E_z \exp\{i(\omega t - k_z \sin\psi_1 - k_y \cos\psi_1\} - - E_z \exp\{i(\omega t + k_z \sin\psi_1 - k_y \cos\psi_1)\}, \quad (23)$$

and

$$E_y \exp\{i(\omega t - k_z \sin\psi - k_y \cos\psi)\} + E_y \exp\{\omega t + k_z \sin\psi - k_y \cos\psi\} + E_y \exp\{i(\omega t - k_z \sin\psi_1 - k_y \sin\psi_1)\} + + E_y \exp\{\omega t + k_z \sin\psi_1 - k_y \cos\psi_1\} \quad (24)$$

yields after simple calculations for the E_z and E_y-fields

$$E_{z\,\mathrm{total}} = -2E_z \sin(k_z \sin\psi)\cos(\omega t - k_y \cos\psi) - 2E_z \sin(k_z \sin\psi_1)\cos(\omega t - k_y \cos\psi_1), \quad (25)$$

and

$$E_{y\,\mathrm{total}} = 2E_y \cos(k_z \sin\psi)\sin(\omega t - k_y \cos\psi) + 2E_y \cos(k_z \sin\psi_1)\sin(\omega t - k_y \cos\psi_1) \quad (26)$$

Fulfilling the conditions derived in the previous section, accelerating channels are produced along the y-direction Left and right of this channel, the E_z-field has different signs in the y-direction [eq (25)] But now the

E_z-component can produce a channel too For a given z-position the E_z-component varies with

$$\cos[\omega t - \tfrac{1}{2}(k_y \cos\psi + k_y \cos\psi_1)] \cos[\tfrac{1}{2}(-k_y \cos\psi + k_y \cos\psi_1)], \tag{27}$$

while the E_y-component varies with

$$\sin[\omega t - \tfrac{1}{2}(k_y \cos\psi + k_y \cos\psi_1)] \cos[\tfrac{1}{2}(-k_y \cos\psi + k_y \cos\psi_1)] \tag{28}$$

Both channels have the same resonance condition, but operate at different phases when the channel is in the phase of accelerating no focussing effect exists and vice versa The focussing effect exists since the fields left and right of the channel have opposite signs So the system works like a conventional quadrupole in conventional accelerators

6. Conclusions

In this paper the possibility of accelerating electrons in the focus of a laser beam is discussed Field patterns able to accelerate and focus electron beams are analysed. The fields are produced with the help of a birefringent crystal outside the focus This seems to be a more practicable suggestion than earlier ones on laser accelerators

But this is still the first phase in thinking on laser linacs Beam dynamics, maximum currents and related topics must be considered in next

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