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LEPTON ENERGY SPECTRA IN e^+e^- ANNIHILATION AND OTHER PROCESSES

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An analysis is presented of the single inclusive lepton spectra expected from the decay of charmed hadrons or heavier particles with new flavors produced in e^+e^- annihilation. Extremely soft spectra are found, in contrast to the heavy lepton signal. Good detection efficiency at low momentum will therefore continue to be important in future experiments. A range of plausible inclusive D meson spectra abstracted from a parton fragmentation picture is employed. The model is applicable to other processes.

1. Introduction

Single low momentum electrons associated with hadrons have recently been observed in e^+e^- annihilation experiments at DORIS at $\sqrt{s} = 4.0 - 4.4$ GeV [1]. The hadron multiplicity in these events is high [1] and the signal is associated with the production of strange particles [2]. These observations invite an interpretation in terms of the production and subsequent semi-leptonic decay of charmed particles (hereafter generically called D). Calculations of two-body channels ($D\bar{D}$, $\bar{D}\overset{*}{D}$, ...), which must dominate in this threshold region, show that this hypothesis is tenable provided the D decay spectrum is rather soft [3].

In this paper we are mainly concerned with the form the lepton spectrum will take at higher energies where two-body $D\bar{D}$ final states will presumably constitute a small fraction of the total charm cross section. In particular, it is interesting to contrast the lepton spectra expected from charm and from the decay of pair produced heavy leptons (L^+L^-). For charmed particles [4] and heavy leptons [5] with masses of order 2 GeV, the lepton spectrum is expected to be softer and the hadron multiplicity higher in the case of charm, the two processes becoming more distinct with increasing energy, as $D\bar{D}$ production becomes increasingly inelastic [6]. However, if more flavors exist they may also yield a relatively hard lepton spectrum near their threshold, and it is interesting to see whether such a contribution could easily be detected.

We shall adopt a model in which charmed quark-antiquark pairs are produced ($e\bar{e} \to c\bar{c}$) and subsequently fragment to charmed hadrons ($c \to D + ...$) which then decay. In sect. 2 we discuss our assumptions for the fragmentation and decay functions and use them to construct a quark-to-lepton fragmentation function in which the D momentum has been integrated out. At energies large compared to M_D, M_c and the transverse momentum in $c \to D + ...$ fragmentation, this function depends only on $\omega = E_{\text{lepton}}/E_B$, where E_B is the beam energy, and can be written analytically. At subasymptotic energies care is necessary since the fragmentation picture is not Lorentz-invariant. However, we have found an ansatz which we believe interpolates plausibly between exclusive $D\bar{D}$ production at threshold and multiparticle production at asymptotic energies.

Our results, presented in sect. 3, show that, regardless of the details of the model, inelastic $c \to D$ fragmentation leads to a lepton spectrum which is very strongly peaked at low ω, and is therefore easily distinguishable from the spectrum from heavy lepton decay. A direct consequence of this is that the angular distribution of the leptons which are the progeny of charmed quarks shows little trace of their parents' $1 + \cos^2\theta$ distribution relative to the beam axis. However, we find that the lepton fragments of heavier flavors may not be so easy to distinguish.

Having introduced a charmed-quark-to-lepton fragmentation function, it is natural to use it to describe in addition the processes

$$\nu N \to \mu^- D + ... \qquad \text{and} \qquad \mu N \to \mu D + ...$$
$$\qquad\quad \hookrightarrow e + ... \qquad\qquad\qquad\qquad \hookrightarrow e + ...$$

and to relate them to

$$e^+e^- \to D + ...$$
$$\qquad\quad \hookrightarrow e + ...$$

Relevant formulae are collected in sect. 4, where we present a brief discussion of how these processes might be treated at finite energies. Our conclusions are summarized in sect. 5.

2. Decay and fragmentation functions

We shall use the following rest frame spectrum for the decay $D \to Xe\nu$ ★

$$\frac{1}{\Gamma}\frac{d\Gamma}{dE_e} = wf(E_e)\,, \tag{1}$$

where

$$f(E_e) = \frac{E_e^2(M_D^2 - M_X^2 - 2M_D E_e)^2}{M_D - 2E_e}\,,$$

$$w = 96(1 - 8m^2 + 8m^6 - m^8 - 12m^4 \ln m^2)^{-1} M_D^{-6}\,,$$

$$m = M_X/M_D\,. \tag{2}$$

This is the form of the spectrum for $D \to Ke\nu$, with $M_X = M_K$ (and also for $c \to se\nu$ with $M_D = M_c$, $M_X = M_s$ and a V – A coupling). It is somewhat more sharply peaked than 3-body phase space and might work reasonably well for multibody states with the invariant mass M_X held fixed in the spirit of an isobar model (e.g. it is a reasonable approximation to the spectrum expected in $D \to K^*e\nu$ for $M_X = M_{K^*}$ [7,8]; possible spectra are discussed further in refs. [3,7–9]). M_X will be kept as a free parameter here.

We assume throughout that the transverse momentum in the fragmentation $c \to D + ...$ is negligible, as must be the case in the fragmentation of non-strange quarks $u, d \to \pi + ...$ if this picture is to describe correctly the jets observed at SPEAR [10] (we expect that $\langle p_T^2 \rangle \ll M_D^2$ and so neglect of $\langle p_T^2 \rangle$ should be a good first approximation). Further we assume that the probability of finding $\boldsymbol{p}_D$ in the range $\boldsymbol{p}_D$ to $\boldsymbol{p}_D + d\boldsymbol{p}_D$ is a function $D(z)|\boldsymbol{p}_c|dz$ of $z = |\boldsymbol{p}_D|/|\boldsymbol{p}_c|$ ★★. Some previous authors have assumed a z^{-1} singularity in $D(z)$ as $z \to 0$ [9,11,12]. We believe that such a term should have a negligibly small coefficient since it represents the associated production of $D\bar{D}$ pairs in the central plateau of rapidity ($c \to D + D\bar{D} + D\bar{D} + ...$) with an implied charmed particle multiplicity growing like $\ln|\boldsymbol{p}_c|$. In models with short-range order this processes should be independent of the flavor of the fragmenting quark; the evident paucity of charmed particle production in pp collisions then suggests that it is negligible. In any case consistency demands that we neglect it since we have neglected associated production in events in which non-charmed quarks are

★ We use the symbol e, but all our results apply equally well to muons. The two-body decays $D \to \mu\nu$, $e\nu$ are negligible if we assume D is pseudoscalar, since the matrix element is then proportional to m_μ, m_e.

★★ Note that $D(z)$ is supposed to include the effects of the production of heavier charmed particles which decay to D.

produced at the primary photon vertex. With one D per c the function $D(z)$ must be normalized to one

$$\int_0^1 D(z)\mathrm{d}z = 1 \,. \tag{3}$$

We adopt the ansatz

$$\begin{aligned} D(z) &= (n+1)(1-z)^n \qquad z>0\,, \\ &= 0 \qquad\qquad\qquad\quad\; z<0\,, \end{aligned} \tag{4}$$

where we expect $n = 1$ or 2 from theoretical considerations. We will investigate the sensitivity to variations in n. At asymptotic energies we should clearly put $|\boldsymbol{p}_\mathrm{c}| = \frac{1}{2}\sqrt{s}$ (in the c.m. system) and we expect that $D(z) \to 0$ as $z \to 1$ where inclusive production turns into exclusive $\mathrm{D\bar{D}}$ production (suggesting $n = 1$ for a smooth exclusive-inclusive connection [13] with monopole form factors and $n = 2$ from more detailed theoretical considerations [14]). We favor $n = 2$ which appears to be supported by data on inclusive hadron production at large momenta at SPEAR [15].

If we wish to extend the model to subasymptotic energies we are then forced to put $|\boldsymbol{p}_\mathrm{c}| = \sqrt{\frac{1}{4}s - M_\mathrm{D}^2}$ so that with $\boldsymbol{p}_\mathrm{D} = z\boldsymbol{p}_\mathrm{c}$, $z \to 1$ always corresponds to the exclusive processes and the vanishing of $D(z)$ in this limit make sense. (With other prescriptions there is the danger of excluding $z = 1$ and violating the normalization condition.) The model then correctly yields D meson production at rest at threshold and is, we believe, likely to work reasonably well in an average sense at all energies. In our view the $\mathrm{e^+e^-} \to \mathrm{c\bar{c}}$ production cross section will interpolate smoothly through any s-channel structure in $\sigma(\mathrm{e^+e^-} \to \mathrm{D} + ...)$ down to low energies (though it does not have the threshold behavior of $\mathrm{e^+e^-} \to \mathrm{D\bar{D}}$). Thus we assume

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z}(\mathrm{e^+e^-} \to \mathrm{D} + ...) = \frac{16\pi\alpha^2}{9s}\sqrt{1 - 4M_\mathrm{D}^2/s}\,(1 + 2M_\mathrm{D}^2/s)\,D\!\left(\frac{2|\boldsymbol{p}_\mathrm{D}|}{\sqrt{s - 4M_\mathrm{D}^2}}\right)\,, \tag{5}$$

taking $Q = \frac{2}{3}$ for the charmed quark and including a factor 3 for color (in fact we are primarily interested in spectra in which multiplicative functions of s play no role but we wish to include our best guess for completeness; obviously this formula and our ansatz for $D(z)$ are subject to direct verification if p_D is measured *via* hadronic decays). For lepton production

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_\mathrm{e}}(\mathrm{e^+e^-} \to \underset{\hookrightarrow \mathrm{e} + ...}{\mathrm{D} + ...}) = B_\mathrm{D}\,\frac{16\pi\alpha^2}{9s}\sqrt{1 - 4M_\mathrm{D}^2/s}\,(1 + 2M_\mathrm{D}^2/s)\,\frac{\mathrm{d}F(p_\mathrm{c}, E_\mathrm{e})}{\mathrm{d}E_\mathrm{e}}\,, \tag{6}$$

where

$$B_\mathrm{D} = \frac{\Gamma(\mathrm{D} \to \mathrm{e} + ...)}{\Gamma(\mathrm{D} \to \mathrm{all})}$$

and the quark-to-lepton fragmentation function $\mathrm{d}F/\mathrm{d}E$ is given by

$$\frac{\mathrm{d}F(p_c, E_e)}{\mathrm{d}E_e} = w \int_{z_{\min}}^{1} \mathrm{d}z\, D(z) \int_{E_e(E_D - p_D)}^{x} \frac{\mathrm{d}(p_D \cdot p_e) M_D}{2|\mathbf{p}_D|(p_D \cdot p_e)} f\left(\frac{p_D \cdot p_e}{M_D}\right), \tag{7}$$

with

$$p_D = z p_c = z\sqrt{\tfrac{1}{4}s - M_D^2},$$

$$z_{\min} = \max\left[0, \frac{4M_D^2 E_e^2 - (M_D^2 - M_X^2)^2}{4p_c E_e (M_D^2 - M_X^2)}\right],$$

$$x = \min\left[E_e(E_D + p_D), \tfrac{1}{2}(M_D^2 - M_X^2)\right]. \tag{8}$$

The $p_D \cdot p_e$ integral is easy to do analytically with our choice of f but we have done the z integral numerically except in the $p_c \to \infty$ limit where we obtain

$$\lim_{p_c \to \infty} p_c \frac{\mathrm{d}F(p_c, E_e)}{\mathrm{d}E_e} = G(\omega)$$

$$= \tfrac{3}{8} w \Big\{ \tfrac{1}{12}(1 - 6M_X^2 + 3M_X^4 + 2M_X^6)(-3 + 4t - t^2 - 2\ln t)$$

$$+ \omega M_X^4 \left(2\ln t + \frac{1}{t} - t\right) + \tfrac{1}{4}\omega^2 (2M_X^2 - 1)\left(3 - \frac{4}{t} + \frac{1}{t^2} - 2\ln t\right)$$

$$+ \tfrac{1}{9}\omega^3\left(-1 + \frac{3}{t} - \frac{3}{t^2} + \frac{1}{t^3}\right) + \tfrac{1}{2}M_X^4 \Big[-2f\left(1 - \frac{1}{\alpha}\right) + 2f(1 - \omega) - \omega$$

$$+ 3\ln M_X^2 + \ln(1 - \omega)(-3 - \omega^2 + 4\omega) + \omega t + 2\ln t \ln M_X^2 - 4t\ln(tM_X^2)$$

$$+ t^2 \ln(tM_X^2) - \ln(t - \omega)(t^2 - \omega^2 - 4t + 4\omega)\Big]\Big\}. \tag{9}$$

The units are such that $M_D = 1$ and $t = \alpha\omega$ with $\alpha^{-1} = 1 - M_X^2$. $f(x)$ is the dilogarithm function as defined in ref. [16], where tables of its values may be found. With $M_X = 0$, this formula reduces to the simple (but probably misleading) form

$$G(\omega) = -14 + 36\omega - 18\omega^2 - 4\omega^3 - 6\ln\omega + 18\omega^2 \ln\omega, \tag{10}$$

which may be useful for exploratory purposes.

3. Results for e^+e^- annihilation

We now present results for various beam energies $E_B = \sqrt{\frac{1}{4}s}$ as a function of our two parameters M_X (eq. (1)) and n (eq. (4)) for $M_D = 1.87$ GeV. For comparison we also give results for a heavy lepton decay $L^- \to \nu_L e^- \bar{\nu}_e$ for V ± A couplings with $M_{\nu_L} = 0$ unless stated otherwise and $M_L = 1.87$ GeV; the V + A case is the same as D decay in our model with $M_X = 0$ and $D(z) = \delta(1 - z)$. For completeness we give the analytic forms of the spectra in the heavy lepton case (see also ref. [17])

$$p_L \frac{d\Gamma^{V\pm A}}{dE_e} = \begin{cases} f^{\pm}(\xi_-)\,, & E_e > \frac{1}{2}(E_B - p_L)\,, \\ f^{\pm}(\xi_-) - f^{\pm}(\xi_+)\,, & E_e < \frac{1}{2}(E_B - p_L)\,, \end{cases} \tag{11}$$

where (in units $M_L = 1$)

$$p_L = \sqrt{E_B^2 - 1}\,, \qquad f^+(\xi) = 2\xi^2(3 - 2\xi)\,,$$

$$\xi_\pm = 1 - 2E_e(E_B \pm p_L)\,, \qquad f^-(\xi) = 2\xi(1 + \tfrac{1}{2}\xi - \tfrac{2}{3}\xi^2)\,. \tag{12}$$

In fig. 1 we show results at $E_B = 2$ GeV for $n = 2$ which illustrate the sensitivity of the spectrum to M_X. The similarity of the V + A heavy lepton and the $M_X = 0$ charm curves shows the extreme insensitivity to $D(z)$ near threshold. The fact that V + A gives a slightly softer spectrum than V − A is easy to understand since the

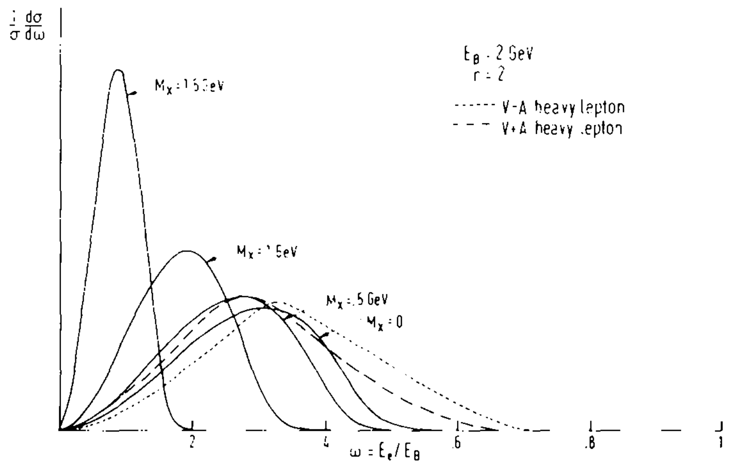

Fig. 1. Lepton energy spectra for beam energy $E_B = 2$ GeV as a function of $\omega = E_e/E_B$ for leptons derived from charmed mesons with mass 1.87 GeV (full lines) for various M_X (eq. (2)) and $n = 2$ (eq. (4)) and for heavy leptons of mass 1.87 GeV with V − A (dotted) and V + A (dot-dashed) couplings.

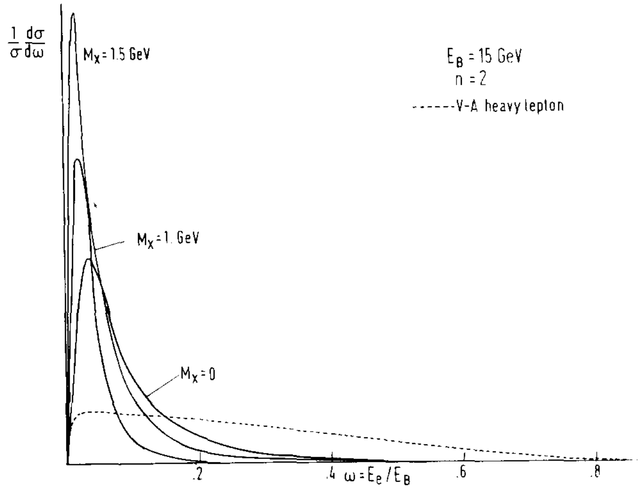

Fig. 2. As for fig. 1, but with E_B = 15 GeV.

configuration in which E_e takes its maximum value is forbidden by angular momentum conservation in the former case.

Fig. 2 is for E_B = 15 GeV and n = 2; note that the marked difference between the heavy lepton and charm induced spectra is very insensitive to M_X at high energy. Fig. 3, which is for E_B = 15 GeV and M_X = 0.5 GeV, shows that this separation is also insensitive to n. Fig. 4 illustrates the comparitive insensitivity of the spectrum in heavy lepton decay to the neutrino mass in the high-energy limit.

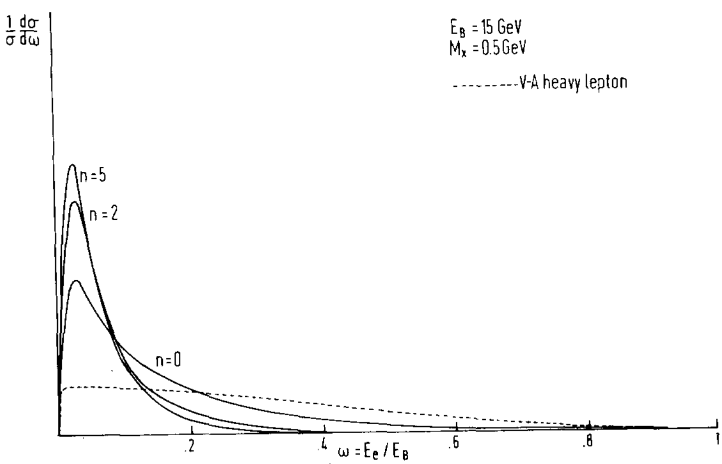

Fig. 3. As for fig. 2, with M_X = 0.5 GeV and various n (eq. (4)).

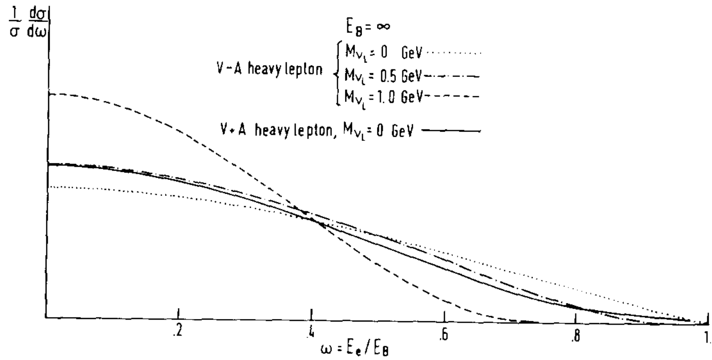

Fig. 4. Heavy lepton decay spectra in the high-energy limit for various values of M_{ν_L}.

In figs. 5–9 we specialize to the case $n = 2$ which we favor slightly and $M_X = 0.5$ GeV, M_K or $M_X = 1$ GeV. (A value of this order is favored by the soft spectra seen at DORIS near threshold.) In addition we include the effects of a new flavor for $E_B = 5$ GeV which we describe by the same model with the parameters arbitrarily given the values $M_{D'} = 4.8$ GeV, $M_{X'} = 2$ GeV. Although a new flavor seems to stand out near threshold (fig. 6), it gradually merges with the charm result as the beam energy increases (figs. 7, 8). An increase of M_X would make this distinction between charm and a new flavor less clear. In addition it must be emphasized that all our spectra have the same normalization; in reality charmed quarks are expected to be produced $\frac{4}{3}$ as copiously as heavy leptons and charge $\frac{1}{3}$ quarks only $\frac{1}{3}$

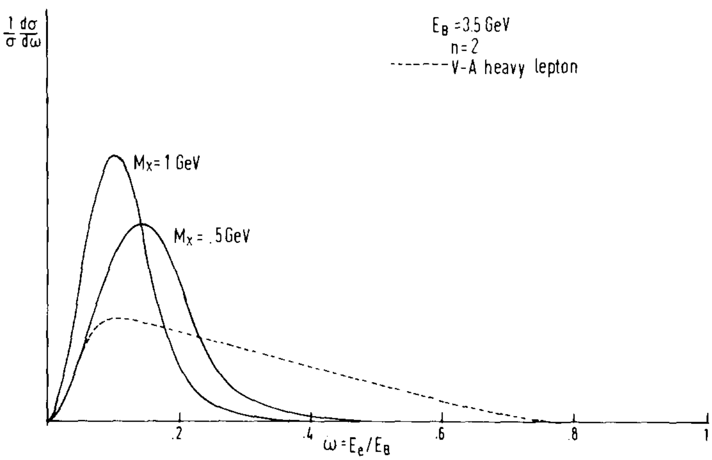

Fig. 5. Spectra at $E_B = 3.5$ GeV for $n = 2$ compared to the V – A heavy lepton signal with $M_{\nu_L} = 0$.

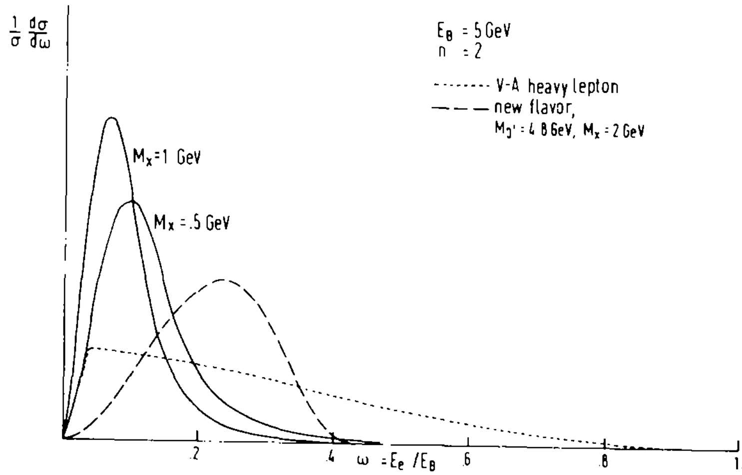

Fig. 6. Spectra at E_B = 5 GeV for n = 2 including a new flavor with M_D = 4.8 GeV and M_X = 2 GeV.

as copiously. Furthermore the semileptonic branching ratio may be less for charmed particles than for heavy leptons.

A quantitative measure of the difference between the spectra in the charm and heavy lepton cases is also interesting. In the decay of any unpolarized particle A $\rightarrow$ a + ... The quantity $\langle E_a \rangle / E_A$ is Lorentz invariant since $\langle p_a \rangle = 0$ in the frame in

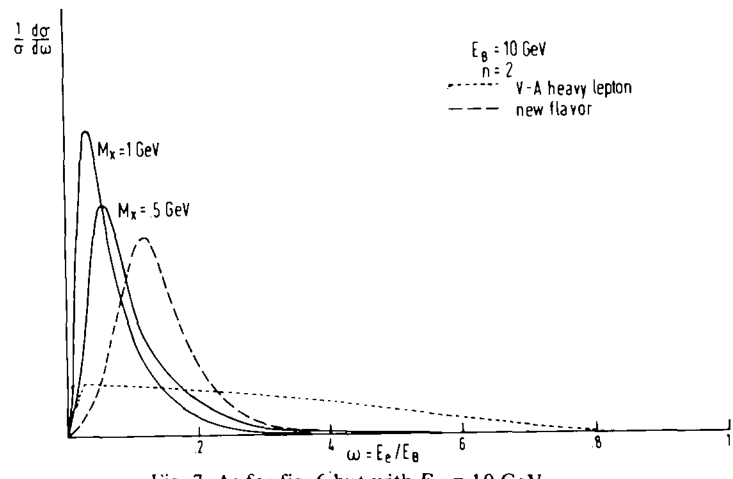

Fig. 7. As for fig. 6 but with E_B = 10 GeV.

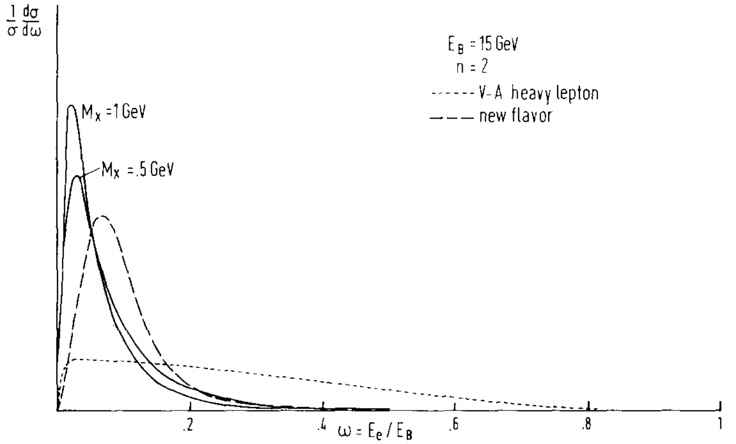

Fig. 8. As for fig. 6 but with E_B = 15 GeV.

which $\boldsymbol{p}_A = 0$. Hence

$$\frac{\langle E_e \rangle}{E_B} = \frac{\langle E_e \rangle_{\text{rest frame}}}{M_D} \frac{\langle E_D \rangle}{E_B} .$$

With our model for the spectrum (eq. (2))

$$\frac{\langle E_e \rangle}{E_D} = \frac{3 - 30r - 20r^2 + 60r^3 - 15r^4 + 2r^5 - 60r^2 \ln r}{10[1 - 8r + 8r^3 - r^4 - 12r^2 \ln r]}$$

$$\approx \tfrac{3}{10}(1 - r) , \tag{13}$$

where $r = M_X^2/M_D^2$. The approximation slightly overestimates the exact result, but

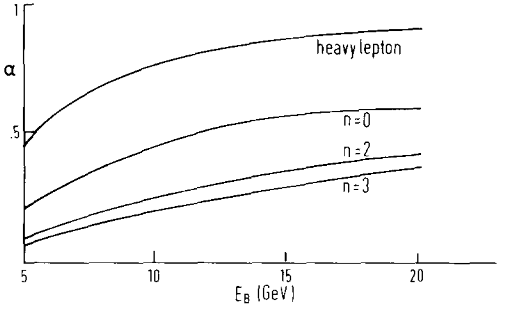

Fig. 9. The angular distribution coefficient α defined in eq. (16) as a function of E_B.

nowhere by more than 0.02. The same formula works for heavy lepton decays in the V + A case with a suitable transposition of symbols; in the V − A case $\langle E_e\rangle/E_L = 7/20$ if $M_{\nu_L} = 0$. Using our D function

$$\frac{\langle E_D\rangle}{E_B} = \int_0^1 \sqrt{z^2E_B^2 + M_D^2(1-z^2)}\,D(z)\,dz/E_B$$

$$\xrightarrow[E_B\to\infty]{} \frac{1}{n+2}\,.$$

Therefore in the high-energy limit

$$\frac{\langle E_e\rangle}{E_B} \approx \frac{3(1-r)}{10(n+2)} \qquad \text{(charm)}\,,$$

$$\approx \tfrac{3}{10} \qquad \text{(V + A heavy lepton)}\,,$$

$$\approx \tfrac{7}{20} \qquad \text{(V − A heavy lepton)}\,. \tag{14}$$

It is clear from the figure that it is not only the average energy which is much less in the charm case; the spectrum also cuts off much more sharply. Using eq. (10) and the high-energy limit of eq. (11) it is easy to calculate the fraction of the cross section contributed by $\omega > \omega_{min}$ in the asymptotic limit when $n = 2$ and $M_X = 0$ (which leads to an overestimation in the charm case). The results are shown in table 1

We conclude that excellent electron and muon identification at low momenta will be very desirable at PETRA and PEP.

Next we consider the angular distribution of the leptons relative to the beam axis. We start from the observation of Pais and Treiman [18] that for a parent D meson of known energy E_D and angular distribution

$$\frac{1}{\sigma_{e^+e^-\to D+\dots}}\frac{d\sigma}{d\cos\theta}(e^+e^-\to D+\dots) = \frac{3(1+\lambda(E_D,s)\cos^2\theta)}{2(3+\lambda(E_D,s))} \tag{15}$$

the integrated angular distribution of the daughter lepton is completely determined if its mass is neglected:

$$\frac{2}{\sigma_{e^+e^-\to D+\dots \,\llcorner\to e+\dots}}\frac{d\sigma}{d\cos\theta_e}(e^+e^-\to D+\dots \,\llcorner\to e+\dots)$$

$$= 1 + \frac{3\lambda(E_D,s)}{3+\lambda(E_D,s)}\left[\frac{3-2\beta^2}{\beta^2} - \frac{3(1-\beta^2)}{3\beta^2}\ln\frac{1+\beta}{1-\beta}\right](\cos^2\theta_e - \tfrac{1}{3})\,, \tag{16}$$

Table 1

$\omega_{\min}$	0.01	0.025	0.1	0.2	0.3	0.4	0.6	0.8	0.9
charm $M_X = 0$	0.80	0.63	0.26	0.09	0.03	0.01	7×10^{-4}	8×10^{-6}	1×10^{-7}
V + A	0.98	0.95	0.80	0.61	0.45	0.30	0.10	0.01	2×10^{-3}
V − A	0.98	0.96	0.83	0.67	0.52	0.39	0.17	0.04	0.01

Fraction of events with $\omega > \omega_{\min}$ in the high-energy limit with $n = 2$ and $M_X = 0$.

where $\beta = p_D/E_D$. If the primary c quarks have a $1 + \cos^2\theta$ distribution, the D meson spectrum will be slightly different due to the non-zero p_T in the fragmentation ($\lambda^{-1} = 1 + 2\langle p_T^2\rangle/p_D^2$ is a satisfactory approximation to the jet model result [19]). However, this is only important at low z for small E_B where the form of the quark distribution itself might be expected to differ from $1 + \cos^2\theta$ due to some finite effective quark mass. In conformity with our recipe for using the fragmentation model near threshold, which appears to imply an effective quark mass M_D, we set

$$\lambda(E_D, s) = \frac{\beta_c^2}{2 - \beta_c^2}\,, \qquad \beta_c = \sqrt{\frac{s - 4M_D^2}{s}} \tag{17}$$

(thus continuing to neglect p_T^2). This form has the behavior we expect as $\beta \to 0$ and $\beta \to 1$ but as we do not believe it in detail we will only present results for $E_B > 5$ GeV, which do not depend sensitively on λ. We integrate over the D momentum to obtain results for α as defined by

$$\frac{1}{\sigma_e}\frac{d\sigma_e}{d\cos\theta_e} = \frac{3(1 + \alpha\cos^2\theta_e)}{2(3 + \alpha)}\,. \tag{18}$$

In all cases the soft spectrum causes a dramatic smearing relative to the angular distribution expected for heavy leptons (fig. 9). However, these results only apply after integration over *all* lepton energies. With a minimum cut on lepton energy α will be larger.

4. Dilepton production in neutrino and muon interactions

These processes have been discussed by several authors [9,11,12,20,21] in quark fragmentation models of the type employed here. The soft spectrum postdicted by the model agrees qualitatively with the data. More speculatively, the same approach might be applied to the spectrum of leptons from charmed particles produced in hadron collisions. At asymptotic energies the D meson and the product lepton emerge (in the laboratory frame) along the direction of the momentum transfer $\boldsymbol{q}$, the cross section being given in terms of our function G (eqs. (9) and (10)) by *

$$\frac{\mathrm{d}\sigma(\nu\mathrm{N} \to \mu\mathrm{e} + ...)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}(p_\mathrm{D} \cdot p_\mathrm{e})} = \frac{B_\mathrm{D}}{q \cdot p} \left(\frac{\mathrm{d}\sigma(\nu\mathrm{N} \to \mu + ...)}{\mathrm{d}x\,\mathrm{d}y}\right)_{\Delta c=1} G\left(\frac{p \cdot p_\mathrm{e}}{q \cdot p}\right),$$

$$\frac{\mathrm{d}\sigma(\mu\mathrm{N} \to \mu\mathrm{e} + ...)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}(p_\mathrm{D} \cdot p_\mathrm{e})} = \frac{B_\mathrm{D}}{q \cdot p} \left(\frac{\mathrm{d}\sigma(\mu\mathrm{N} \to \mu + ...)}{\mathrm{d}x\,\mathrm{d}y}\right)_{\substack{\text{associated}\\ \text{charm}\\ \text{production}}} G\left(\frac{p \cdot p_\mathrm{e}}{q \cdot p}\right), \tag{19}$$

in standard notation. G is measured directly in $\mathrm{e}^+\mathrm{e}^-$ annihilation, and so measurements of the electron spectrum at fixed x and y in high-energy neutrino and muon experiments will provide a direct test of the underlying picture. Since in the electromagnetic case the y distribution is fixed and the cross section concentrated at small x, it is predicted essentially uniquely in terms of $\mathrm{e}^+\mathrm{e}^-$ data according to our model.

In this picture the D meson carries a fraction z of the momentum $x\boldsymbol{p} + \boldsymbol{q}$ of the ejected quark. At subasymptotic energies this is a frame dependent quantity but, as in the $\mathrm{e}^+\mathrm{e}^-$ case, the ambiguity should be resolved by demanding that the limit $z \to 1$ corresponds to the exclusive process with the lowest threshold, in conformity with the behavior of $D(z)$, and so that $z_{\max} = 1$ in all cases and the normalization (eq. (3)) of one charmed particle per charmed quark is maintained **. I.e. we must choose a frame (most conveniently with p and q parallel) in which for given x and y $\boldsymbol{p}_\mathrm{D} = x\boldsymbol{p} + \boldsymbol{q}$ in the exclusive process $\nu\mathrm{N} \to \mu\mathrm{DN}$ and $\mu\mathrm{N} \to \mu\mathrm{Dc}$. Furthermore for hadronic final states of relatively low invariant mass it is necessary to choose some modified scaling variable (e.g. as done in refs. [20,22]) which causes the cross section to vanish smoothly at threshold. In view of this extra ambiguity we give only asymptotic formulae here, extrapolation to low energy being altogether more complicated and more dangerous than in the $\mathrm{e}^+\mathrm{e}^-$ case.

5. Conclusions

Our most important result is that in $\mathrm{e}^+\mathrm{e}^-$ annihilation at very high energy, leptons derived from the decay of charmed particles or heavier particles with new flavors are likely to be very soft (see figs. 1–7 and table 1). This result is very insensi-

* We consider only the products of the charmed quark which is struck by the virtual photon in the electromagnetic case. We are not aware of any serious attempt to treat the spectator (anti) charmed quark's products, which presumably give rise to soft leptons in the hole fragmentation region.

** Derman [20] applied his model at finite energies in such a way that $z_{\max}$ becomes less than one, violating the normalization condition, but it seems likely that this was not important numerically since $D(z)$ is small for z near one.

tive to the details of the model and, we believe, has a much more general validity than the parton fragmentation picture which we used to construct a reasonable inclusive D spectrum. We repeat our earlier conclusion that excellent electron and muon detection at low energy will be very desirable in experiments at PETRA/PEP energies.

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