COMMENTS ON CHARMED BARYON PAIR PRODUCTION IN e^+e^--ANNIHILATION

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We show that the symmetric quark model calculation of charmed baryon pair creation in e^+e^--annihilation has an incorrect threshold structure. The $c\bar{c}$-pair creation model possesses the correct s-wave threshold behaviour, if the $c\bar{c}$-pair and diquark pair are created in orbital s-waves. We show that the two models become identical for a specific orbital configuration and discuss implications for the relative production rates of $\Lambda_c\bar{\Lambda}_c : \Sigma_c\bar{\Sigma}_c : \Sigma_c\bar{\Sigma}_c^* + \Sigma_c^*\bar{\Sigma}_c : \Sigma_c^*\bar{\Sigma}_c^*$.

The discovery of the charmed baryon $\bar{\Lambda}_c$ (2.24) in the Fermilab photo production experiment [1] has given rise to expectations that the observation of charmed baryon pairs in e^+e^--annihilation is imminent. The production rate of charmed baryon pairs can be expected to be reasonably large due to form factor enhancement effects of close-by $c\bar{c}$-vector mesons [2, 3], and, in fact, the authors of ref. [3] have argued that the SPEAR data is already showing some evidence of charmed baryon pair production around $\sqrt{q^2} \approx 4.8$ GeV. In addition to the Λ_c (2.24) the Fermilab data also shows some circumstantial evidence of strongly decaying charmed baryon states Σ_c (2.41) and Σ_c^*(2.48) [4] as predicted in the charmonium picture [5].

Neglecting mass difference effects the authors of ref. [3] have suggested that one can expect production rates of

$$\sigma_{\Lambda_c\bar{\Lambda}_c} : \sigma_{\Sigma_c\bar{\Sigma}_c} : \sigma_{\Sigma_c\bar{\Sigma}_c^*+\Sigma_c^*\bar{\Sigma}_c} : \sigma_{\Sigma_c^*\bar{\Sigma}_c^*} = 3 \cdot 1 : 16 : 10 \tag{1}$$

for the e^+e^--production of charmed baryon pairs. Their model is based on a picture in which the electro-magnetic current first creates a free pair of charmed quarks $c\bar{c}$ which then pick up the missing light quarks from a diquark pair created independently from the vacuum. Subsequent authors generalized this result using the same basic production picture, allowing, however, for more general orbital configurations [4, 6].

In the case of the creation of charmed meson pairs the results of using such a $c\bar{c}$-pair creation model were basically corroborated by an analysis based on a $SU(2)_w$ quark model approach [7].

We shall in this note use the same $SU(2)_w$ approach to calculate production rates of charmed baryon pairs. We find that the $SU(2)_w$ model results are not applicable in the threshold region of baryon pair production since they do not have the correct threshold structure. We then demonstrate that the $c\bar{c}$-pair creation model has the correct threshold structure provided the charmed quark pair and the light diquark pair are created in s-wave states. Finally we show how the $SU(2)_w$ model results are recovered for a specific orbital configuration of the $c\bar{c}$-pair creation approach.

Helicity amplitudes and helicity state counting. In order to enumerate the helicity amplitudes which contribute to the e^+e^- pair creation processes considered here we define as usual [8, 9] c.m. helicity amplitudes $F^{\lambda\lambda'}$ labelled by the particle helicities λ and λ'. The differential cross section is given by

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$$\frac{d\sigma}{d\cos\theta} = \frac{2\pi\alpha^2 p_c}{3(q^2)^{5/2}} \left(\tfrac{3}{8}(1+\cos^2\theta) \sum_\lambda (|F^{\lambda+1,\lambda}|^2 + |F^{\lambda-1,\lambda}|^2 + \tfrac{3}{4}\sin^2\theta \sum_\lambda |F^{\lambda\lambda}|^2 \right), \tag{2}$$

where p_c is the magnitude of the c.m. momenta.

For the three cases one has the following contributions

$$(\tfrac{1}{2}^+\overline{\tfrac{1}{2}^+}) \qquad F^{\frac12 -\frac12} = F^{-\frac12 \frac12}, \qquad F^{\frac12 \frac12} = F^{-\frac12 -\frac12}, \tag{3}$$

$$(\tfrac{1}{2}^+\overline{\tfrac{3}{2}^+} + \tfrac{3}{2}^+\overline{\tfrac{1}{2}^+}) \qquad F^{\frac12 \frac32 *} = -F^{-\frac12 -\frac32 *} = F^{\frac32 * \frac12} = -F^{-\frac32 * \frac12}, \qquad F^{\frac12 -\frac12 *} = -F^{-\frac12 \frac12 *} = -F^{\frac12 * -\frac12} = F^{-\frac12 * \frac12},$$
$$F^{\frac12 \frac12 *} = -F^{-\frac12 -\frac12 *} = F^{\frac12 * \frac12} = -F^{-\frac12 * -\frac12}, \tag{4}$$

$$(\tfrac{3}{2}^+\overline{\tfrac{3}{2}^+}) \qquad F^{\frac12 \frac32} = F^{-\frac32 -\frac12} = F^{\frac32 \frac12} = F^{-\frac12 -\frac32}, \qquad F^{\frac12 -\frac12} = F^{-\frac12 \frac12},$$
$$F^{\frac32 \frac32} = F^{-\frac32 -\frac32}, \qquad F^{\frac12 \frac12} = F^{-\frac12 -\frac12}, \tag{5}$$

where we have starred the $J = \frac{3}{2}$ helicities in eq. (4). The relations (3)–(5) follow from discrete symmetries [8, 9].

We note in passing that a tally of the accessible helicity and charge states results in the ratio $4 \cdot 12 : 36 : 30$, where the transverse and longitudinal parts contribute as $(2:6:24:18)_T$ and $(2:6:12:12)_L$. None of these ratios agree with the result of eq. (1). This is contrary to the case of charmed meson pair production, where the naive counting of helicity states gives the same result as the free $c\bar{c}$-creation approach [7]. In fact, we shall see in the next section that threshold conditions preclude equal weighting of the helicity population at least in the limit $\eta \to 1$ ($\eta \equiv q^2/(m+m')^2$).

LS-amplitudes and threshold conditions. In order to discuss the threshold properties of two-particle production it is convenient to define amplitudes Γ_{LS} that correspond to two-particle states of definite spin S ($\boldsymbol{S} = (\boldsymbol{S}_1 + \boldsymbol{S}_2)$) [10].

In the $\frac{1}{2}^+\frac{1}{2}^+$ case one obtains for the matrix connecting $(\Gamma_{01}, \Gamma_{21})$ with $(F^{\frac12 \frac12}, F^{\frac12 -\frac12})$ ‡

$$\begin{pmatrix} 2/3 & 4/3 \\ -4/3 & 2/3 \end{pmatrix} \tag{6}$$

and for $\frac{1}{2}^+\overline{\frac{3}{2}^+}$ for the matrix connecting $(\Gamma_{01}, \Gamma_{21}, \Gamma_{22})$ with $(F^{\frac12 \frac32}, F^{\frac12 -\frac12}, F^{\frac12 \frac12})$

$$\begin{pmatrix} 1 & 1/3 & 2/3 \\ 1/2 & 1/6 & -4/3 \\ -1/2 & 3/2 & 0 \end{pmatrix} \tag{7}$$

and finally for the matrix connecting $(\Gamma_{01}, \Gamma_{21}, \Gamma_{23}, \Gamma_{43})$ with $(F^{\frac12 \frac32}, F^{\frac12 -\frac12}, F^{\frac32 \frac32}, F^{\frac12 \frac12})$ in the $\frac{3}{2}^+\overline{\frac{3}{2}^+}$ case one has

$$\begin{pmatrix} 8/5 & -8/15 & 3/5 & -1/15 \\ 4/5 & -4/15 & -6/5 & 2/15 \\ 32/35 & 24/35 & 3/35 & 27/35 \\ 24/35 & 18/35 & 4/35 & 36/35 \end{pmatrix}. \tag{8}$$

We have normalized the *LS*-amplitudes such that the total cross section is given by the sums of squares of the *LS*-amplitudes.

‡ In the next three matrices we omit square root signs; thus e.g. $-\frac{4}{3}$ reads $-\sqrt{4/3}$. The matrices are unitary when the dimensionality of the helicity base is taken into account.

In the limit $\eta \to 1$ one has the threshold behaviour

$$\Gamma_{L+2,S}/\Gamma_{L,S(S')} \propto (1-\sqrt{\eta}) \tag{9}$$

which follows either from using the general results of refs. [9, 10] or more directly by expanding the Γ_{LS} in terms of constraint free from factors (see e.g. ref. [2, 11]). And thus, at threshold $\eta = 1$, the helicity amplitudes have to conspire to satisfy

$$\begin{array}{llll} (\tfrac{1}{2}^+\overline{\tfrac{1}{2}^+}) & F^{\frac{1}{2}-\frac{1}{2}} = \sqrt{2}F^{\frac{1}{2}\frac{1}{2}}\,, & (\tfrac{1}{2}^+\overline{\tfrac{3}{2}^+}) & F^{\frac{1}{2}-\frac{1}{2}} = \sqrt{\tfrac{1}{3}}F^{\frac{1}{2}\frac{3}{2}} = \sqrt{\tfrac{1}{2}}F^{\frac{1}{2}\frac{1}{2}} \\ (\tfrac{3}{2}^+\overline{\tfrac{3}{2}^+}) & F^{\frac{1}{2}-\frac{1}{2}} = -\sqrt{\tfrac{4}{3}}F^{\frac{1}{2}\frac{3}{2}} = -\sqrt{\tfrac{8}{9}}F^{\frac{3}{2}\frac{3}{2}} = \sqrt{8}F^{\frac{1}{2}\frac{1}{2}}\,. & & \end{array} \tag{10}$$

This verifies the assertion made in the foregoing section. The above set of constraint equations can of course also be satisfied by evasion if all helicity amplitudes are identically zero at threshold.

We should mention that in the case of the charmed meson pair production discussed in ref. [7] one is dealing only with p-wave production, except for the $D^*\overline{D^*}$ where there is also one $L = 3$ amplitude. Thus there is altogether only one constraint at threshold compared to the many constraints of eq. (10) in the charmed baryon case.

Multipole amplitudes and symmetric quark model. The results of the symmetric quark model are simple when expressed in terms of multipole amplitudes rather than in terms of helicity amplitudes. In analogy to the formalism developed in ref. [9] we define multipole amplitudes by

$$(\tfrac{1}{2}^+\overline{\tfrac{1}{2}^+}) \quad \Gamma_E = \sqrt{2}F^{\frac{1}{2}\frac{1}{2}}\,; \quad \Gamma_M = -F^{\frac{1}{2}-\frac{1}{2}}; \qquad \text{at } \eta = 1 \quad \Gamma_E = -\Gamma_M\,, \tag{11a, b}$$

$$\begin{aligned} (\tfrac{1}{2}^+\overline{\tfrac{3}{2}^+}) \quad & \Gamma_M = -\tfrac{1}{2}\sqrt{3}F^{\frac{1}{2}\frac{3}{2}} + \tfrac{1}{2}F^{\frac{1}{2}-\frac{1}{2}}, \quad \Gamma_Q = \tfrac{1}{2}F^{\frac{1}{2}\frac{3}{2}} + \tfrac{1}{2}\sqrt{3}F^{\frac{1}{2}-\frac{1}{2}}; \quad \Gamma_C = \sqrt{2}F^{\frac{1}{2}\frac{1}{2}}, \\ & \text{at } \eta = 1 \quad \Gamma_Q = -\sqrt{3}\Gamma_M = \tfrac{1}{2}\sqrt{3}\Gamma_C; \end{aligned} \tag{12a, b}$$

$$\begin{aligned} (\tfrac{3}{2}^+\overline{\tfrac{3}{2}^+}) \quad & \Gamma_E = F^{\frac{1}{2}\frac{1}{2}} + F^{\frac{3}{2}\frac{3}{2}}, \quad \Gamma_M = +\sqrt{\tfrac{6}{5}}F^{\frac{1}{2}\frac{3}{2}} + \sqrt{\tfrac{2}{5}}F^{\frac{1}{2}-\frac{1}{2}}, \quad \Gamma_Q = F^{\frac{3}{2}\frac{3}{2}} - F^{\frac{1}{2}\frac{1}{2}}; \quad \Gamma_O = \sqrt{\tfrac{4}{5}}F^{\frac{1}{2}\frac{3}{2}} - \sqrt{\tfrac{3}{5}}F^{\frac{1}{2}-\frac{1}{2}}, \\ & \text{at } \eta = 1 \quad \Gamma_E = \sqrt{5}\Gamma_M = \tfrac{1}{2}\Gamma_Q = \sqrt{\tfrac{5}{24}}\Gamma_O\,, \end{aligned} \tag{13a. b}$$

where the Γ_E, Γ_M, Γ_Q, Γ_C and Γ_O denote charge, magnetic dipole, electric quadrupole, Coulombic quadrupole and magnetic octupole couplings, respectively. In eqs. (11b), (12b) and (13b) we have written down the threshold conditions for the multipole amplitudes. The total annihilation cross section is given by twice the sum of squares of transverse multipole and once the sum of squares of longitudinal multipole amplitudes.

The results of an $SU(2)_w$ quark model analysis based on the transformation between current and constituent quarks [12] can now be expressed in a very concise way. Setting all baryon masses equal one obtains in the $c\bar{c}$-sector

$$\Gamma_E^{\Lambda_c\bar{\Lambda}_c}(q^2) = \Gamma_E^{\Sigma_c\bar{\Sigma}_c}(q^2) = \frac{1}{\sqrt{2}}\Gamma_E^{\Sigma_c^*\overline{\Sigma_c^*}}(q^2) = \frac{2\sqrt{2}}{3}G_E(q^2)\,, \tag{14}$$

$$\Gamma_M^{\Lambda_c\bar{\Lambda}_c}(q^2) = 3\Gamma_M^{\Sigma_c\bar{\Sigma}_c}(q^2) = \frac{3}{2\sqrt{2}}\Gamma_M^{\Sigma_c\overline{\Sigma_c^*}} = \frac{3}{\sqrt{10}}\Gamma_M^{\Sigma_c^*\overline{\Sigma_c^*}} = \frac{2\sqrt{2}}{3}\sqrt{\eta}\,G_M(q^2)\,, \tag{15}$$

and

$$\Gamma_Q^{\Sigma_c\overline{\Sigma_c^*}}(q^2) = \Gamma_C^{\Sigma_c\overline{\Sigma^*}} = \Gamma_Q^{\Sigma_c^*\overline{\Sigma_c^*}}(q^2) = \Gamma_O^{\Sigma_c^*\overline{\Sigma_c^*}}(q^2) = 0\,, \tag{16}$$

where $G_E(q^2)$ and $G_M(q^2)$ denote longitudinal and transverse $SU(2)_w$ photon couplings which are in general independent functions of q^2. For the $I = 1$ cases, eqs. (14)–(16) apply to any one of the charge states. Barring the trivial case that all multipole amplitudes are zero at threshold (evasion), it is clear that the results of the $SU(2)_w$

quark model eqs. (14)–(16) are in contradiction to the threshold relations (11b), (12b) and (13b). This seems to be a manifestation of the basic non-relativistic nature of the quark model which does not exhibit the threshold structure following from relativistic invariance. Let us briefly remark that relativistic formulations of SU(6) [13] and the quark model [14] get around this impasse by predicting an evasive solution for the threshold constraints, i.e. all multipole amplitudes have dynamical zeros at threshold. We shall not further advocate this possibility, which could, however, be checked directly e.g. in the case of $\Sigma_c\overline{\Sigma}{}_c^*$ production by measuring the angular distribution close to threshold. Whereas s-wave dominance gives a flat $\cos\theta$-distribution, eq. (16) predicts a $(1+\cos^2\theta)$-distribution right down to threshold. Away from threshold there is no a priori reason to discard the quark model results eqs. (14)–(16). For the ratio eq. (1) one obtains

$$3\eta|G_M|^2+\tfrac{3}{2}|G_E|^2 : \eta|G_M|^2+\tfrac{9}{2}|G_E|^2 : 16\eta|G_M|^2 : 10\eta|G_M|^2+9|G_E|^2 . \tag{17}$$

One notes the agreement of the transverse contributions with the result of eq. (1). Before continuing with the discussion of eq. (17) we briefly recapitulate and extend the results of refs. [3, 6].

$c\bar{c}$-creation model. In the $c\bar{c}$-creation model the photon creates a $c\bar{c}$-pair with longitudinal and transverse polarization amplitudes a and b ($|a|^2+|b|^2=1$) and a light diquark pair is created from the vacuum with s- and d-wave amplitudes c_s and c_d ($|c_s|^2+|c_d|^2=1$) [6]. Alternatively one can also describe the $c\bar{c}$-creation by s- and d-wave amplitudes $a_s=\sqrt{\frac{1}{3}}a+\sqrt{\frac{2}{3}}b$ and $a_d=-\sqrt{\frac{2}{3}}a+\sqrt{\frac{1}{3}}b$ and the diquark creation by helicity ±1 and helicity 0 amplitudes $c=\sqrt{\frac{2}{3}}c_s+\sqrt{\frac{1}{3}}c_d$ and $d=-\sqrt{\frac{1}{3}}c_s+\sqrt{\frac{2}{3}}c_d$. Then, by assuming the c-quark and the diquark to be in a baryon s-wave state one considers matrix elements of various angular momentum components and arrives at ($\Lambda_c\overline{\Lambda}_c$-production is not directly related in this approach)

$$(\Sigma_c\overline{\Sigma}_c) \qquad F^{\frac12\frac12}=\frac{a}{\sqrt{27}}(c_s+\sqrt{8}c_d) \qquad F^{\frac12-\frac12}=\frac{b}{\sqrt{27}}(c_s-\sqrt{2}c_d), \tag{18}$$

$$(\Sigma_c\overline{\Sigma}{}_c^*) \qquad F^{\frac12\frac12}=\frac{a}{\sqrt{27}}(\sqrt{8}c_s-c_d) \qquad F^{\frac12-\frac12}=-\sqrt{\frac{2}{27}}\,b(c_s-\sqrt{2}c_d)$$
$$F^{\frac12\frac32}=-\tfrac{1}{3}b(\sqrt{2}c_s+c_d), \tag{19}$$

$$(\Sigma_c^*\overline{\Sigma}{}_c^*) \qquad F^{\frac12\frac12}=-\frac{a}{\sqrt{54}}(\sqrt{2}c_s-5c_d) \qquad F^{\frac12-\frac12}=-2\frac{b}{\sqrt{27}}(c_s-\sqrt{2}c_d)$$
$$F^{\frac32\frac32}=\frac{a}{\sqrt{6}}(\sqrt{2}c_s+c_d) \qquad F^{\frac12\frac32}=\frac{b}{\sqrt{18}}(\sqrt{2}c_s+c_d). \tag{20}$$

Note that the transverse transitions $F^{\frac12-\frac12}$ and $F^{\frac12\frac32}$ go only via the helicity ±1 and helicity 0 components c and d, respectively, of the diquark pair. For pure s-wave production of the $c\bar{c}$- and diquark pair, i.e. $a_d=c_d=0$, the above helicity amplitudes satisfy the appropriate particle threshold conditions and also one recovers the $I=1$ part of the prediction eq. (1). In a dynamical picture this situation obtains if the combined threshold of the constituent production coincides with particle threshold. One should thus be careful in arguing for the presence of substantial $c\bar{c}$ d-wave components close to particle threshold.

A clue as to how one obtains correspondence with the $SU(2)_W$ results can be read off eq. (19) by demanding that $\Sigma_c\overline{\Sigma}{}_c^*$ is purely transverse, i.e. setting $c_s=\frac{1}{3}$ and $c_d=\frac{2}{3}\sqrt{2}$. The same s- and d-wave admixture is arrived at by direct construction using quark model wave functions. Then by fixing the s- and d-wave mixture of the $c\bar{c}$-pair at $b/a=\sqrt{2\eta}G_M/G_E$, i.e. the transverse-longitudinal coupling ratio of a free spin $\frac{1}{2}$ particle, one has complete correspondence between the two schemes as can be seen by comparing eqs. (14–16) and eqs. (18–20).

In the $c\bar{c}$-creation model the relative $\Lambda_c\overline{\Lambda}_c$-production rate is usually determined by using SU(6) arguments in addition. From the above it is clear that this prediction of eq. (1) must be treated with caution, since SU(6) realizes only in a situation where the d-wave production of diquark pair dominates ($|c_d|^2/|c_s|^2=8$) which is presumably far above threshold, whereas the result eq. (1) has been derived assuming s-wave dominance of diquark production.

In conclusion we have shown that the $SU(2)_w$ symmetric quark model is too restrictive to be applicable in the threshold region of charm baryon pair production, unless one is willing to accept extra dynamical zeros of all form factors at threshold. This could be checked experimentally by measuring differential cos θ-distributions close to threshold in particular for $\Sigma_c\overline{\Sigma}_c^*$-production. The $c\bar{c}$-pair creation model is less restrictive and possesses correct threshold behaviour if the threshold for constituent pair creation coincides with the particle threshold. In the threshold region the predicted $\Lambda_c\overline{\Lambda}_c$ rate, which uses extra SU(6) input, must be viewed with caution. At higher q^2, the $c\bar{c}$-pair creation model becomes less predictive, since one does not know the q^2-dependence of the various s- and d-wave admixtures.

We have shown that the $SU(2)_w$ model corresponds to a specific orbital configuration of the $c\bar{c}$-pair creation model. If this specific $SU(2)_w$ choice of s- and d-wave admixture is valid in the higher q^2-region, and if $G_E(q^2)/q^2G_M(q^2) \xrightarrow{q^2\to\infty} 0$ as expected in the quark parton model, the (transverse) production cross sections can be expected to be in the ratio 3 : 1 : 16 : 10.

Finally we would like to mention that the above conclusions could be considerably altered if form factor effects due to the coupling of ψ, ψ' ... resonances strongly affect the calculated quark model rates [15]. Hopefully experiments will soon provide an answer to this question.

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