A POSSIBLE IDENTIFICATION OF $\chi(3.45)$ WITH A "TIME-LIKE" $c\bar{c}$ EXCITATION

H. KRASEMANN
II. Institut für Theoretische Physik der Universität Hamburg and Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

and

M. KRAMMER
Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

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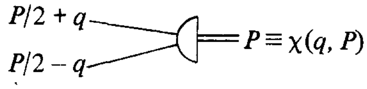
We identify the four observed intermediate states between J/ψ and ψ' with the four $j^{PC} = j^{++}$ P waves of a relativistic bound state model. Assuming a point-like quark photon vertex we calculate bounds on their radiative couplings to J/ψ and ψ' by the help of four-dimensional dipole sum rules. These bounds also imply upper bounds on the total widths.

We suggest that the pions are true relativistic $c\bar{c}$ bound states with the special emphasis that in this case there appear extra states which have no analogue in the nonrelativistic charmonium model. These extra states are excitation in relative time and thus vanish in the nonrelativistic limit. Also the experimental situation possibly indicates the need to go beyond the nonrelativistic description.

Three intermediate states between J/ψ and ψ', $\chi(3.41)$, $P_c(3.51)$ and $\chi(3.55)$ are consistent, in production and decay, with the three nonrelativistic $c\bar{c}$ bound states ^3P_j, $j^{PC} = 0^{++}, 1^{++}, 2^{++}$. The fourth intermediate state [1], $\chi(3.45)$, also is a $c\bar{c}$ state as is strongly suggested by its large coupling to J/ψ. For the X(2.83) [2] on the other hand the $c\bar{c}$ nature is not clear because its coupling to J/ψ is very small and couplings to other $c\bar{c}$ states are not yet observed. Both states, however, the X(2.83) and the $\chi(3.45)$, are welcome as the expected pseudoscalars η_c and η'_c. But both states immediately cause problems for charmonium [3], i.e. the nonrelativistic perturbative treatment of QCD. In particular the fact that one forbidden M1 transition is not seen [4]. $\Gamma(\psi' \to X(2.83)\gamma) < 2\% \cdot (225 \pm 56)$ keV, but another forbidden M1 transition is a main decay mode of $\chi(3.45)$: $B(\chi(3.45) \to J/\psi\gamma) \geq 0.27 \pm 0.13$ [1, 4], would set a very small upper limit on the total width of the $\chi(3.45)$ (< 11 keV) [5].

In relativistic bound state models [6] the first extra $C = +$ state is a "time-like" "P" wave, degenerate with the "space-like" P waves, and its quantum numbers are either $j^{PC} = 0^{++}$ or 1^{++}. The two physical 0^{++} (or 1^{++}) states in principle are mixtures of this time-like P wave and the corresponding space-like P wave. Therefore the $\chi(3.45)$ might either be the second 0^{++} or 1^{++} state, the transition $\chi \to J/\psi + \gamma$ is an electric instead of a magnetic transition and it is not forbidden by any selection rule. Subsequently we sketch the occurrence of time-like excitations in a relativistic Bethe salpeter (BS) model with heavy quarks and strong binding [6]. We further give estimates of γ transitions within this model assuming that the photon couples point-like to the quarks. Our main emphasis, however, are not the detailed features of the model but rather the occurrence of an extra $C = +$ state on the P wave level with $j^{PC} = 0^{++}$ or 1^{++}. Thus it is in principle clearly distinguishable from the η'_c of charmonium by general selection rules, i.e.: i) $\chi \not\to 2$ pseudoscalars if $j^P = 0^-, 1^+, 2^-, \ldots$; ii) $\chi \not\to \gamma\gamma$ if $j = 1$; iii) $(e^+e^- \to \gamma_1\chi \to \gamma_1\gamma_2 J/\psi \to \gamma_1\gamma_2\ell^+\ell^-) \sim (1 + \cos^2\theta_1)(1 + \cos^2\theta_\ell)$ if $j = 0$, where θ_1 is the angle between γ_1 and the beam, θ_ℓ is the angle between the lepton pair and γ_2.

As a guide to the relativistic $c\bar{c}$ bound state problem we take the conventional field theoretical description of mesons by BS amplitudes built from quark fields ψ



$$= \mathcal{F}\langle 0|T\{\psi(x/2),\ \bar{\psi}(-x/2)\}|\text{meson}\rangle$$

which satisfy the homogeneous BS equation

$$S_F^{-1}(P/2+q)\chi(q,P)\bar{S}_F^{-1}(P/2-q)$$
$$= \mathrm{i}\int d^4q'\,\mathcal{K}(P,\lambda;q,q')\chi(q',P)\,.$$

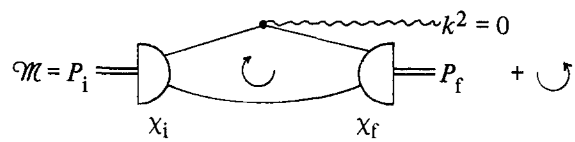
The explicit model by Böhm, Joos and Krammer assumes i) heavy quarks ($2m_q \gg M_{\text{bound state}}$), ii) a convolution type kernel $\mathcal{K} = \lambda\cdot\mathcal{K}(q-q')$ with the coupling strength λ (which corresponds to strong binding in a smooth potential) and free quark propagators S_F, iii) a spin structure which reproduces the singlet triplet structure of the mesons. Technical inputs are the Wick rotation to Euclidean space, the O(4) expansion and the oscillator approximation to the potential-like kernel. A Dirac structure + $(\gamma_5\times\gamma_5)$ of the interaction kernel leads to the leading components of the BS amplitudes $\chi^0 = P + T + S = \gamma_5\Phi_P + \sigma_{\mu\nu}\Phi_T^{\mu\nu} + \Phi_S$ and $-(\gamma_5\times\gamma_5)$ to $\chi^0 = A + V = \gamma_5\gamma_\mu\Phi_A^\mu + \gamma_\mu\Phi_V^\mu$ and to the scalar equation (in Euclidean space)

$$-(q^2 + m_q^2 - M^2/4)\Phi(q) = \int d^4q\,K(q-q')\Phi(q')\,. \quad (1)$$

This equation allows separation of the Φ's into O(4) spherical harmonics Y_{nlm} and the hyperradial part. The Y_{nlm} are labelled by the O(4) quantum number n and the ordinary angular momentum quantum numbers l, m with the restriction $n \geq l \geq 0$. $(n-l)$ counts the nodes in relative time. For a given n the degeneracy is $(n+1)^2$ as compared to $2l+1$ in the O(3) case. Although part of the time excited states are ruled out by the field theoretical normalization condition [7] there remain more states than in nonrelativistic models. The lowest ones of the remainder have $n = 1$ and $l = 0$. On the next excitation level $n = 2$ there are six time excited states, among those two vector mesons [8]. In table 1 we give the S and P wave covariants for both models, which have to be multiplied with the hyperradial wavefunction to yield the full amplitude $\chi^0(q, P)$.

The occurrence of a fourth $C = +$ "P" wave state leads us to tentatively identify the χ(3.45) as a member of the "P" family. In order to further investigate this possibility we study the radiative transitions among $c\bar{c}$ states.

The simplest ansatz for radiative transitions is a point-like quark photon coupling. In the BS model gauge invariance of the point coupling is guaranteed [9] for light-like photons ($k^2 = 0$) by taking the BS amplitudes at the meson poles.



$$k^\mu\epsilon_\mu(\gamma) = 0, \qquad P_i + P_f + k = 0\,.$$

Evaluating this matrix element according to the Feynman rules gives in the $-\gamma_5\times\gamma_5$ model ‡1

$$\mathcal{M} = \mathrm{Tr}(\mathrm{SU}_4)e(2\pi)^{-9/2}\mathrm{i}\int d^4q$$
$$\times\, 2\,[\epsilon^\mu(\gamma)(2q_\mu + P_{i\mu})\Phi_i\Phi_f + (k^\nu\epsilon^\mu(\gamma) - k^\mu\epsilon^\nu(\gamma)\Phi_{i\mu}\Phi_{f\nu}] \quad (2)$$

if Φ_i and Φ_f are out of the same Dirac sector V or A and

$$\mathcal{M} = \mathrm{Tr}(\mathrm{SU}_4)\,e(2\pi)^{-9/2}$$
$$\times\,\mathrm{i}\int d^4q\,2\epsilon_{\alpha\beta\gamma\delta}k^\alpha\epsilon^\beta(\gamma)\,\Phi_f^\gamma\Phi_i^\delta \quad (3)$$

‡1 Similar formulae hold in the $+\gamma_5\times\gamma_5$ model.

Table 1
S and P wave covariants in the models $\pm\gamma_5\times\gamma_5$. $\epsilon_\mu(\epsilon_{\mu\nu})$ denote the polarization vector (tensor). In the two lowest lines the extra states are found which vanish in the nonrelativistic limit.

	J^{PC}	Covariants $(-\gamma_5\times\gamma_5)$	Covariants $(+\gamma_5\times\gamma_5)$
$n=0$	0^{-+}	$\gamma_5\hat{P\!\!\!/}$	γ_5
	1^{-+}	$\epsilon\!\!/$	$\hat{P\!\!\!/}\epsilon\!\!/$
	1^{+-}	$2\gamma_5\hat{P\!\!\!/}\epsilon_\mu\hat{q}^\mu$	$2\gamma_5\epsilon_\mu\hat{q}^\mu$
$n=1$	0^{++}	$2/\sqrt{3}\,(\hat{q\!\!\!/} - (\hat{q}\cdot\hat{P})\hat{P\!\!\!/})$	$2/\sqrt{3}\hat{P\!\!\!/}(\hat{q\!\!\!/} - (\hat{q}\cdot\hat{P})\hat{P\!\!\!/})$
$l=1$	1^{++}	$2/\sqrt{2}\epsilon^{\mu\nu\rho\sigma}\epsilon_\mu\hat{q}_\nu\hat{P}_\rho\gamma_\sigma$	$2/\sqrt{2}\hat{P\!\!\!/}\epsilon^{\mu\nu\rho\sigma}\epsilon_\mu\hat{q}_\nu\hat{P}_\rho\gamma_\sigma$
	2^{++}	$2\epsilon_{\mu\nu}\hat{q}^\mu\gamma^\nu$	$2\hat{P\!\!\!/}\epsilon_{\mu\nu}\hat{q}^\mu\gamma^\nu$
$n=1$	1^{+-}	$2\gamma_5\epsilon\!\!/(\hat{q}\cdot\hat{P})$	0^{+-} $2(\hat{q}\cdot\hat{P})$
$l=1$	0^{++}	$2(\hat{q}\cdot\hat{P})\hat{P\!\!\!/}$	1^{++} $2\gamma_5\hat{P\!\!\!/}\epsilon\!\!/(\hat{q}\cdot\hat{P})$

if not. Here Tr (SU$_4$) = 4/3 for $c\bar{c}$ and

$$\Gamma = \frac{4}{4M_\mathrm{i}^2} \cdot \frac{1}{2j_\mathrm{i}+1} \cdot \int \frac{\mathrm{d}\Omega}{4\pi} \sum_{\mathrm{i,f}} |\mathcal{M}|^2 . \tag{4}$$

In (2) the first term is a dipole term, while the second one gives the expected relativistic corrections [10]. These are of the order of k/M and numerically add up to 10 to 20% in the rates. Keeping these relativistic corrections in mind we will only discuss the dipole term. In the dipole approximation the $n-l$ = even excitations do not couple to the $n-l$ = odd ones. This is easily seen in the matrix element involving vector states $\Phi^\nu \sim \epsilon^\nu$ and the scalar state $\Phi_\nu \sim (\hat{q}\cdot\hat{P})\hat{P}_\nu$: $\mathcal{M} \sim \epsilon^\nu \hat{P}_\nu \epsilon^\mu(\gamma) I_\mu$. Here I_μ denotes the loop integral. Gauge invariance requires $k^\mu I_\mu = 0$, thus $I_\mu \sim k_\mu$ and therefore $\mathcal{M}$ vanishes. However, the ordinary scalar P state $\Phi_\nu \sim \hat{q}_\nu - (\hat{q}\cdot\hat{P})\hat{P}_\nu$ couples to $\Phi^\nu \sim \epsilon^\nu$. Since the physical states in principle are mixtures of these two degenerate scalars ‡2 they may both couple to the vector state.

In order to calculate the matrix elements (2) one could start assuming a special "potential" and then solve the hyperradial equation (1) to gain the amplitudes Φ which enter eq. (2). Instead we will apply dipole sum rules [11] generalized to four dimensions ‡3:

$$4 = \sum_\mathrm{f} \tfrac{1}{4}(M_\mathrm{i}^2 - M_\mathrm{f}^2)|x|_\mathrm{fi}^2 ;$$

$$\frac{-n^2}{4(n+1)} = \sum_{f,\, n_\mathrm{f}=n_\mathrm{i}-1} \tfrac{1}{4}(M_\mathrm{i}^2 - M_\mathrm{f}^2)|x|_\mathrm{fi}^2 ; \tag{5}$$

$$\frac{(n+2)^2}{4(n+1)} = \sum_{f,\, n_\mathrm{f}=n_\mathrm{i}+1} \tfrac{1}{4}(M_\mathrm{i}^2 - M_\mathrm{f}^2)|x|_\mathrm{f}^2 ;$$

‡2 In the $+\gamma_5 \times \gamma_5$ model there are two degenerate axial vectors instead of scalars. The same arguments hold for these.

‡3 We derive the four dimensional analogue of the nonrelativistic dipole sum rules with the full (squared) Hamiltonian $H^2 = P^2 + M^2$.

to the width formula (4) in the corresponding shape

$$\Gamma = \tfrac{1}{3}\, \mathrm{Tr}^2(\mathrm{SU}_4)\, \alpha (2j_\mathrm{f}+1) k_\mathrm{fi}^3 |x|_\mathrm{fi}^2 .$$

Now we use the property of the harmonic oscillator that for radiative transitions i) $1\mathrm{P}_j \to 1\mathrm{S}_1$ saturates the sum rule for $1\mathrm{S} \to \Sigma f$ and ii) $2\mathrm{S}_1 \to 1\mathrm{P}_j$ together with the former one saturates $1\mathrm{P}_j \to \Sigma \mathrm{S}_1 (n_\mathrm{f} = n_\mathrm{i} - 1)$. Since the first transition is maximal and the sum is negative, also the second transition is maximal. We now replace $(M_\mathrm{i}^2 - M_\mathrm{f}^2)|x|_\mathrm{fi}^2 = 2M_\mathrm{i} k_\mathrm{fi} |x|_\mathrm{fi}^2$ by its oscillator value, thus saturating the sum rules (5), and obtain

$$\Gamma(1\mathrm{P}_j \to 1\mathrm{S}_1\gamma) < \tfrac{1}{3}\,\mathrm{Tr}^2(\mathrm{SU}_4)\, \alpha\, 2\frac{k_{1\mathrm{S},1\mathrm{P}}}{M_{1\mathrm{P}}} ,$$

$$\Gamma(2\mathrm{S}_1 \to 1\mathrm{P}_j\gamma) < \tfrac{1}{3}\,\mathrm{Tr}^2(\mathrm{SU}_4)\, \alpha\, \frac{2j+1}{3} \frac{k_{1\mathrm{P},2\mathrm{S}}^2}{M_{2\mathrm{S}}} . \tag{6}$$

The same result holds – to this approximation – in the $+\gamma_5 \times \gamma_5$ model. The numerical values are given in table 2.

The experimental information on $\psi' \to \chi(3.45) + \gamma$ allows to determine the admixture of the space like P wave covariant to the time-like covariant. This admixture also accounts for the $\chi(3.45) \to \mathrm{J}/\psi + \gamma$ decay and allows to derive limits on the total width of $\chi(3.45)$ which may well be in the range of 100 to 500 keV, see table 2.

For the magnetic transitions the same model yield from (3)

$$\Gamma(\mathrm{V_r} \to \mathrm{PS_{r'}}\gamma) = \tfrac{1}{3}\,\mathrm{Tr}^2(\mathrm{SU}_4)\, \alpha \frac{4}{M^2} k_\mathrm{fi}^3 \delta_\mathrm{r}^{\mathrm{r}'} \tag{7}$$

Table 2

Limits on the radiative widths $\Gamma_1^1 \equiv \Gamma(\psi' \to \mathrm{P_c}/\chi\gamma)$ and $\Gamma_2 \equiv \Gamma(\mathrm{P_c}/\chi \to \mathrm{J}/4\gamma)$ and on the total widths of the $\mathrm{P_c}/\chi$ states. The columns a(b) refer to the $(\mp)\gamma_5 \times \gamma_5$ model. B_exp is taken from refs. [1,4].

	Γ_1 (keV)	Γ_2 (keV)	$\Gamma_{\mathrm{P_c}/\chi} = \frac{\Gamma_2}{B_\mathrm{exp}}$ (MeV) a	b
$\chi(3.55)$	< 34	< 460	< 1.7 ± 0.8	< 1.7 ± 0.8
$\mathrm{P_c}(3.51)$	}b < 34	}b < 380	< 0.63 ± 0.16	< 0.63 ± 0.16
$\chi(3.45)$	}a	}a	< 0.45 ± 0.23	< 0.15 ± 0.08
$\chi(3.41)$	< 27	< 230	< 1.0 ± 0.6	< 1.0 ± 0.6

where $M = M_{PS}(M_V)$ for the $(\mp)\gamma_5 \times \gamma_5$ model. As before the quark masses do not show up in this formula but only the masses of the physical particles, in contrast to charmonium calculations.

We have discussed the possibility that the $\chi(3.45)$ is a fourth P wave $c\bar{c}$ state as present in relativistic models. We found that its experimental properties lead to reasonable total widths in our model. The pseudoscalar η_c' has still to be found. The pseudoscalar η_c is to be found, too, if the X(2.83) is no $c\bar{c}$ state. On the other hand, if the X(2.83) is the η_c, the smallness of its radiative coupling to J/ψ has te be explained. Unfortunately our simple ansatz would not do this; eq. (7) yields numerical results similar to nonrelativistic charmonium calculations. We believe that this problem could only be resolved by dropping the point-like quark photon coupling, which, on the other hand, is even numerically successful in the case of the electric transitions to the $c\bar{c}$ P waves.

Finally we emphasize that – not only in the case of $\chi(3.45)$ – the search for a relativistic degree of freedom in the hadron spectrum is important.

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References

[1] SLAC-LBL-Collaboration: J.S. Whitaker et al., Phys. Rev. Lett. 37 (1976) 1596;
PLUTO-Collaboration: V. Blobel in Proc. XII Rencontre de Moriond (Flaine 1977) (ed. Tran Thanh Van).
[2] DASP-Collaboration: W. Braunschweig et al., DESY 77/02, Phys. Lett., to be published.
[3] T. Appelquist and H.D. Politzer, Phys. Rev. Lett. 34 (1975) 43;
A. De Rújula and S.L. Glashow, Phys. Rev. Lett. 34 (1975) 46;
E. Eichten et al., Phys. Rev. Lett. 34 (1975) 369.
[4] Upper limits on $J/\chi \to \gamma X$, $\psi' \to \gamma\chi, \gamma X$:
MP^2S^3-Collaboration: reported by E. Hilger at the Aachen Conference (1977). Even for $M_{\eta_c} \neq 2.83$ GeV but < 3.1 GeV the limit $B(\psi' \to \gamma\eta_c) < 2\%$ holds.
[5] M.S. Chanowitz and F.J. Gilman, Phys. Lett. 63B (1976) 178.
[6] For a review see:
M. Böhm, H. Joos and M. Krammer, Act. Phys. Austr. Suppl. XI (1973) 3; Ref. TH-1949 CERN;
M. Krammer, Act. Phys. Austr. 40 (1974) 187.
[7] S. Mandelstam, Proc. Roy. Soc. A233 (1955) 248;
R.E. Cutkowsky and M. Leon, Phys. Rev. 135B (1964) 1445.
[8] M. Böhm, H. Joos and M. Krammer, Act. Phys. Austr. 38 (1973) 123.
[9] J.M. Golden, Nucl. Phys. B23 (1970) 19.
[10] See e.g. Y. Tosa, Nagoya Univ. preprint DPNU-34 (1976).
[11] H.A. Bethe and E.E. Salpeter, Quantum mechanics of one- and two-electron atoms, New York 1957;
J.D. Jackson, Phys. Rev. Lett. 37 (1976) 1107.