Nuclear Physics B138 (1978) 345–364

SU(4) WEAK CURRENTS

T.C. YANG
Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany
and
Department of Physics and Astronomy, University of Massachusetts, Amherst, MA, USA *

Received 15 August 1977
(Revised 27 February 1978)

We suggest $SU_L(4) \otimes U(1)$ as the gauge symmetry of weak and electromagnetic interactions for quartets of quarks and leptons. We analyze how the (additional) $SU_L(4)$ weak currents (besides the $SU_L(2)$ subgroup) could affect the weak interactions of ordinary particles, the atomic parity violation, the neutral-current neutrino reactions and the decays of the τ heavy lepton and the charmed mesons. The suppression of neutral-current parity violation in atomic experiments can be naturally incorporated in this model while at the same time the success of the Weinberg-Salam model with respect to the inclusive neutral current data is kept. The model has limited freedom and therefore many definite predictions.

1. Introduction

In this paper we present a gauge model of unified weak and electromagnetic interactions based on $SU_L(4) \otimes U(1)$ symmetry.

The $SU_L(2) \otimes U(1)$ model of Weinberg and Salam [1] (WS) predicts parity violation (due to neutral currents) in heavy atoms. The atomic bismuth experimental results ** are about an order of magnitude smaller than that predicted by the WS model using the atomic calculations [3]. Thus, insofar as the atomic physics calculations are believable, the atomic bismuth results imply modification of the basic Weinberg-Salam model. The Weinberg-Salam model, on the other hand, is in good agreement with the inclusive neutral current neutrino data. The $SU_L(4) \otimes U(1)$ model presented here preserves this success and incorporates the atomic bismuth results. It also has many other definite predictions which were previously reported elsewhere [4].

* Present address. Work supported in part by the National Science Foundation.
** The latest results, reported by Sanders [2] are $R_{876\mathrm{nm}} = (-0.7 \pm 3.2) \times 10^{-8}$ (Univ. of Washington experiment) and $R_{648\,\mathrm{nm}} = (2.7 \pm 4.7) \times 10^{-8}$ (Oxford experiment). The Weinberg-Salam model with the atomic physics calculation predicts $R_{876\,\mathrm{nm}} \sim -23 \times 10^{-8}$ and $R_{648\,\mathrm{nm}} = -30 \times 10^{-8}$.

A detailed analysis of SU(4) symmetry breaking and the constraints induced by such symmetry breaking is presented here.

We suggest in this model a new domain of weak interactions. The $SU_L(4) \otimes U(1)$ model predicts strangeness-changing neutral currents in certain decay channels of the heavy leptons. We realize that strangeness-changing neutral currents are very much suppressed in the weak interactions of ordinary particles. This can be naturally explained with the GIM mechanism [5] (a phenomena associated with diagonal neutral currents). This feature is preserved in this model. The difference is that the off-diagonal neutral currents carrying strangeness (associated with the U-spin raising and lowering operators), which are inevitable in any model where the u, d, s, c quarks belong to one representation *, do not participate in the weak interactions of the ordinary particles, yet could contribute to the decays of the heavy leptons. This observation could be made more general beyond the present model. It could be experimentally tested. The suppression of strangeness-changing neutral currents in the ordinary sector is "natural" [6], that is, independent of the parameters of the Lagrangian. The W bosons associated with the U-spin raising and lowering currents are not assigned with super-heavy masses. Even though the model may not reflect the real world, this aspect itself warrants sufficient study.

Another feature of this model which is different from the others is the large number of heavy leptons predicted here. This is forced upon us by the cancellation of the triangle anomalies. Specifically, we have three quartets of leptons: (e^-, ν_e, ν_τ, τ^-) with lepton number as of the electron; (μ^-, ν_μ, M^0, M^-) with lepton number as of the muon; and a third quartet with a new lepton number. The τ^- and its associated neutrino ν_τ are identified with the heavy lepton observed at SLAC [7] and DESY [8]. In this model τ is an excited electron. The decays of τ turn out to be very much like a sequential lepton, but being an excited electron, it has coupling to the electron (although much suppressed) which will reveal its identity. The particular decay modes $\tau^- \to eK_S$, e^-K^{*0}, $e^-K\pi\pi$, ... should be searched for experimentally. The existence of such decay modes demonstrates (a) the weak interactions of strangeness-changing neutral currents, (b) the existence (e.g. the eK_S invariant-mass plot) and the lepton number of τ, which so far have been based on indirect experimental evidence. The decays of M^0 and M^- show similar characteristics. Neutrino reactions were reported which suggest heavy leptons of the muon type as the origin. Such reactions are anticipated in this model.

The model is based on several theoretical observations discussed in sect. 2. We present a heavy-lepton mass relation and a Cabibbo angle formula. Sect. 3 is an

* We argue that the approximate strong interaction symmetry is for members of the same representation. New heavy quarks, if they exist, form another SU(4) quartet (t, b, h, g) and we anticipate an approximate SU(3) symmetry for the t, b, h quarks as well. If (u, d), (c, s) quarks belong to different representations, as in the $SU(2) \otimes U(1)$ model, the strong SU(3) symmetry of the u, d, s quarks appears to us as an accident.

analysis of the Higgs mechanism. This section contains the results of an extensive study of the gauge SU(4) symmetry breaking which is itself an interesting topic. Sect. 4 contains the effective interaction Lagrangian. Sect. 5 deals with the suppression of strangeness-changing neutral currents in the ordinary weak interaction. Sect. 6 confronts the model with present data. Sect. 7 gives the model predictions for τ and charmed meson decays. We give our conclusions in sect. 8. The Higgs potential is dealt with in the appendix.

2. The fermion representations

Under the $SU_L(2)$ symmetry the left-handed leptons and quarks (ν_e, e^-), (ν_μ, μ^-), (u, d) and (c, s) form doublets. With respect to the $SU_L(4)$ symmetry, the two quark doublets form a quartet, namely (u, d, s, c) (neglecting the Cabibbo angle for the moment), which transforms like the 4 representation. If we assign the leptons to an SU(4) quartet like the quarks, i.e. $(\nu_e, e^-, \mu^-, \nu_\mu)$, many of the W bosons which mediate either the lepton-number non-conserving transitions or strangeness-changing neutral currents (e.g. $\overline{\psi}_L \gamma_\mu \lambda_{6+i7} \psi_L$) will have to be extremely heavy, since such phenomena are untolerable in the weak interactions of ordinary particles [9]. We regard this lepton representation unsatisfacotry also for several other reasons. First, the quark-lepton analogy would imply similar mass breaking for the quarks and the leptons. Note that the quarks and leptons have the same flavor interactions under SU(4). However, ν_μ is almost massless whereas the c quark is much heavier than the ordinary quarks. Secondly, this assignment leaves no room for a "heavy lepton" τ whose properties have been extensively studied at SLAC and DESY. The third reason is that this assignment is not anomaly-free with respect to the full SU(4) gauge symmetry unless one introduces in addition a set of 16 mirror fermions to cancel the triangle anomalies; which seems to us to be arbitrary. The last reason is that it predicts large parity violation in heavy atoms such as bismuth, as does the WS model.

We propose to assign (e^-, ν_e) and (μ^-, ν_μ) to two different SU(4) multiplets. The arguments can be seen by first considering the triangle anomalies.

2.1. Cancellation of triangle anomalies and the lepton representation

For an SU(n) symmetry, $n > 2$, the leptons must belong to representations conjugate to those of the quarks in order to cancel the triangle anomalies. The quark and lepton charges must sum up to zero if the anomalies associated with the U(1) currents are to be canceled. This results in a proliferation of the lepton families. The charges of the lepton quartets can only be in sequence as $(-1, 0, 0, -1)$ or $(0, +1, +1, 0)$, barring doubly charged particles. The electron and muon family can only take the first charge assignment. We have the following quark and lepton representations:

$$\begin{bmatrix} u^R & u^G & u^B \\ d^R & d^G & d^B \\ s^R & s^G & s^B \\ c^R & c^G & c^B \end{bmatrix}, \qquad \begin{bmatrix} e^- & \mu^- & \ell^0 \\ \nu_e & \nu_\mu & \ell^+ \\ \nu_\tau & M^0 & L^+ \\ \tau^- & M^- & L^0 \end{bmatrix},$$

where R, G, B denote the colors of the quarks, ℓ(L) represents a family of sequential leptons. The quarks have unbroken color symmetry. If the leptons also have a "color" symmetry of their own kind, the lepton color symmetry must be badly broken since they have integer charges and unequal masses. We note that the electron and muon must have charges opposite to the proton, a consequence of the anomaly concellation. Experimental evidence shows that τ can only be either a sequential lepton or an excited electron * (ortholepton). This limits τ to have the electron quantum number, noticing that ℓ^-, L^- decay *via* V + A interactions and thus cannot be the τ lepton.

2.2. The fermion masses

The fact that fermions get their masses from the Higgs mechanism provides us with a universal mass breaking for the quarks and the leptons. We assume, for simplicity, that the same set of Higgs scalars couple to the quarks and leptons. We then expect roughly $m_e, m_{\nu_e} < m_{\nu_\tau} << m_\tau$, as we know m_u^{bare}, $m_d^{bare} < m_s^{bare} << m_c^{bare}$ Indeed the relation between the quarks and the electron family may be an approximate equality: $m_e, m_{\nu_e}, m_u^{bare}, m_d^{bare} \sim 0$ is consistent with strong PCAC, and experimentally one has $m_\tau \sim m_c^{bare} \sim m_D$. We therefore conjecture that $m_{\nu_\tau} \sim m_s^{bare}$ might also be true. (m^{bare} denotes the mass that appears in the Lagrangian, i.e. the current-algebra mass.)

(Using the mass relation $m_{\nu_\tau}/m_\tau \simeq m_s^{bare}/m_c^{bare}$, we could in fact argue that the τ and D meson masses are close to each other, based on our knowledge about m_s^{bare} and m_{ν_τ}. A current algebra plus quark model estimate gives m_s^{bare} in the range of a couple of hundred MeV. m_{ν_τ} is experimentally limited to below 600 MeV, and in this model ν_τ has to be heavier than 350 MeV, a limit obtained from the absence of $K^+ \to \pi^+ \nu_\tau \bar{\nu}_e$ etc. modes **. This means m_τ/m_D is O(1).)

For the same reason, we expect $m_\mu, m_{\nu_\mu} < m_{M^0} << m_{M^-}$ and the mass relation $m_{\nu_\tau}/m_\tau \simeq m_{M^0}/m_{M^-}$ should be true. (The mass relation cannot be used for light leptons, such as electron, muon, etc., since the radiative corrections could be comparable to the bare masses themselves. For heavy leptons, $m^{physical} \simeq m^{bare}$ may not be a bad approximation.) Interpretation of the trimuon events with the heavy

* For the latest review see ref. [10].

** It has been brought to our attention that the experimental detection efficiency was sharply cut off for $E_\pi < 250$ MeV in the experiment for $K \to \pi\nu\nu$ [22]. This in turn means a lower bound $m_{\nu_\tau} > 130$ MeV.

lepton chain decay hypothesis leads to an estimate of their masses consistent with our formula [11a] *.

2.3. The Cabibbo angle

The main reason for introducing charm is to cancel the strangeness-changing neutral currents within the SU(2) ⊗ U(1) sector [4]. When we extend the gauge symmetry beyond SU(2) ⊗ U(1), we need to study this question regarding the "new" currents. We find that it restricts severely the structure of the weak currents.

With respect to the SU(2) sub-symmetry, the quarks are in doublets $\binom{u}{d}$ and $\binom{c}{s}$; the Cabibbo angle could appear either as a mixing between the d and s states (the GIM scheme) or between the u and c states (namely,

$$u(\theta) = u \cos\theta_C - c \sin\theta_C , \qquad c(\theta) = u \sin\theta_C + c \cos\theta_C) .$$

They are equivalent to each other: both give the same SU(2) ⊗ U(1) weak currents. However, in a fully gauged SU(4) model, the two schemes have quite different phenomenological implications. The origin of the Cabibbo angle also appears quite different.

The first thing we note is that if the d and s quarks are mixed, then a neutral W boson coupling to a diagonal neutral current, e.g.

$$\bar{\psi}_L \gamma_\mu (\sqrt{\tfrac{2}{3}}\lambda_8 + \sqrt{\tfrac{1}{3}}\lambda_{15})\psi_L \sim \bar{\psi}_L \gamma_\mu \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \psi_1$$

would generate strangeness-changing neutral currents and induce $K \to \pi ee$ decays. Thus, assuming GIM mixing, this neutral W boson would have to be very heavy. This choice is not acceptable, if we do not want large parity violation in heavy atoms such as bismuth, since the masses of the W bosons are then constrained. This difficulty is absent if the Cabibbo mixing appears in the u and c quark sector (and d, s remain unmixed). This is what we advocate. It leads to an interesting alternative interpretation of the Cabibbo angle.

We note that for GIM mixing the Cabibbo angle does not exist in the SU(3) limit. The usual interpretation of the Cabibbo angle is expressed through the chiral SU(3) symmetry breaking. In contrast, the Cabibbo angle expressed in u and c quark mixing cannot be rotated away, even in the SU(3) limit, since $m_u \neq m_c$. The Cabibbo angle in such a theory is characterized by the different orders of symmetry breaking expressed in terms of m_u and m_c. Since the breaking of the strong interaction symmetry appears only in the quark mass term, the large charmed-quark mass manifests a badly broken SU(4) symmetry to the level of SU(3) symmetry. A second step of symmetry breaking (of the SU(3) symmetry), on a much smaller scale, is responsible for the u-quark mass and mixing between the u and c quarks (Δm_{uc}). If that is the

* Other evidence for a possible neutral heavy lepton M^0 is reported in ref. [11b].

case, one finds after diagonalization of the mass matrix (m_u, $\Delta m_{uc} << m_c$)

$$\sin\theta_C \sim \frac{\Delta m_{uc}}{m_c - m_u} \sim \frac{m_u}{m_c - m_u},$$

where m_u, m_c refer to the "physical" quark masses. Taking $m_u \sim \frac{1}{3}m_N \sim 0.31$ GeV, $m_c \sim \frac{1}{2}m_{J/\psi} \sim 1.55$ GeV, as commonly assumed, we arrive at $\sin\theta_c \sim 0.24$, which is quite encouraging. If the origin of the second symmetry breaking is due to radiative corrections, the Cabibbo angle should be calculable. This is an interesting possibility which remains to be investigated.

3. The breaking of the SU(4) gauge symmetry

The weak interaction phenomenology is crucially determined by the masses (eigenvalues) and the eigenstates of the W bosons. This is dictated by spontaneous symmetry breaking. Even though SU(4) symmetry breaking is more complicated than the SU(2) case, much of the results can in fact be understood from group decomposition. Nevertheless it is still a non-trivial matter to explicitly demonstrate the Higgs mechanism. The latter often has more constraints.

3.1. The SU(2) subgroup

The SU(2) symmetry here refers to the symmetry of the conventional charged currents à la Weinberg-Salam model. In terms of the quartet representation ($u(\theta_c)$, d, s, $c(\theta_c)$), where $u(\theta_c) = u\cos\theta_c - c\sin\theta_c$, $c(\theta_c) = u\sin\theta_c + c\cos\theta_c$, the SU(2) consists of the following generators:

$$\lambda_{W^+} \equiv \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & K & 0 \end{bmatrix} = \tfrac{1}{2}[\lambda_{1+i2} + K\lambda_{13-i14}] = [\lambda_{W^-}]^+ ,$$

$$\lambda_3 = \tfrac{1}{2}[\lambda_{W^+}, \lambda_{W^-}] = \tfrac{1}{2}[\lambda_3 + \sqrt{\tfrac{1}{3}}\lambda_8 - \sqrt{\tfrac{2}{3}}\lambda_{15}] = \begin{bmatrix} \frac{1}{2} & & & \\ & -\frac{1}{2} & & \\ & & -\frac{1}{2} & \\ & & & \frac{1}{2} \end{bmatrix}, \qquad (3.1)$$

where $K = \pm 1$; λ's are the SU(4) λ-matrices. The sign of K is not determined experimentally.

3.2. The hierarchy of the SU(4) symmetry breaking

First, we note that the SU(2) of (3.1) is formed by the direct sum of the generators of two SU(2) groups operating in the (u, d) and (s, c) spaces. The SU(4) group contains another SU(2) subgroup in addition to (3.1) which contains the following generators:

$$\lambda_{(K\bar{D})^0} \quad \begin{bmatrix} 0 & 0 & 0 & -h \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \tfrac{1}{2}[\lambda_{6+i7} - h\lambda_{9+i10}] = [\lambda_{(\bar{K}D)^0}]^+ ,$$

$$\lambda_Y = \tfrac{1}{2}[\lambda_{(K\bar{D})^0}, \lambda_{(\bar{K}D)^0}] = \sqrt{\tfrac{1}{3}}\lambda_8 + \sqrt{\tfrac{1}{6}}\lambda_{15} = \begin{bmatrix} \frac{1}{2} & & & \\ & \frac{1}{2} & & \\ & & -\frac{1}{2} & \\ & & & -\frac{1}{2} \end{bmatrix} , \tag{3.2}$$

where $h = \pm 1$. The SU(4) has only one simple subgroup which contains the SU(2) of (3.1) as a subgroup. This group is O(5) (or SP(4)), which consists of the generators given in (3.1) and (3.2) ($h = K = \pm 1$), and in addition the following generators:

$$\lambda_{K^+} \equiv \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \tfrac{1}{2}\lambda_{4+i5} = [\lambda_{K^-}]^+ ,$$

$$\lambda_{D^+} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \tfrac{1}{2}\lambda_{11-i12} = [\lambda_{D^-}]^+ . \tag{3.3}$$

The two SU(2) subgroups of (3.1) and (3.2) with $K = h = \pm 1$ are subgroups of O(5). However, the two subgroups with $K = -h = \pm 1$ are subgroups of SU(4) but not O(5).

The rest of the SU(4) generators are given by

$$\lambda_{H^+} \equiv \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -K & 0 \end{bmatrix} = \tfrac{1}{2}[\lambda_{1+i2} - K\lambda_{13-i14}] = [\lambda_{H^-}]^+ ,$$

$$\lambda_{H^0} \equiv \begin{bmatrix} 0 & 0 & 0 & h \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \tfrac{1}{2}[\lambda_{6+i7} + h\lambda_{9+i10}] = [\lambda_{\bar{H}^0}]^+ , \tag{3.4}$$

and

$$\lambda_X \equiv \begin{bmatrix} \frac{1}{2} & & & \\ & -\frac{1}{2} & & \\ & & \frac{1}{2} & \\ & & & -\frac{1}{2} \end{bmatrix} = \tfrac{1}{2}[\lambda_3 - \sqrt{\tfrac{1}{3}}\lambda_8 + \sqrt{\tfrac{2}{3}}\lambda_{15}] . \tag{3.5}$$

From the above remarks, we conclude with the following three typical routes of breaking the SU(4) symmetry.

(i) SU(4) → O(5) → SU(2). The "super" symmetry breaking provides us with SU(4) – O(5) = 5 W bosons of superheavy masses and O(5) – SU(2) = 7 W bosons of intermediate masses which are heavier than the SU(2) W bosons. The breaking of SU(2) with the additional U(1) gives the Weinberg-Salam model.

(ii) SU(4) → SU(2) ⊗ SU(2) → SU(2). The "super" symmetry breaking provides us with SU(4) – $[SU(2)]^2$ = 9 W bosons of superheavy masses and $[SU(2)]^2$ – SU(2) = 3 W bosons of intermediate masses. The breaking of SU(2) with the additional U(1) leads to the Weinberg-Salam model.

(iii) SU(4) → U(1) ⊗ SU(2) → SU(2). In this case, we have 11 super-heavy W bosons and one medium-heavy W boson (which is neutral, i.e. Y) plus the W bosons of the Weinberg-Salam model.

In all the above three cases, we have at least one medium-heavy W boson coupling to the diagonal currents, namely the Y boson. The W bosons coupling to (3.4) are estimated to be heavier than 500 GeV in order to preserve the weak interaction universality and can be forgotten for practical purposes.

3.3. The Higgs scalars

We introduce first two Higgs scalar multiplets, M and N, which transform under SU(4) as anti-symmetric second-rank tensors and another Higgs scalar multiplet, L which transforms as a symmetric second-rank tensor under SU(4). Denoting the SU(4) transformation as R, then they transform under SU(4) as

$$\phi_{ij} \to R_{il} R_{jm} \phi_{lm} \,, \quad \text{where} \quad \phi = M, N, L \,.$$

Let g and W^i_μ (i = 1, ..., 15) be the gauge coupling and the gauge bosons. The gauge-invariant Lagrangian can be written as

$$\mathcal{L} = -\tfrac{1}{2} \sum_{\phi=M,N,L} \mathrm{Tr}\,|\partial_\mu \phi + \tfrac{1}{2} ig(\lambda_i W^i_\mu)\phi + \tfrac{1}{2} ig\phi(\lambda^{\mathrm{T}}_i W^i_\mu)|^2 + \ldots \quad . \tag{3.6}$$

We find the following.

(i) The first route of SU(4) symmetry breaking can be done with two antisymmetric 2nd-rank tensors *. SU(4) is broken to O(5) with the following non-zero vacuum expectation value:

$$\langle M \rangle = A \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -h \\ -1 & 0 & 0 & 0 \\ 0 & h & 0 & 0 \end{bmatrix} , \qquad h = \pm 1 \,. \tag{3.7}$$

* SU(4) symmetry breaking with two antisymmetric second-rank tensors has been remarked upon by Elias and Swift [12].

O(5) is broken to SU(2) with $\langle N\rangle$ given by

$$\langle N\rangle = a\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.8}$$

(ii) SU(4) is broken to SU(2) ⊗ SU(2) (with $h = -K = \pm 1$) *via* the Higgs scalar L:

$$\langle L\rangle = B\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -h \\ 1 & 0 & 0 & 0 \\ 0 & -h & 0 & 0 \end{bmatrix}, \qquad h = \pm 1 . \tag{3.9}$$

A remark: In general, we would expect that the two matrix elements in (3.7) or (3.9) need not be equal (i.e. $|h| \neq 1$). However, by minimizing the most general potential of M and N, for example, (which is even under $M \to -M$, or $N \to -N$) we find that the solutions require that $h^2 = 1$. If $h^2 \neq 1$, we would find that the mass relation of the Weinberg-Salam model, namely, $m_Z = m_W/\cos\theta_w$ is spoiled. Because of the minimization condition, the above mass relation turns out to hold also in this $SU_L(4) \otimes U(1)$ model. What this means is that the group-theoretical analysis of the SU(4) symmetry breaking can be realized. The above mass relation has indirect experimental support from the inclusive neutral-current neutrino reactions.

To break the SU(2) ⊗ U(1) symmetry à la Weinberg-Salam model, we need 4 Higgs scalars each transforming as 4 representation with the following vacuum expectation values:

$$\langle\phi_1\rangle = \begin{bmatrix} \lambda \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \langle\phi_2\rangle = \begin{bmatrix} 0 \\ \lambda \\ 0 \\ 0 \end{bmatrix}, \quad \langle\phi_3\rangle = \begin{bmatrix} 0 \\ 0 \\ \lambda \\ 0 \end{bmatrix}, \quad \langle\phi_4\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \lambda \end{bmatrix}, \tag{3.10}$$

where ϕ_1, ϕ_4 have charge in the order $(0, -1, -1, 0)$ and ϕ_2, ϕ_3 have charge $(1, 0, 0, 1)$. The gauge-invariant Lagrangian is given by

$$\mathcal{L}_\phi = -\tfrac{1}{2}\sum_j |\partial_\mu\phi_j + \tfrac{1}{2}ig(\lambda_i W_\mu^i)\phi_j \pm \tfrac{1}{2}ig' B_\mu\phi_j|^2 + \ldots \qquad \begin{matrix} + \text{ for } j = 2, 3 \\ - \text{ for } j = 1, 4 \end{matrix}, \tag{3.11}$$

where g' is the coupling constant for U(1), and B the corresponding boson.

From (3.6)–(3.11), we obtain the following grand formulas for the eigenstates and masses of the W bosons (for definiteness, we take $K = 1$):

$$W_{H^\pm} \equiv \tfrac{1}{2}[W_{1\pm i2} - W_{13\mp i14}], \qquad m^2_{W_{H^\pm}} = g^2(A^2 + B^2 + \tfrac{1}{4}\lambda^2),$$

$$W^\pm \equiv \tfrac{1}{2}[W_{1\pm i2} + W_{13\mp i14}], \qquad m^2_{W^\pm} = \tfrac{1}{4}g^2\lambda^2,$$

$$\tfrac{1}{2}[W_{6\pm i7} + W_{9\pm i10}]\,, \qquad m^2 = g^2(A^2 + \tfrac{1}{4}(a^2 + \lambda^2))\,,$$

$$\tfrac{1}{2}[W_{6\pm i7} - W_{9\pm i10}]\,, \qquad m^2 = g^2(B^2 + \tfrac{1}{4}(a^2 + \lambda^2))\,. \tag{3.12a}$$

(To conform with the previous notation, we define

$$W_{(K\bar{D})^0,(\bar{K}D)^0} \equiv \tfrac{1}{2}[W_{6\pm i7} - hW_{9\pm i10}]\,,$$

$$W_{H^0,(\bar{H}^0)} \equiv \tfrac{1}{2}[W_{6\pm i7} + hW_{9\pm i10}]\,, \qquad \text{with } h = \pm 1\ .)$$

$$W_{K^\pm} \equiv \sqrt{\tfrac{1}{2}}\, W_{4\pm i5}\,, \qquad m^2_{W_{K^\pm}} = g^2(B^2 + \tfrac{1}{4}(a^2 + \lambda^2))\,,$$

$$W_{D^\pm} \equiv \sqrt{\tfrac{1}{2}}\, W_{11\mp i12}\,, \qquad m^2_{W_{D^\pm}} = g^2(B^2 + \tfrac{1}{4}(a^2 + \lambda^2))\,,$$

$$X \equiv \sqrt{\tfrac{1}{2}}\,(W_3 - \sqrt{\tfrac{1}{3}}\,W_8 + \sqrt{\tfrac{2}{3}}\,W_{15})\,, \qquad m^2_X = g^2(A^2 + B^2 + \tfrac{1}{4}\lambda^2)\,,$$

$$Y \equiv \sqrt{\tfrac{2}{3}}\,W_8 + \sqrt{\tfrac{1}{3}}\,W_{15}\,, \qquad m^2_Y = g^2(\tfrac{1}{2}a^2 + \tfrac{1}{4}\lambda^2)\,,$$

$$Z = A_3 \cos\theta_W - B\sin\theta_W\,, \qquad m^2_Z = m^2_W/\cos^2\theta_W\,,$$

$$A = A_3 \sin\theta_W + B\cos\theta_W\,, \qquad m^2_A = 0\,, \tag{3.12b}$$

where $\tan^2\theta_W = g'^2/(\tfrac{1}{2}g^2)$, $A_3 \equiv \sqrt{\tfrac{1}{2}}\,(W_3 + \sqrt{\tfrac{1}{3}}\,W_8 - \sqrt{\tfrac{2}{3}}\,W_{15})$.

From (3.12), we immediately see that:

(i) $SU(4) \to O(5)\ (h = K = 1) \to SU(2)$ is realized with $A \gg (a, B) > \lambda$.

(ii) $SU(4) \to SU(2) \otimes SU(2)\ (h = -K = -1) \to SU(2)$ is realized with $B \gg (A, a) > \lambda$.

(iii) $SU(4) \to U(1) \otimes SU(2) \to SU(2)$ is realized with $(A, B) \gg a > \lambda$.

The potential of the Higgs scalars and the stability of the vacuum are discussed in the appendix.

4. SU(4) weak currents

The interaction of the gauge W boson with the quarks and leptons is prescribed by the following gauge-invariant Lagrangian:

$$\begin{aligned}\mathcal{L} = {}& \bar{\psi}^q_L\gamma_\mu(\partial^\mu + \tfrac{1}{2}ig\lambda_i W^\mu_i + \tfrac{1}{2}ig'y^q B^\mu)\psi^q_L + \bar{\psi}_R\gamma_\mu(\partial^\mu + ig'QB^\mu)\psi^q_R \\ & + \bar{\psi}^\ell_L\gamma_\mu(\partial^\mu + \tfrac{1}{2}ig'(-\lambda^T_i)W^\mu_i - \tfrac{1}{2}ig'y^\ell B^\mu)\psi^\ell_L + \bar{\psi}^\ell_R\gamma_\mu(\partial^\mu + ig'QB^\mu)\,\psi^\ell_R\,,\end{aligned} \tag{4.1}$$

where $\psi_{L,R} = \tfrac{1}{2}(1 \mp \gamma_5)\psi$. The y^q, y^ℓ are constants (matrices) determined from the charges of the quarks and leptons, i.e. $y = Q - \tfrac{1}{2}(\lambda_3 + \sqrt{\tfrac{1}{3}}\lambda_8 - \sqrt{\tfrac{2}{3}}\lambda_{15})_L$. Thus $y = \tfrac{1}{3}$ for the quarks and $y = 1$ for the electron and muon families. Q is the charge matrix. Note that ψ^ℓ (the lepton representations) transforms as 4 under SU(4). In terms of the eigenstates of the W bosons given in eqs. (3.8)–(3.10), one finds the following couplings for the off-diagonal weak currents:

$$\mathcal{L}_{\rm int}^{\rm off\text{-}diagonal} = \tfrac{1}{2} g W_{\mu}^{-} (\bar{u}_{\theta}\gamma_{\rm L}^{\mu} d + \bar{c}_{\theta}\gamma_{\rm L}^{\mu} s - \bar{\nu}_{\rm e}\gamma_{\rm L}^{\mu} e^{-} - \bar{\nu}_{\tau}\gamma_{\rm L}^{\mu}\tau^{-} - \bar{\nu}_{\mu}\gamma_{\rm L}^{\mu}\mu^{-}$$

$$- \bar{M}^{0}\gamma_{\rm L}^{\mu} M^{-}) + \tfrac{1}{2} g W_{(\bar{\rm K}{\rm D})^{0},\mu} (-h\bar{u}_{\theta}\gamma_{\rm L}^{\mu} c_{\theta} + \bar{d}\gamma_{\rm L}^{\mu} s - \bar{\nu}_{\tau}\gamma_{\rm L}^{\mu}\nu_{\rm e} + h\bar{\tau}^{-}\gamma_{\rm L}^{\mu} e^{-}$$

$$- \bar{M}^{0}\gamma_{\rm L}^{\mu}\nu_{\mu} + h\bar{M}^{-}\gamma_{\rm L}^{\mu}\mu^{-}) + \sqrt{\tfrac{1}{2}}\, g\, W_{{\rm K}^{-},\mu} (\bar{u}_{\theta}\gamma_{\rm L}^{\mu} s - \bar{\nu}_{\tau}\gamma_{\rm L}^{\mu} e^{-} - \bar{M}^{0}\gamma_{\rm L}^{\mu}\mu^{-})$$

$$+ \sqrt{\tfrac{1}{2}}\, g\, W_{{\rm D}^{-},\mu} (\bar{c}_{\theta}\gamma_{\rm L}^{\mu} d - \bar{\nu}_{\rm e}\gamma_{\rm L}^{\mu}\tau^{-} - \bar{\nu}_{\mu}\gamma_{\rm L}^{\mu} M^{-}) + \text{hermitian conjugates}$$

$$+ \ldots , \tag{4.2}$$

where $\gamma_{\rm L}^{\mu} \equiv \gamma^{\mu}\, \tfrac{1}{2}(1 - \gamma_5)$, u_{θ} and c_{θ} stand for $u \cos\theta_{\rm C} - c \sin\theta_{\rm C}$, and $u \sin\theta_{\rm C} + c \cos\theta_{\rm C}$, respectively, with $\theta_{\rm C}$ the Cabibbo angle. The interactions of super-heavy W bosons of eq. (3.8) are omitted from (4.2). The coupling of the diagonal neutral currents can be expressed as

$$\mathcal{L}_{\rm int}^{\rm diagonal} = eA^{\mu} [j_{\mu}^{\rm em,q} + j_{\mu}^{\rm em,\ell}] + \sqrt{\tfrac{1}{2}}\, g\, Y^{\mu} [j_{\mu}^{\rm Y,q} - j_{\mu}^{\rm Y,\ell}]$$

$$+ \sqrt{\tfrac{1}{2} g^2 + g'^2}\, Z^{\mu} \{ j_{\mu}^{\rm A_3,q} - \sin^2\theta_{\rm w} j_{\mu}^{\rm em,q} - j_{\mu}^{\rm A_3,\ell} - \sin^2\theta_{\rm w} j_{\mu}^{\rm em,\ell} \} , \tag{4.3}$$

where $j_{\mu}^{\rm em}$ is just the electromagnetic current, $j_{\mu}^{\rm A_3} \equiv \bar{\psi}\gamma_{\rm L}^{\mu}\lambda_3\psi$, $j_{\mu}^{\rm Y} \equiv \bar{\psi}\gamma_{\rm L}^{\mu}\lambda_{\rm Y}\psi$ (λ_3 and $\lambda_{\rm Y}$ are defined in (3.1) and (3.2), respectively). The quark and lepton currents coupling to $\rm W^{\pm}$ and Z are identical to the WS model. (The weak angle in terms of SU(4) and U(1) coupling constants is given by $\tan^2\theta_{\rm w} = g'^2/(\tfrac{1}{2}g^2)$). In the rest of this paper, we examine the weak interactions induced by (4.2) and (4.3).

5. For the ordinary particles

5.1. Strangeness-changing neutral currents

The reasons that the strangeness-changing neutral currents are absent in the sector of ordinary particles become transparent from the interaction Lagrangian obtained above. (i) The strangeness-changing neutral currents (coupling to $\rm W^0_{(KD)}$) couple to at least one "new" lepton in semileptonic processes (see 4.2), thus we will not see any strangeness-changing neutral currents in the weak interactions of ordinary particles if the "new" leptons are heavy enough. An immediate consequence of this assumption is the prediction of strangeness-changing neutral currents in the decays of the heavy leptons (see sect. 7). Note from (4.2) that the strangeness-changing neutral currents (off-diagonal) are not coupled to the diagonal neutral currents (which couple to ee, $\mu\mu$, $\nu\nu$, for example) thus $\rm K^+ \not\to \pi^+$ ee, etc. (ii) The diagonal neutral currents do not change strangeness (from 4.3). This has been discussed above in connection with the Cabibbo structure. (iii) The non-leptonic interactions to the order G_W have no $|\Delta S| = 2$ transition. This follows from the fact that the strangeness-changing neutral currents are not coupled to the diagonal neutral currents. For the question of strangeness-changing neutral-current processes and the K_1-K_2 mass-difference in higher-order weak interactions, we must examine the box diagram with two charged W exchange, etc. The GIM cancellation between the u-quark and c-quark intermediate states works here as it should. Not too surprisingly, the same kind of cancellation works for the additional SU(4) (charged) currents.

As remarked above, the "new" leptons must be heavy enough in order that strangeness-changing neutral currents do not appear in the decays of ordinary particles, (otherwise the $W^0_{(KD)}$ boson coupling to this current will have to be extremely heavy). This means a lower limit on m_{ν_τ}, namely, $m_{\nu_\tau} \gtrsim (m_{K^+} - m_{\pi^+})$, from the absence of $K^+ \to \pi^+ \nu_\tau \bar{\nu}_e$ etc. From the absence of $K^+ \to e^+ \nu_\tau$, we conclude that either $m_{\nu_\tau} \sim m_{K^+} \sim 500$ MeV or the weak SU(4) symmetry must be badly broken to SU(2) ⊗ SU(2) such that $W_{K^\pm}$ (coupling to $e\nu_\tau$) is extremely heavy. These alternatives have been discussed in sect. 3. Experimentally, $m_{\nu_\tau} \sim 500$ MeV is not ruled out at present. Allowing medium-heavy W bosons, we note that the mass of ν_τ is very much restricted in this model which could be easily checked by the experiments.

5.2. Modified non-leptonic interactions

From (4.2) we see that the $|\Delta S| = 1$ non-leptonic interactions of the ordinary particles have a new piece due to $W^0_{(KD)}$ exchange. After Fierz transformations, one has

$$\mathcal{L}_{\text{eff}}^{|\Delta S|=1} = \sqrt{\tfrac{1}{2}}\, G_F \left(1 + \frac{m_W^2}{m_{W'}^2}\right) \{\bar{s}\gamma_\mu(1-\gamma_5)u \cdot \bar{u}\gamma^\mu(1-\gamma_5)d$$

$$- \bar{s}\gamma_\mu(1-\gamma_5)c \cdot c^-\gamma^\mu(1-\gamma_5)d + \text{h.c.}\}\,,$$

where $m_{W'} \equiv m_{W^0_{(KD)}}$.

6. Low-energy neutral-current reactions

In this section we examine the experimental consequences of the diagonal neutral-current interactions given in (4.3). Note that in addition to the neutral currents coupling to the Z boson as in the WS model, we have other neutral currents coupling to the Y boson. Therefore our prediction will differ from the WS model. The mass of the Y boson is not known, and has to be determined experimentally. This we do by first calculating the parity violation effect in atomic bismuth.

6.1. Electron-hadron neutral-current interactions

From (4.3), one obtains the following effective Lagrangian (neglecting the Cabibbo angle):

$$\tfrac{1}{2}\sqrt{\tfrac{1}{2}}G_F\{[\bar{u}\gamma_\mu(1-\gamma_5)u - \bar{d}\gamma_\mu(1-\gamma_5)d - 4X_w(\tfrac{2}{3}u\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d)]$$
$$\times(\bar{e}\gamma^\mu(1-4X_w-\gamma_5)e) + \left(\frac{m_W^2}{m_Y^2}\right)[\bar{u}\gamma_\mu(1-\gamma_5)u + \bar{d}\gamma_\mu(1-\gamma_5)d]$$
$$\times(e^-\gamma^\mu(1-\gamma_5)e)\}\ ,$$

where $X_w \equiv \sin^2\theta_w$. We note that the second term of order m_w^2/m_Y^2 could significantly modify the WS model prediction for deep-inelastic electron scattering and the parity-violation effect in atoms. For example, for the bismuth atom, where the dominant contribution comes from the hadronic vector current and electron axial-vector current interference, the experimentally measured effect is proportional to

$$Q^w = Z(1-4X_w) - N + \left(\frac{m_W^2}{m_Y^2}\right)3(Z+N)$$

$$\sim -129 + \left(\frac{m_W^2}{m_Y^2}\right)627\ ,$$

where, for definiteness, we take $X_w \sim 0.26$. We note that the two terms have opposite signs. As we know from the symmetry arguments above, m_Y is greater than m_W. We find that if $m_Y \sim 2m_W$, then Q^w is indeed very small compared with the WS model prediction (the first term). We have no reason to expect that there should be absolutely no parity violation in atoms. A consistency check for the above Lagrangian will have to wait for the parity-violation experiments in hydrogen, where definite prediction can be made with m_Y already determined. At present, we look for other predictions in neutrino reactions.

6.2. Neutral currents in neutrino reactions

The effective neutral current couplings for the neutrino reactions are given in table 1. We present the numerical results for the inclusive processes, elastic $\bar{\nu}_\mu$e, ν_μe cross sections, and elastic $\bar{\nu}_\mu$p, ν_μp cross sections in figs. 1, 2, 3 and 4. The elastic $\bar{\nu}_\mu$p, ν_μp cross sections are calculated in a similar way as in ref. [13], which involves model assumptions. We note the following features.

(i) The hadronic neutral current coupling to the Y boson is pure isoscalar. Compared with the WS model, this will change the prediction for $I = 0$ and $I = 1$ interference, a quantity which can be measured experimentally [14]. The present data is too crude to test this.

(ii) The inclusive neutral-current neutrino cross sections remain almost the same

Table 1
Values of C_V and C_A for various neutral-current couplings of neutrinos

Processes	C_V	C_A
$\nu_\mu e \to \nu_\mu e$	$-\frac{1}{2} + 2X_W + \frac{1}{2}\rho$	$\frac{1}{2} - \frac{1}{2}\rho$
$\nu u \to \nu u$	$-\frac{1}{2} + \frac{4}{3}X_W + \frac{1}{2}\rho$	$\frac{1}{2} - \frac{1}{2}\rho$
$\nu d \to \nu d$	$\frac{1}{2} - \frac{2}{3}X_W + \frac{1}{2}\rho$	$-\frac{1}{2} - \frac{1}{2}\rho$

C_V and C_A are defined by

$$H_W = \sqrt{\tfrac{1}{2}}\, G_F\, \bar{\nu}\gamma_\mu(1 - \gamma_5)\nu\bar{\psi}\gamma^\mu(C_V + C_A\gamma_5)\psi\ ,$$

$$X_W = \sin^2\theta_W, \qquad \rho \equiv m_W^2/m_{W'}^2\ .$$

as the WS model. (Both are in agreement with the data.) Expressing $R^{\nu(\bar{\nu})} \equiv \sigma_{NC}^{\nu(\bar{\nu})}/\sigma_{CC}^{\nu(\bar{\nu})}$ as $R^{\nu(\bar{\nu})} = R^{\nu(\bar{\nu})} - \Delta$, we find $\Delta = \frac{1}{2}(m_W^2/m_Y^2)(m_W^2/m_Y^2 - \frac{2}{3}X_W)$, which is a very small correction for $m_Y^2 \sim 4m_W^2$ and $X_W \sim 0.3$.

(iii) The elastic $\nu_\mu e$, $\bar{\nu}_\mu e$ cross sections do show some deviation from the WS

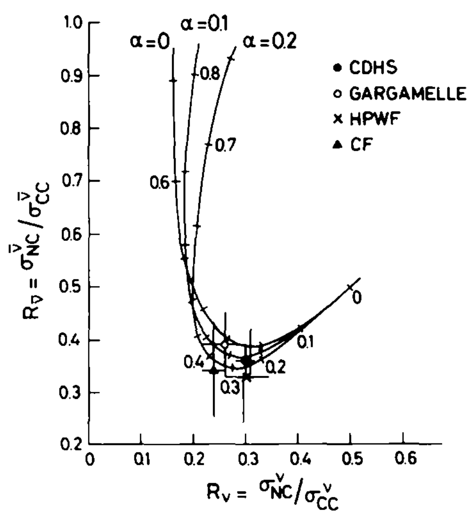

Fig. 1. The ratios of neutral-current cross sections *versus* charged-current cross sections for neutrino and antineutrino reactions as functions of $\sin^2\theta_W$ (labeled beside the curves). The difference between the prediction of this model and that of the Weinberg-Salam model is negligibly small for $\sin^2\theta_W$ of interest. α is the ratio of the antiquark *versus* quark content inside the isoscalar target. The data are from ref. [19].

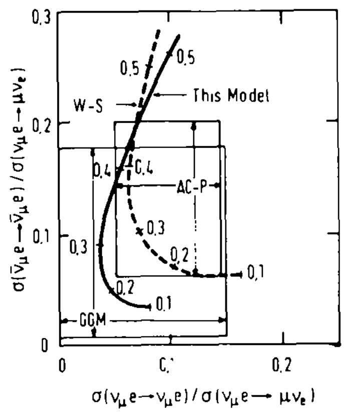

Fig. 2. The elastic $\nu_\mu e$, $\bar{\nu}_\mu e$ cross sections as functions of $\sin^2\theta_W$ (labeled beside the curves). The data are from ref. [20].

model, if $\sin^2\theta_W \lesssim 0.3$ (see fig. 2). The difference begins to disappear as $\sin^2\theta_W$ increases above 0.3.

7. Implications for τ and D meson decays

Here is an essential test for the off-diagonal neutral-current interactions given by (4.2).

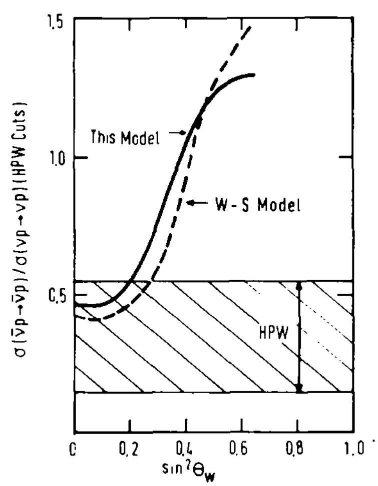

Fig. 3. The elastic $\nu_\mu p$ cross section as a function of $\sin^2\theta_W$. The data are from ref. [21].

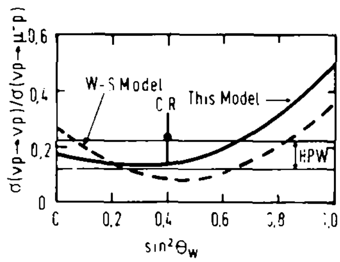

Fig. 4. The ratio of elastic $\bar{\nu}_\mu$p *versus* ν_μp cross sections as a function of $\sin^2\theta_W$. The data are from ref. [21].

The dominant decay modes of the τ lepton are obviously *via* the charged currents, therefore τ behaves almost like a true sequential heavy lepton. The off-diagonal neutral currents lead to $\tau \to e$ + hadron modes. As the analysis of ref. [15] shows, in order to distinguish whether τ is a sequential heavy lepton or an excited electron (ortholepton), such decay modes have to be searched for experimentally.

From (4.2), it is clear that $\tau^- \to e^-(e^+e^-)$ or $e^-(\mu^+\mu^-)$ is forbidden to the lowest order of weak interaction. $\tau^- \to e^-\gamma$ or $\mu^-\gamma$ is about the order of a second-order weak process.

The off-diagonal neutral currents (4.2) give the following predictions. (For definiteness, assume $m^2_{W'} \equiv m^2_{W^0_{(KD)}} \sim m^2_Y \sim 4m^2_W$; see sect. 3.)

(i) The electronic and muonic branching ratios of the τ decay will not be the same. In fact

$$\Gamma(\tau^- \to e^-\nu_\tau\bar{\nu}_e)/\Gamma(\tau^- \to \mu^-\nu_\tau\bar{\nu}_\mu) = \left(1 \pm \frac{m^2_W}{m^2_{W'}}\right)^2 \sim 0.6 \text{ or } 1.5.$$

This is consistent with the Pluto data [8], taking $m_{\nu_\tau} \gtrsim 300$ MeV, and also consistent with the SPEAR data within the error bar, $\sigma(ee)/\sigma(e\mu) \sim 0.57 \pm 0.3$ [16].

(ii) τ decays to electron plus strange particles. We estimate the decay branching ratios B.R.$(\tau^- \to e^-K_S)$ and B.R.$(\tau^- \to e^-K^{*0})$ to be of the order 0.5% and 1–2%, respectively. They are suppressed in comparison with decays $\tau \to \nu\pi$ and $\tau \to \nu\rho$ by a factor $(m^2_W/m^2_{W'})$ and phase spaces. Searching for such modes requires experimentally large statistics. Since $m(eh) = m_\tau$, these events are more or less free from charm background. Experimental searches for these decay modes are crucial tests for the off-diagonal neutral-current structure.

(iii) The corresponding predictions for D mesons are $\Gamma(D^+ \to \pi^+\bar{\nu}_\tau\nu_e)/\Gamma(D \to Ke\bar{\nu}) \sim 0.07$, and $\Gamma(D^0 \to \bar{\nu}_\tau\nu_e)/\Gamma(D \to Ke\bar{\nu}) \sim 0.02$, assuming the same constant form factors for the three-body decay modes and $f_D \sim f_\pi$. With large amounts of data, these modes could be searched for. Note that the off-diagonal charm-changing neutral currents coupling to $W^0_{(KD)}$ do not induce $D^0, \bar{D}^0$ mixing; $D^0, \bar{D}^0$ mixing in this model (due to u and c mixing) is suppressed and consistent with the present experimental limit.

8. Conclusions

The main result of this paper is the effective weak interaction Lagrangian given by (4.2) and (4.3). In addition to the usual $SU_L(2) \otimes U(1)$ symmetry, we have proposed a new $SU_L(2)$ weak symmetry composed of three charge-neutral generators given by (3.2). (A larger subgroup of SU(4) could involve also the generators of (3.3).) We call this symmetry $SU^N(2)$. The raising and lowering generators of the $SU^N(2)$ group couple the recently "discovered" charmed quark and the τ lepton to the known particles, whereas the third component modifies the weak neutral currents. The latter manifests itself in the small parity-violation effect recently reported in atomic bismuth. A definite test will be provided by the parity-violation search in hydrogen in the future. Examining the neutrino data, we find that the difference between the predictions of this model and of the WS model is not very big. Elastic $\bar{\nu}_\mu e$ and $\nu_\mu e$ cross section and isospin interference measurement in inclusive processes should provide more meaningful tests [17]. Obviously, crucial tests of the $SU^N(2)$ symmetry lie in the sector of the τ lepton and the charmed mesons, such as measuring the τ-neutrino mass, the ratio of the τ electronic and muonic branching ratios and searching for $\tau \to e$ + hadrons decay modes. These are difficult experiments because the predicted effects are quite small. Nevertheless, they are doable experiments and we anticipate definite results in the near future. Among these, we feel that $\tau^- \to e^- K_S$, $\tau^- \to e^- K^{*0}$ searches are the most interesting ones.

The model predicts many new leptons. The electron family is already complete, i.e. $(e^-, \nu_e, \nu_\tau, \tau^-)$. Evidence for the new members of the muon family $(\mu^-, \nu_\mu, M^0, M^-)$ is already being suggested experimentally [11]. The dominant semileptonic decay modes of M^0 will involve strangeness-non-zero final states (see eq. (4.2)). In contrast to the τ-lepton case, this is a striking prediction which can be easily checked experimentally. The most direct check is looking for resonances in $\mu^- K^+$, $\mu^- K^{+*}$ invariant-mass distributions. Indirect clues could be found in the same-sign dimuon data. This happens *via* the production of M^-, which decays into $\mu^- \bar{\nu}_\mu M^0$ with M^0 decaying subsequently into $\mu^- K^+$, $u^- K^{+*}$, etc. Thus we expect the same-sign dimuon events to be associated with strange particles. Phenomena involving trimuons are more complicated and will be discussed elsewhere. e^+e^- annihilation experiments are ideal places for searching for charged heavy leptons which decay into the neutral ones, including the third quartet having a new lepton number, which otherwise will be difficult to produce experimentally.

Recent data from pN collision have discovered two or more resonances $\Upsilon(9.4)$, $\Upsilon'(10.0)$ ... [18]. This could be evidence for more heavy quarks. In this framework, the natural assignment of heavy quarks will be a new quartet of heavy quarks (t, b, h, g) *. For that matter, a different quark representation (u, d, b, t), (c, s, h, g) is also acceptable. The latter suggestion has been discussed in ref. [17].

In conclusion, we have suggested a much richer weak interaction than ordinarily

* See the footnote on page 346.

pictured. In contrast to the many gauge models which have populated the literature, this model contains many clean predictions. Despite the fact that it may be an incorrect description of nature, one might still benefit from such a study.

I wish to thank S. Yamada for discussions on the properties of the τ lepton which lead to the present assignment of the τ lepton. Thanks are due to G. Mikenberg, T. Kahl, G. Knies and H. Meyer for discussing the experimental feasibility of $\tau \to eK_S$ searches. I have benefited from the help of M. Kugler on the properties of the O(5) group, and from H. Joos, M. Krammer, A. Swift and many of my other colleagues for various comments on the manuscript.

Appendix

The Higgs potential

For simplicity, we consider first the most general quartic potential for two Higgs scalar multiplets M and N, both transforming as antisymmetric second-rank tensors under SU(4). We assume reflection symmetry (i.e. invariance under $M \to -M$ or $N \to -N$). The potential can be written as

$$\begin{aligned} P(M, N) = {} & a_1 \operatorname{Tr}(\bar{M}M\bar{M}M) + a_2 \operatorname{Tr}(\bar{N}N\bar{N}N) + b_1 [\operatorname{Tr}(\bar{M}M)]^2 + b_2 [\operatorname{Tr}(\bar{N}N)]^2 \\ & + c[\operatorname{Tr}(\bar{M}M\bar{N}N) + \operatorname{Tr}(M\bar{M}N\bar{N})] + d \operatorname{Tr}(\bar{M}M) \operatorname{Tr}(\bar{N}N) + e_1 \operatorname{Tr}(\bar{M}N) \operatorname{Tr}(\bar{N}M) \\ & + e_2 [(\operatorname{Tr} \bar{M}N)^2 + (\operatorname{Tr} \bar{N}M)^2] + f_1 \operatorname{Tr}(\bar{M}M) + f_2 \operatorname{Tr}(\bar{N}N) \end{aligned} \tag{A.1}$$

Let us assume that M and N have the following non-vanishing vacuum expectation values:

$$\langle M \rangle = \begin{bmatrix} 0 & 0 & x & 0 \\ 0 & 0 & 0 & y \\ -x & 0 & 0 & 0 \\ 0 & -y & 0 & 0 \end{bmatrix}, \qquad \langle N \rangle = \begin{bmatrix} 0 & z & 0 & 0 \\ -z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We find that the extreme point of the potential corresponds to x, y, z satisfying the following equations:

$$2a_1 x^2 + 4b_1(x^2 + y^2) + (c + 2d)z^2 + f_1 = 0 , \tag{A.2}$$

$$2a_1 y^2 + 4b_1(x^2 + y^2) + (c + 2d)z^2 + f_1 = 0 , \tag{A.3}$$

$$2a_2 z^2 + 4b_2 z^2 + (c + 2d)(x^2 + y^2) + f_2 = 0 . \tag{A.4}$$

From (A.2) and (A.3) one finds $x^2 = y^2$. The above equations can be simplified as

$$(2a_1 + 8b_1)x^2 + (c + 2d)z^2 + f_1 = 0 , \tag{A.5}$$

$$(2a_2 + 4b_2)z^2 + 2(c + 2d)x^2 + f_2 = 0 . \tag{A.6}$$

Thus the vacuum expectation values are determined. The implication of the constraint $x = \pm y$ has been discussed in sect. 3.

We have to ask next whether the extreme point of the potential obtained above remains stable under small perturbations, for otherwise the vacuum expectation values do not correspond to a minimum of the potential and will shift under renormalization. Let $M = \langle M\rangle + \Delta M$ and $N = \langle N\rangle + \Delta N$. Since the first derivatives of the potential vanish at $\langle M\rangle$ and $\langle N\rangle$, we find after some algebra that

$$\begin{aligned} P(\langle M\rangle + \Delta M, \langle N\rangle + \Delta N) - P(\langle M\rangle, \langle N\rangle) &= \tfrac{1}{2}c\{\mathrm{Tr}\,|\alpha + \beta|^2 + \mathrm{Tr}\,|\alpha^{\mathrm{T}} + \beta^{\mathrm{T}}|^2\} \\ &+ \tfrac{1}{2}d[\mathrm{Tr}\,\alpha + \mathrm{Tr}\,\beta]^2 + (a_1 - \tfrac{1}{2}c)\,\mathrm{Tr}\,|\alpha|^2 + (a_2 - \tfrac{1}{2}c)\,\mathrm{Tr}\,|\beta|^2 \\ &+ (b_1 - \tfrac{1}{2}d)|\mathrm{Tr}\,\alpha|^2 + (b_2 - \tfrac{1}{2}d)\,|\mathrm{Tr}\,\beta|^2 \\ &+ e_1\,|\mathrm{Tr}\,\gamma|^2 + e_2\,[(\mathrm{Tr}\,\gamma)^2 + (\mathrm{Tr}\,\gamma^{+})^2]\,, \end{aligned} \tag{A.7}$$

where α and β are hermitian matrices given by

$$\alpha \equiv \Delta\bar{M}\langle M\rangle + \langle\bar{M}\rangle\,\Delta M + \Delta\bar{M}\Delta M\,,$$

$$\beta \equiv \Delta\bar{N}\langle N\rangle + \langle\bar{N}\rangle\,\Delta N + \Delta\bar{N}\Delta N\,, \tag{A.8}$$

$$\gamma \equiv \Delta\bar{M}\langle N\rangle + \langle\bar{M}\rangle\Delta N + \Delta\bar{M}\Delta N\,. \tag{A.9}$$

The stability condition requires that the coefficients of each term in (A.7) should all be positive, i.e.

$$c, d, e_1, e_2 \geqslant 0\,,$$

$$a_1 \geqslant \tfrac{1}{2}c, \quad a_2 \geqslant \tfrac{1}{2}c, \quad b_1 \geqslant \tfrac{1}{2}d, \quad b_2 \geqslant \tfrac{1}{2}d\,. \tag{A.10}$$

There obviously exists a finite range where (A.10) are satisfied. The potential retains its most general form and the renormalizability condition is satisfied.

We can extend the same calculation to include the third multiplet L, which transforms as symmetric second-rank tensor under SU(4), and so on.

References

[1] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
Abdus Salam, *in* Elementary particle physics, ed. N. Svartholm (Almquist and Wikskell, Stockholm, 1968) p. 367.
[2] P.G.H. Sanders, Int. Symp. on lepton and photon interactions at high energies, Hamburg, 1977.
[3] E.M. Henley and L. Wilets, Phys. Rev. A14 (1976) 1411;
M. Brimicombe, C.E. Loving and P.G.H. Sanders, J. of Phys. B9 (1976) L237;
I.B. Khriplovich, JETP Lett. 20 (1974) 315.
[4] T.C. Yang, Phys. Lett. 70B (1977) 239.
[5] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285.
[6] S. Glashow and S. Weinberg, Phys. Rev. D15 (1977) 1958.

[7] M.L. Perl et al., Phys. Rev. Lett. 35 (1975) 1489;
M.L. Perl et al., Phys. Lett. 63B (1976) 466;
G.J. Fledman et al., Phys. Rev. Lett. 38 (1977) 117.
[8] J. Burmester et al., Phys. Lett. 68B (1977) 297, 301.
[9] J.C. Pati and A. Salam, Phys. Rev. D8 (1973) 1243; D10 (1974) 275;
V. Elias and A.R. Swift, Phys. Rev. D7 (1977) 1937;
N.G. Deshpande, R.C. Hwa and P.D. Mannheim; Univ. of Oregon preprint OITS-74 (1977).
[10] M.L. Perl, Talk presented at 1977 Int. symp. on lepton and photon interactions at high energies, Hamburg, 1977, SLAC-PUB-2022.
[11] (a). A. Benvenuti et al., Phys. Rev. Lett. 38 (1977) 1110.
(b). H. Faissner et al., Aachen preprint PITHA-94 (1977);
D.S. Baranov et al., Phys. Lett. 70B (1977) 269.
[12] V. Elias and A. Swift, Phys. Rev. D13 (1976) 2083.
[13] J.D. Bjorken, Lectures given at 1976 SLAC Summer Institute, SLAC-report-198 (1976).
[14] P.Q. Hung and J.J. Sakurai, Phys. Lett. 63B (1976) 295.
[15] A. Ali and T.C. Yang, Phys. Rev. D14 (1975) 3052.
[16] M.L. Perl, Talk given at 12th Rencontre de Moriond, Flaine, 1977.
[17] T.C. Yang, Parity-violation effect in atomic physics and the structure of neutral currents in gauge theories, 1977 Cargése Summer Institute.
[18] S.W. Herb et al., Phys. Rev. Lett. 39 (1977) 252;
W.R. Innes et al., Phys. Rev. Lett. 39 (1977) 1240.
[19] J. Steinberger, Talk presented at the Budapest Conf., Hungary, 1977;
D. Buchholz, Presented at Aachen Neutrino Conf., 1977;
J. Blietschau et al., Nucl. Phys. B118 (1977) 218;
P. Wanderer et al., HPWF preprint 77/1 (1977).
[20] P. Musset, Presented at 18th Int. Conf. on high-energy physics, Tbilisi, USSR, 1976.
[21] W. Lee et al., Phys. Rev. Lett. 37 (1976) 186;
C. Cline et al., Phys. Rev. Lett. 27 (1976) 252, 648.
[22] J.H. Klems et al., Phys. Rev. D4 (1971) 66.