# THREE GLUON JETS AS A TEST OF QCD

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As a test of quantum chromodynamics (QCD), we suggest looking for gluon jets in the decay of a heavy quark-antiquark bound state produced in $e^+e^-$ annihilation, $Q\bar{Q} \to$ 3 gluons → 3 gluon jets. In particular, we point out that these events form a jet Dalitz plot, and we calculate the gluon or jet distributions (including the effect of polarized $e^+e^-$ beams). This process affords a test of the gluon spin. It is the analogue of two-jet angular distributions in $e^+e^- \to q\bar{q} \to$ 2 quark jets. We also estimate multiplicities and momentum distributions of hadrons in $Q\bar{Q} \to$ 3 gluons → hadrons, using the recently discovered $\Upsilon(9.4)$ as an example.

There is at present much interest in the theory of quarks with color and flavor interacting with an octet of massless vector gauge bosons carrying color only. This theory is called quantum chromodynamics (QCD) [1]. Colored quarks have never been seen in deep inelastic processes. Jets of hadrons are seen, however, and in $e^+e^-$ annihilation two back-to-back jets occur with an angular distribution just that for $e^+e^- \to q\bar{q}$ with spin 1/2 quarks [2]. It is physically reasonable that a colored gluon with large momentum will also lead to a jet of hadrons carrying the total three-momentum of the gluon (but no color) [3,4]. The angular and momentum distribution of this jet then reproduces the original gluon angular and momentum distribution. Since gluons have no flavor, gluon jets have no net flavor (unlike quark jets, which carry isospin and other flavors).

As a test of QCD we suggest looking for the existence of gluon jets in the decay of a heavy $^3S_1$ quark-antiquark state [5] ‡1, $Q\bar{Q} \to$ 3 gluons → 3 gluon jets. This is the mechanism for the direct decay of such a heavy state in QCD [6]. It is a generalization of the $3\gamma$ decay of orthopositronium in QED, and it is this fact which allows it to be calculated. This also means that $Q\bar{Q} \to 3g$ is a sensitive test of the belief that g is a massless vector particle. The search for gluon jets as a QCD correction to $e^+e^- \to 1\gamma \to q\bar{q}$, namely $e^+e^- \to 1\gamma \to q\bar{q}g$, has been suggested in [3]. Searching for gluon jets in the decay of heavy $Q\bar{Q}$ resonances may be preferable – especially if objects with mass above ~20 GeV are found.

Of course, in addition to the 3 gluon jets, $Q\bar{Q} \to 1\gamma \to q\bar{q}$ gives rise to two back-to-back jets in the final state. Off the narrow $Q\bar{Q}$ resonance the sole process is $e^+e^- \to 1\gamma \to q\bar{q}$, and only the two-jet configuration results.

To be concrete, we suggest looking for 3 jet decays of the recently found $\Upsilon(9.4)$ [7] ‡2, assuming it to be a $Q\bar{Q}$ state. Assuming that below $\Upsilon$, $R = \sigma(e^+e^- \to \text{had})/\sigma(e^+e^- \to \mu^+\mu^-) = 5$ (including $\tau^+\tau^-$), we find the ratios

$$\Upsilon \to e^+e^- + \mu^+\mu^- : \Upsilon \to \gamma \to q\bar{q} + \tau\bar{\tau} \to 2\text{jets}:$$
$$\Upsilon \to 3g \to 3\text{jets} \approx 2:5:5 \quad \text{for charge } e_Q = 2/3\,, \qquad \approx 2:5:20 \quad \text{for charge } e_Q = -1/3\,, \tag{1}$$

using [6]

$$\Gamma(\Upsilon \to 3g)/\Gamma(\Upsilon \to e\bar{e}) = 10(\pi^2 - 9)\alpha_s^3/81\pi\alpha^2 e_Q^2\,, \tag{2}$$

and choosing the QCD coupling $\alpha_s(M_\Upsilon) = 0.15$.

There is an important consistency check on $\Upsilon \to 3g$ being the sole direct decay. Since it is very improbable

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‡1 We denote the new heavy quark by Q and all lighter ones by q.

‡2 If $\Upsilon \to$ 3 jets is seen, a more appropriate symbol might be the German upsilon $\mathfrak{Y}$.

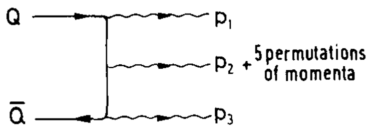


Fig. 1. The amplitude for $Q\bar{Q} \to 3$ gluons; we do not explicitly indicate the gluons' color.

for a gluon to fragment to a heavy charmed quark pair we expect that

$$\Gamma(\Upsilon \to \underset{\hookrightarrow \bar{e}^- + \ldots}{c\bar{c}} + \ldots)/\,\Gamma(\Upsilon \to e\bar{e}) \approx R\,(e^+e^- \to \underset{\hookrightarrow \bar{e}^- + \ldots}{c\bar{c}} + \ldots),$$

and not very much larger as would be the case if $\Upsilon$ decayed by mixing with charm ($Q\bar{Q} \to c\bar{c}$). Of course, this relation holds trivially for a heavy lepton $\tau$. The gluon (jet) momentum distribution can be computed in QCD. The mean energy of a gluon jet will be $M_\Upsilon/3$, the jets will be coplanar and form a sixfold symmetric jet Dalitz plot in terms of the jet momenta $|\boldsymbol{P}_1|$, $|\boldsymbol{P}_2|$, $|\boldsymbol{P}_3|$ or the relative angles. This distribution can be obtained by a straightforward but nontrivial calculation like that for the decay of the $^3S_1$ state of positronium to $3\gamma$. An essential complication is the polarization of $\Upsilon = Q\bar{Q}$ produced by $e^+e^-$ beams. The photon in $e^+e^- \to 1\gamma \to Q\bar{Q}$ transfers the polarization of the $e^+e^-$ state to $Q\bar{Q}$. Introducing the spin vectors $s_\mu$ (for Q) and $\bar{s}_\mu$ (for $\bar{Q}$) and the polarization tensor

$$S_{\mu\nu} = \tfrac{1}{2}(s_\mu \bar{s}_\nu + \bar{s}_\mu s_\nu)\,, \tag{3}$$

(for transversely polarized $e^+e^-$ beams we find $2S_{xx} = -1 - P^2$, $2S_{\Upsilon\Upsilon} = -1 + P^2$, the other elements being zero; $P$ is the $e^\pm$ polarization; the $x$-axis is perpendicular to the ring plane and the $\mathfrak{y}$ axis is in the ring plane, but perpendicular to $\boldsymbol{p}_{e^-}$). We evaluate the six contributing Feynman diagrams for $Q\bar{Q} \to 3g$ in the $Q\bar{Q}$ rest frame, neglecting the binding energy (fig. 1). This and the masslessness of the gluons give the following conditions for the jet (= gluons) momenta $\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3$,

$$\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}_3 = 0, \quad |\boldsymbol{p}_1| + |\boldsymbol{p}_2| + |\boldsymbol{p}_3| = 2M_Q = M_\Upsilon\,. \tag{4}$$

The trace calculation has been done by computer (using REDUCE) and the resulting 3 gluon jet distribution is given by the five-fold differential decay rate

$$\frac{d^5 T}{dx_1\, dx_2\, d^3R} = \frac{\text{const}}{x_1^2 x_2^2 x_3^2}\, W(x_1, x_2, x_3)\,, \tag{5}$$

where we define $\boldsymbol{x}_i = \boldsymbol{p}_i/M_Q$, $x_i = |\boldsymbol{x}_i|$. The variables $x_1, x_2, x_3$ parametrize the jet Dalitz plot ($x_1 + x_2 + x_3 = 2$). In addition, three angles $\alpha, \beta, \gamma$ with $d^3R = d\alpha\, d\cos\beta\, d\gamma$ describe the orientation of the gluon jet plane relative to the $e^+e^-$ beam axis and storage ring plane. Defining

$$S_{ij} \equiv S(\boldsymbol{x}_i, \boldsymbol{x}_j) = x_i^\mu S_{\mu\nu} x_j^\nu\,, \quad S = S_\mu{}^\mu = s \cdot \bar{s}\,, \tag{6}$$

the distribution function is given by

$$\begin{aligned} W(x_1,x_2,x_3) &= 4[x_1^2(1-x_1)^2 + x_2^2(1-x_2)^2 + x_3^2(1-x_3)^2] \\ &\quad + 2S(x_1-1)(x_2-1)(x_3-1)[x_1^2 + x_2^2 + x_3^2] \\ &\quad + \sum_{i=1}^{3} S_{ii} F_i(x_1, x_2, x_3) + \sum_{i<j=1}^{3} S_{ij} F_{ij}(x_1, x_2, x_3), \end{aligned} \tag{7}$$

where the 6 coefficient functions are cyclic permutations of

$$\begin{aligned} F_3(x_1,x_2,x_3) &= 2(x_1^2 + x_2^2) + 4(x_1^2 x_2 + x_1 x_2^2) \\ &\quad + x_1^2 x_2^2 - 6x_1 x_2\,, \\ F_{12}(x_1,x_2,x_3) &= 2(x_1 + x_2)(x_3^2 + x_3) \\ &\quad + x_1 x_2 (4 - x_3^2 - 8x_3)\,. \end{aligned} \tag{8}$$

Formula (7) for $S_{\mu\nu} = 0$ was already calculated by Ore and Powell in 1949 [8].

Experimentally, it might be best to search for 3 jet events (and distinguish them from $e^+e^- \to 1\gamma \to q\bar{q}$ jets) by calorimetry. Equivalently, weighting each seen particle by its momentum reduces the jet smearing effect due to uncorrelated low momentum particles. This weighting is also necessary in order to construct the total jet momentum needed for comparison with eq. (7). It should be kept in mind that eq. (7) sometimes

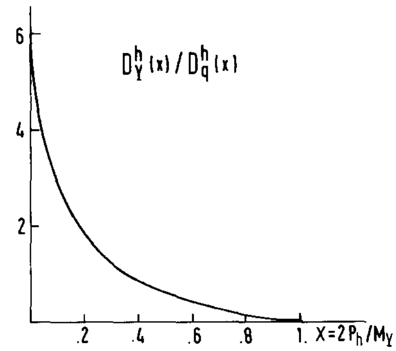


Fig. 2. The ratio $D^h_\Upsilon/D^h_q(x)$.

leads to a configuration with two nearly parallel gluon momenta and an apparent two jet structure. Possible confusion with $\Upsilon \to 1\gamma \to q\bar{q}$ jets can be dealt with by using data on $e^+e^- \to 1\gamma \to q\bar{q}$ jets off resonance [‡3]. Of course, the most favorable conditions for a 3 jet search occur if $e_Q = -1/3$, as the $1\gamma$ background is then small.

It may be that $\Upsilon(9.4)$ is too low in mass to produce satisfactory 3 jet events. One can then imagine excluding 2 jet $q\bar{q}$ events (they should have an axis perpendicular to which momenta are small), and studying the remainder. In these remaining events there will exist a plane perpendicular to which the momenta are bounded. The distribution of this plane is given by eq. (7), integrated over all or a part of the jet Dalitz plot. Another possibility is to search for 3 jet events in a subset of the data with low multiplicity (and high average particle momentum). Or one can pick events with a high momentum particle, and study the distribution of the remainder.

We now attempt to estimate particle multiplicities and momentum distributions for $\Upsilon \to 3g \to$ hadrons. We bracket the expected multiplicity by two extreme cases. In the first case, a gluon fragments with the same multiplicity as a single quark jet. Then we find from $\langle N^{ch}\rangle \approx 4.5$ in $e^+e^-$ annihilation at $E_{CM} = 6$ GeV [2],

$$\langle N^{ch}\rangle_{\Upsilon\to 3g} \approx \tfrac{3}{2}\left[\langle N^{ch}\rangle_{e^+e^-}; E_{CM} = 6\ \mathrm{GeV}\right] \approx 7\,, \tag{9}$$

(because there are three jets of average momentum 3 GeV, rather than two as in $e^+e^-$). If on the other hand (and this we believe more likely) a gluon materializes into a $q\bar{q}$ pair before fragmenting to hadrons, then

$$\langle N^{ch}\rangle_{\Upsilon\to 3g} \approx 3\left[\langle N^{ch}\rangle_{e^+e^-}; E_{CM} = 3\ \mathrm{GeV}\right] \approx 10\,. \tag{10}$$

Thus we expect $7 \leqslant \langle N^{ch}\rangle_{\Upsilon\to 3g} \leqslant 10$ [‡4]. For comparison, extrapolation of measured $e^+e^-$ multiplicities gives $\langle N^{ch}\rangle \approx 5$ off resonance in the $\Upsilon$ region. Clearly, $\Upsilon \to 3g$ may lead to many hadrons of low momentum, making it hard to see jets in individual events.

How will a gluon fragment to hadrons? Because it has integer color charge, it cannot simply fragment into a string of $q\bar{q}$ pairs ordered in rapidity from zero to the rapidity of the gluon. Such a one-dimensional string cannot transfer an integer color charge to $y = 0$. We suggest that gluon fragmentation takes place through gluon $\to q\bar{q}$, followed by independent fragmentation of each quark. Most of the time the $q\bar{q}$ will not have large invariant mass, so momenta perpendicular to $\boldsymbol{p}_{\mathrm{gluon}}$ will be bounded. We ignore the rare events where $q\bar{q}$ have a large invariant mass. (These will lead to a tail of high transverse momenta, or even $g \to q + \bar{q} \to 2$ jets.) The distribution of the virtual q and $\bar{q}$ should be that of a two body decay. Then taking the scaling limit and defining $x = p_h/p_q$; $z = p_h/p_g$ we obtain from the parent-child relation [9] [‡5]

$$D_g^{h^\pm}(z) = \int_z^1 \frac{dx}{x} D_q^{h^\pm}(x) D_g^q\left(\frac{z}{x}\right) + \int_z^1 \frac{dx}{x} D_{\bar{q}}^{h^\pm}(x) D_g^{\bar{q}}\left(\frac{z}{x}\right). \tag{11}$$

With our Ansatz, $D_g^{q,\bar{q}}(z/x) = 1$ and putting $xD_q^{h^\pm}(x) = a + b(1-x)^2$ we obtain the gluon fragmentation function

$$zD_g^{h^\pm}(z) = 2[a + b(1+z)](1-z) + 4bz\ln z\,. \tag{12}$$

We will put $a = 0.05$ and $b = 1.05$ from Sehgal's parametrization of $D_u^{\pi^\pm}$ [11]. In order to calculate the fragmentation distribution from $\Upsilon$, we need the gluon momentum distribution. For simplicity we use the distribution for $S_{\mu\nu} = 0$, and make a simple observation. The function

$$D_\Upsilon^g(\eta) = a\,\eta\,, \quad \text{with} \quad \eta = 2P_g/M_\Upsilon\,, \tag{13}$$

fits the exact expression [8] to 5% except at $\eta \to 1$, where it is 15% too small; we will set $a = 6$, to take account of the three gluons in $\Upsilon \to 3g$. Hence

$$xD_\Upsilon^{h^\pm}(x) = x\int_x^1 \frac{dz}{z} D_g^{h^\pm}(z) D_\Upsilon^g\left(\frac{x}{z}\right) \tag{14}$$

$$= 6\{[a + b + (5b - a)x](1-x) + 2b(2+x)x\ln x\},$$

(the last line follows from eqs. (12), (13)). In fig. 2 we have plotted the ratio $D_\Upsilon^h/D_q^h$, so as to give an idea how rapidly the hadron distribution from $\Upsilon \to 3g$ decreases relative to a quark jet distribution. We expect that the ratio

‡3 Also, note that for light $q$, $e^+e^- \to q\bar{q}$ or $e^+e^- \to \Upsilon \to 1\gamma \to q\bar{q}$ is suppressed at $\theta_{jet} = \phi_{jet} = 90°$ with transversely polarized $e^+e^-$ beams. This may be useful.

‡4 The *total* multiplicity must also include a part from $\Upsilon \to 1\gamma \to$ hadrons.

‡5 The $D_a^b(z)$ are Feynman's fragmentation functions for an object $a$ to give $b$ carrying momentum fraction $z$ [10];

$$\frac{\mathrm{d}\Gamma(\Upsilon \to 3\mathrm{g} \to \mathrm{h}^{\pm} + ...)/\mathrm{d}x}{[\mathrm{d}\sigma(\mathrm{e}^{+}\mathrm{e}^{-} \to 1\gamma \to \mathrm{h}^{\pm} + ...)/\mathrm{d}x]_{\text{off resonance}}}, \qquad (15)$$

will resemble fig. 2.

We conclude with some comments. (1) To the extent that scaling is valid, $D^{\mathrm{g}}_{\Upsilon}(\mathfrak{y}) \propto \mathfrak{y}$ allows us to extract $D^{\mathrm{h}^{\pm}}_{\mathrm{g}}$ directly from $\Upsilon \to 3\mathrm{g} \to$ hadrons data. From eq. (14) follows immediately

$$D^{\mathrm{h}^{\pm}}_{g}(z) = \mathrm{const} \cdot z^2 \frac{\partial}{\partial z} \left( \frac{1}{z} \frac{\mathrm{d}\sigma(\Upsilon \to \mathrm{h}^{\pm} + ...)}{\mathrm{d}z} \right)_{\Upsilon \to 3\mathrm{g\ events}}$$

(2) If the leading fragments of g tend to match its quantum numbers, we expect directly produced $\pi^0$, $\eta$, $\rho^0$ to be disfavored compared to $\omega$, $\phi$. As one example, we expect the inclusive ratios $\rho^0/\omega$ or $\rho^0/\phi$ to vanish as $|\boldsymbol{p}| \to M_{\Upsilon}/2$. There may be events of the form $\Upsilon \to 3\phi$, $\omega\phi\phi$, etc. We estimate that the probability for a gluon to yield $\phi$ is of order $4\pi\alpha_{\mathrm{s}}(M_{\phi})/f^2_{\phi}$ ($f_{\phi}$ is the $\phi$-$\gamma$ coupling), giving $\sim 10^{-3}$ as the probability for $\Upsilon \to 3\phi$. Of course, we expect gluon jets to be globally neutral [3]. Gluons are isoscalar, so that $\pi^0$ and $\pi^{\pm}$ distributions are the same (apart from isospin violating $\eta$ decays, which increase $\pi^0$).

(3) The decay $\Upsilon \to \gamma\mathrm{gg}$ will also take place with branching ratio [12]

$$\frac{\Gamma(\Upsilon \to \gamma 2\mathrm{g})}{\Gamma(\Upsilon \to 3\mathrm{g})} = \frac{16}{5} \frac{\alpha}{\alpha_{\mathrm{s}}} \frac{9}{4} e^2_{\mathrm{Q}} \approx \begin{cases} 0.05 & \text{for } e_{\mathrm{Q}} = -1/3\,, \\ 0.2 & \text{for } e_{\mathrm{Q}} = 2/3\,. \end{cases}$$

The photon spectrum is approximately linear in $z_{\gamma} = 2p_{\gamma}/M_{\Upsilon}$, and we expect average multiplicities 2/3 of those for $\Upsilon \to 3\mathrm{g}$. This process offers a way of obtaining gluon jets with a much suppressed contribution from $1\gamma$ decays of $\Upsilon$. $\Upsilon \to 1\gamma \to \mathrm{q}\bar{\mathrm{q}}\gamma$ has a rate much smaller than the above, and the photons have predominantly low energy.

(4) While we doubt that any of our detailed considerations can be applied to the decay of $\mathrm{J}/\psi \to 3\mathrm{g} \to$ hadrons, it does seem likely that $\mathrm{J}/\psi$ is a poorer source of high momentum hadrons than the $\mathrm{e}^{+}\mathrm{e}^{-}$ continuum nearby (especially after subtracting the hadron distribution from $\mathrm{J}/\psi \to 1\gamma \to$ hadrons).

(5) If the 3 gluon jet decay of a heavy $\mathrm{Q}\bar{\mathrm{Q}}$ state is found, it will in our opinion provide a striking confirmation of QCD, and we emphasize that this test does not involve comparing QCD radiative correction effects to experiment.

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