TWO-GLUON JETS FROM $\Upsilon'(10.0)$

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We propose to search for two-gluon jets with $E_{\text{jet}} \simeq 5$ GeV in $e^+e^- \to \Upsilon' \to \gamma + {}^3P_2 \to \gamma + 2\,g \to \gamma + 2$ jets. The gluonic origin of the jets can be tested by measuring the angular correlation. The rate and the energy of the monochromatic photon of this and of the competing processes are estimated.

Quantum Chromo Dynamics [1] predicts eight coloured massless gluons, which mediate the strong interaction and show up in Zweig forbidden hadronic decays [2]. Suggestions how to see these gluons were made by Ellis, Gaillard and Ross [3], who studied hard gluon bremsstrahlung from two-quark jets in e^+e^- annihilation, and by Koller and Walsh [4], who proposed to look, again in e^+e^-, for three-gluon jets in the decays of ${}^3S_1$ bound states of new heavy quarks, like $\Upsilon(9.4)$ [5].

It is unclear how much energy per gluon is required to let gluons show up as jets. Two extreme assumptions [4] would be (i) a gluon fragments similar to a quark or (ii) a gluon fragments into a $q\bar{q}$ pair first. In case (i) 2.5–3 GeV per gluon are sufficient for a jet to be formed, whereas in case (ii) 5–6 GeV per gluon are required, which would make it difficult to identify three-gluon jets from Υ decay.

We want to point out that in any case in the Υ' region there is a source of gluon jets with fixed gluon energy of ~ 5 GeV:

$$e^+e^- \to \Upsilon'(10.0) \to \gamma + {}^3P_2(9.8) \to \gamma + 2\,g \to \gamma + 2\,\text{jets}\,. \tag{1}$$

In this process the angular correlation allows to differentiate clearly two-gluon jets from two-quark jets. If these proposed gluon jets are found they would provide a threefold test. The existence of a dominant two-gluon decay of the ${}^3P_2$ state tests the perturbation series of QCD, the angular correlation tests the non-relativistic character of the bound state dynamics and the helicities of the gluons. Before we turn to the angular correlation let us estimate the quality of this source in discussing the P-wave splittings and the branching ratios.

The experimental mass difference $\Upsilon'(10.0) - \Upsilon(9.4)$ is approximately equal to $\psi'(3.7) - J/\psi(3.1)$ (for reviews of the J/ψ family, see ref. [6]). We therefore expect for the Υ system $M(2^3S_1) - M(^3P) \simeq 160$ MeV, as in charmonium. For the P-wave splittings, on the other hand, we estimate from the Fermi spin–orbit potential $\sim m^{-2}\langle r^{-1}\partial_r V(r)\rangle$ an m^{-1} scaling [‡1], where m denotes the quark mass. In the standard model [7] only the Coulombic term $-(4/3)\alpha_s/r$ contributes to spin-dependent forces, but the length scale is determined by the linear confinement potential, $\langle r^{-3}\rangle \sim m$. Incidentally m^{-1} scaling arises also in a pure logarithmic potential [8] [‡2]. We thus find for the Υ system

‡1 Amusingly, the implied constancy of ΔM^2 (M = bound state mass) is approximately satisfied even when comparing $\chi(3.55)$, $P_c(3.51)$, $\chi(3.41)$ with $A_2(1.31)$, $A_1(1.15)$, $\delta(0.98)$.

‡2 Quigg and Rosner have recently assembled scaling properties in potential models [9].

Table 1
Estimates of Υ′(10.0) and $^3P_2$(9.8) partial widths and branching ratios. Explanations see text.

	ψ′(3.7) partial decay widths input [6] (keV)	scaling behaviour	Υ′(10.0) partial decay widths [branching ratios] Q = 2/3 (keV)	Q = −1/3 (keV)
e^+e^-	1.9	$Q^2 m^{-\beta}, \frac{1}{2} \geq \beta \geq 0$	1.1 [3%] − 1.9 [4%]	0.27 [1.2%]−0.47 [1.7%]
"γ"			7.5−13	1.9−3.2
$^3P_2 + \gamma$	15	$Q^2 k^2 m^{-1}$	6.2 [16%−13%]	1.5 [7%−5.5%]
$^3P_1 + \gamma$	15	$Q^2 k^2 m^{-1}$	4.5	1.1
$^3P_0 + \gamma$	15.5	$Q^2 k^2 m^{-1}$	2.5	0.6
3 g [a)]	20	$\alpha_s^3 m^{-\beta}, \frac{1}{2} \geq \beta \geq 0$	5.6−10	5.6−10
$\pi\pi\, ^3S_1$	105	m^{-2}	11	11
$\Gamma_{tot}(\Upsilon')$			38−48	22−28
	$^3P_2$(3.5) branching ratios input [6]	scaling behaviour	$^3P_2$(9.8) branching ratios Q = 2/3	Q = −1/3
$^3S_1 + \gamma$	15%	$Q^2 k^2 m^{-1}$	50%−10%	20%− 2%
2 g [a)]	85%	$\alpha_s^2 m^{-\delta}$, $2 \geq \delta \geq 0$	50%−90%	80%−98%

a) This is a theoretical input.

$M(^3P_2) - M(^3P_0) \simeq 50$ MeV or, more explicitly, $M(\Upsilon') - M(^3P_2) \simeq 150$ MeV, $M(\Upsilon') - M(^3P_1) \simeq 170$ MeV and $M(\Upsilon') - M(^3P_0) \simeq 195$ MeV.

The rate for process (1) depends on the product of branching ratios $B(\Upsilon' \to \gamma + {}^3P_2) \cdot B(^3P_2 \to 2$ gluons). We assume the main contributions to the Υ′ width to be (i) the second-order electromagnetic decay $\Gamma_{\text{“}\gamma\text{”}} = \Gamma_{e^+e^-}(3 + R_{hadr})$, (ii) the three-gluon annihilation Γ_{3g}, (iii) the ππ cascade $\Gamma_{\pi\pi 1^3S}$, (iv) and the radiative decays $\Gamma_{\gamma 3P}$. Again we estimate these partial widths by using scaling arguments with ψ′ as input.

$\Gamma_{e^+e^-}$ scales as $Q^2 |\varphi(0)|^2 m^{-2}$. Here $|\varphi(0)|^2 \sim m^3$ for Coulombic wave functions and $\sim m$ in a linear potential. Detailed numerical calculations with the standard potential yield $|\Upsilon'(0)|^2/|\psi'(0)|^2 \lesssim (5\,\text{GeV}/1.6\,\text{GeV})^2$ [‡3]. The logarithmic potential gives $|\varphi(0)|^2 \sim m^{3/2}$. We therefore safely assume that $\Gamma_{e^+e^-}$ scales as $Q^2 m^{-\beta}$ with $1/2 \geq \beta \geq 0$. Γ_{3g} has the scaling behaviour $\alpha_s^3 m^{-\beta}$, where β is the same value as in $\Gamma_{e^+e^-}$ according to standard QCD. For $M(\Upsilon') - M(\Upsilon) = M(\psi') - M(J/\psi)$ the ππ cascade scales as m^{-2} [10]. We derive the scaling behaviour of the radiative widths from the dipole sum rules [11] $\Gamma_{\gamma 3P} \sim Q^2 k^2 m^{-1}$.

‡3 This relation reflects an equality of $\Gamma(V \to e^+e^-)/Q^2$ for V = ψ′ and Υ′ or J/ψ and Υ respectively. Experimentally $\Gamma(V \to e^+e^-)/\langle Q_V \rangle^2$ is even approximately equal for all ground state vector mesons ρ, ω, φ, J/ψ.

Last not least the rate depends on the branching ratio of $^3P_2$ into two gluons. We assume $\Gamma_{tot} \simeq \Gamma(^3P_2 \to \gamma + {}^3S_1) + \Gamma(^3P_2 \to 2\,g)$, where the radiative width again scales as $Q^2 k^2 m^{-1}$, and we feel safe with $\Gamma(^3P_2 \to 2\,g) \sim \alpha_s^2 m^{-\delta}$, $0 \leq \delta \leq 2$.

All our estimates are summarized in table 1. We find that $B(\Upsilon' \to \gamma + {}^3P_2)$ is of the order of 6% to 16% and $B(^3P_2 \to 2\,g)$ ranges from 98% to 50%. One should keep in mind that all these estimates have their uncertainties. We only want to stress one point. In ψ′ decays the radiative widths are known to be a factor of 2–3 below the sum rule limits. This cannot be due to the radiative transitions from 1^3P to higher S-waves alone, which only contribute a small fraction to the dipole sum rules. It must have its main source in relativistic corrections. These corrections, however, may substantially change, if we go from the J/ψ to the Υ system, letting the radiative widths increase above the estimated values of table 1. The sum rules set upper limits of $\Gamma(\Upsilon' \to \gamma + {}^3P_j) = (22, 18, 8)$ keV for $Q = 2/3$ and $j = (2, 1, 0)$ with the phase space as given above.

We now turn to the angular correlations in process (1). Because of the relatively small photon energy we may neglect the recoil. The two-gluon jets are almost collinear. The photon–gluon angular correlations are analogous to the photon–photon correlations in the cascade decay $2^3S_1 \to \gamma + {}^3P_2 \to \gamma\gamma + 1^3S_1$, which have been extensively studied in the literature [12]. We fix the helicity amplitudes for $2^3S_1 \to \gamma + {}^3P_2$ by assuming a pure E1 transition. In QCD the ${}^3P_2 \to 2\,g$ transition is (up to colour factors) analogous to the corresponding two photon decay in positronium [13]. We write the amplitudes for this transition as $T^{\lambda_1-\lambda_2} = T^{\mu\nu}\epsilon_\mu^{\lambda_1}\epsilon_\nu^{\lambda_2}$, where $\lambda_1, \lambda_2 = \pm 1$ are the photon helicities, and find that in positronium only the helicities $\lambda_1 - \lambda_2 = \pm 2$ contribute, since

$$T^{\lambda_1-\lambda_2} = T^{\mu\nu}\epsilon_\mu^{\lambda_1}\epsilon_\nu^{\lambda_2} = 0 \quad \text{for } \lambda_1 = \lambda_2 \,. \tag{2}$$

We obtain

$$W(\theta, \varphi, \theta_{\gamma j}) \sim (1 + \cos^2\theta_{\gamma j})^2(1 + \cos^2\theta) + 2(1 - \cos^4\theta_{\gamma j}) \sin^2\theta + (1 + \cos^2\theta_{\gamma j}) \times (\sin 2\theta_{\gamma j} \sin 2\theta \cos\varphi + \sin^2\theta_{\gamma j} \sin^2\theta \cos 2\varphi) \,, \tag{3 a}$$

where $\theta_{\gamma j}$ is the angle between the photon and the gluon jets, while θ, φ give the direction of the beam axis in the coordinate system where the photon defines the z-axis and the jets define $\varphi = 0$.

When the jets originate from spin 1/2 (quarks) or spin 0 the helicity ± 2 state of ${}^3P_2$ cannot contribute to the decay. In this case we write the angular distribution with the relative weight $|A|^2$ for helicity +1 and −1 over helicity 0 (for spin 0 $|A|^2 = 0$):

$$\begin{aligned} W(\theta, \varphi, \theta_{\gamma j}) \sim{} & [|A|^2(2 + 2\cos^2\theta_{\gamma j} - 4\cos^4\theta_{\gamma j}) \\ & + (10/3 - 8\cos^2\theta_{\gamma j} + 6\cos^4\theta_{\gamma j})](1 + \cos^2\theta) \\ & + [|A|^2(2 - 6\cos^2\theta_{\gamma j} + 8\cos^4\theta_{\gamma j}) \\ & + 12(\cos^2\theta_{\gamma j} - \cos^4\theta_{\gamma j})]\sin^2\theta \\ & + [|A|^2(1 - 4\cos^2\theta_{\gamma j}) - (4 - 6\cos^2\theta_{\gamma j})] \\ & \times \sin 2\theta_{\gamma j} \sin 2\theta \cos\varphi \\ & - [|A|^2\, 4\cos^2\theta_{\gamma j} + (2 - 6\cos^2\theta_{\gamma j})] \\ & \times \sin^2\theta_{\gamma j} \sin^2\theta \cos 2\varphi \,. \end{aligned} \tag{4a}$$

Integration over θ, φ gives

$$W(\theta_{\gamma j}) \sim 1 + \cos^2\theta_{\gamma j} \,, \tag{3b}$$

for gluon jets, whereas

$$W(\theta_{\gamma j}) \sim 1 - [(6 + 3|A|^2)/(10 + 9|A|^2)]\cos^2\theta_{\gamma j} \,, \tag{4b}$$

for quark jets or spin 0 jets ($|A|^2 = 0$). Thus the gluon jets are clearly distinguishable from quark jets or spin 0 jets. Of course such a distinction is not possible when the decaying $Q\bar{Q}$ state has spin 0, like the ${}^3P_0$ state.

Summarizing, in e^+e^- annihilation Υ' yields gluons with a distinct lab energy of ~5 GeV which should form jets. The jets are accompanied by a monochromatic photon of ~150 MeV from the transition $\Upsilon' \to \gamma + {}^3P_2$. Independently of the hadron fragmentation picture one can identify the gluonic origin of these jets by the angular distribution alone. The photon has to be resolved [‡4] from the competing monochromatic photons in the decays $\Upsilon' \to \gamma + {}^3P_1$ and $\gamma + {}^3P_0$ which we estimate to have $E \approx 170$ MeV and 195 MeV respectively. The branching ratio for the process $\Upsilon' \to \gamma + {}^3P_2 \to \gamma + 2\,g \to \gamma + 2$ jets is estimated to lie between 5% and 15%.

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‡4 This may be very hard experimentally even for the new generation of detectors at PETRA and PEP, as W. Wallraff pointed out to us.

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