A POLARIZATION PREDICTION FROM TWO-GLUON EXCHANGE FOR $1^{--}(Q\bar{Q}) \to \gamma + 2^{++}(q\bar{q})$ ☆

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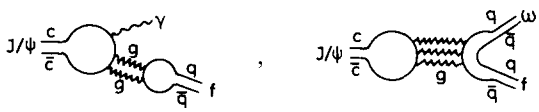
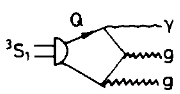
An exclusive Zweig suppressed radiative decay of heavy 1^{--} vector mesons is considered in the approximation ${}^3S_1(Q\bar{Q}) \to \gamma + [g+g \to]\, {}^3P_2(q\bar{q})$ with on-shell gluons. The 2^{++} meson will appear to be in the helicity-zero state if the mass ratio $M_{2^{++}}/M_{1^{--}}$ is small. A comparison with the decays $J/\psi(3.1)$, $\psi'(3.7)$, $\Upsilon(9.4)$, ... $\to \gamma + f(1.3)$, $f'(1.5)$,... is suggested.

The experimental observations of the decay $J/\psi \to \gamma + f$ [1,2] with a rate similar to the decay $J/\psi \to \omega + f$ [3,4] nicely support the gluon counting rules which are abstracted from the asymptotic freedom limit of QCD [5] and contradict superficial VDM estimates. The strength of Zweig suppressed decays of heavy $Q\bar{Q}$ bound states increases with decreasing minimal number of gluons. Zweig suppressed decays involving an "isoscalar" photon may thus become "large" compared to the vector dominance estimate: $\Gamma(J/\psi \to \gamma f)/\Gamma(J/\psi \to \omega f) = \pi\alpha/\gamma_\omega^2 \simeq 10^{-3}$, as might be seen from the comparison of the following graphs:



(1a, b)

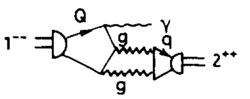
The necessary additional gluon suppresses ω-dominance of the photon. For this reason an exploration of the typical QCD graph (1a) seems worthwhile. Conceptually QCD is a theory like QED, however such diagrams are calculable only in principle. For instance, $1^{--}(Q\bar{Q})$ annihilation into one photon and two gluons has been studied with appropriate approximation of the $Q\bar{Q}$ bound state:



+ permutations , (2)

in particular its application to the decay width $J/\psi \to \gamma$ + all old hadrons [6] or the decay distribution $\Upsilon \to \gamma$ + two gluon jets [7].

In this paper I consider the diagram



+ permutations , (3)

which is related to the decays $J/\psi \to \gamma + f$, $\psi' \to \gamma + f$, etc. The $1^{--} \to \gamma + 2^{++}$ transition depends, for a real photon, on three amplitudes which are independent by kinematics but which get specified by dynamics. In this spirit I shall obtain a polarization prediction from lowest-order QCD which I suggest to apply to these decays, to be tested in

☆ I would like to dedicate this paper to Dr. Kurt Mellentin, who set up the High Energy Physics Index and the Scientific Retrieval System at DESY, and who headed the Scientific Documentation here until his sudden death, in December 1977.

$$e^+e^- \to J/\psi,\ \psi',\ (\Upsilon), \ldots \to \gamma + f,\ f',\ (X), \ldots \to \gamma + \pi\pi,\ K\bar{K}\,. \tag{4}$$

The dynamical assumptions and the procedure of my calculation are as follows.

(i) A nonrelativistic ${}^3S_1(Q\bar{Q})$ state annihilates into one photon and two gluons. This leads to the familiar QCD analogue in QED [8], the decay amplitudes $\mathcal{M}_1$ take the form [‡1]

$$\begin{aligned}\mathcal{M}_1^*(k_1,k_2,k_3;\epsilon_1,\epsilon_2,\epsilon_3;K,\epsilon) \propto{} & [(k_1+k_2,k_3)(k_2+k_3,k_1)(k_3+k_1,k_2)]^{-1}\\ & \times\{(\epsilon_1\epsilon_2)\,[-(k_1k_3)(\epsilon_3k_2)(\epsilon^*k_1) - (k_1k_3)(k_2k_3)(\epsilon^*\epsilon_3) - (k_2k_3)(\epsilon_3k_1)(\epsilon^*k_2)]\\ & + (\epsilon^*\epsilon_3)\,[(k_3k_1)(\epsilon_1k_2)(\epsilon_2k_3) - (k_1k_2)(\epsilon_1k_3)(\epsilon_2k_3) + (k_3k_2)(\epsilon_2k_1)(\epsilon_1k_3)] + 1 \leftrightarrow 3 + 2 \leftrightarrow 3\}\,,\end{aligned} \tag{5}$$

where the k_i denote the photon or gluon four-momenta, $\epsilon_i^{(\lambda_i)}$ their polarization four-vectors and where $K = k_1 + k_2 + k_3$, $K^2 = M_1^2$, and $\epsilon^{(\lambda)}$ refers to the vector meson, $(\epsilon K) = 0$.

If I consider eq. (5) for real quanta $(k_i^2 = 0)$ and take the limit $M_X^2 = (k_1+k_2)^2 \to 0: k_2 \to \lambda k_1$, $(\epsilon_1 k_2) \to 0$, $(\epsilon_2 k_1) \to 0$, then only the first term $(\epsilon_1\epsilon_2)\,[---]$ survives. The physical rule is that the pair (1,2) of the quanta has $J_z = 0$:

,

(→ denotes helicity). Note, that for this pair $J^{PC} = 0^{-+}$ is forbidden!

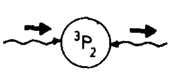
Clearly, any calculation for $M_X^2 \neq 0$ has to be carried through with the full amplitude (5). However, the above limiting case, when combined with the selection rule in (ii) (see below), might be heuristically useful for the limiting case (b) (see below).

(ii) The gluons create a quark pair in the nonrelativistic ${}^3P_2$ state. (${}^3P_2(Q\bar{Q})$ annihilation into gluons [9,10] has, again, its analogue in the positronium decay into $\gamma\gamma$ [11].) The $g + g \to 2^{++}$ vertex $\mathcal{M}_2$ becomes:

$$\begin{aligned}\mathcal{M}_2(k_1,k_2;\epsilon_1,\epsilon_2;P,E) \propto{} & (k_1k_2)^{-2}E^*_{\mu\nu}\\ & \times\{4(k_1k_2)\,\epsilon_1^\mu\epsilon_2^\nu - (\epsilon_1\epsilon_2)(k_1-k_2)^\mu(k_1-k_2)^\nu + 2(k_1\epsilon_2)\,\epsilon_1^\mu(k_1-k_2)^\nu + 2(k_2\epsilon_1)\,\epsilon_2^\mu(k_2-k_1)^\nu\}\,,\end{aligned} \tag{6}$$

where $P = k_1 + k_2$, $P^2 = M_2^2$, and where $E^{(\lambda)}_{\mu\nu}$ denotes the symmetric, traceless spin-two polarization tensor, $E^\mu_\nu P_\mu = 0$.

From eq. (6) follows the selection rule that two on-shell gluons couple to the spin-two meson in the $J_z = \pm 2$ states only [10]:



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The non-relativistic approximation for the 2^{++} mesons made of light quarks, i.e. the f and f′, might be here a disputable dynamical assumption.

(iii) The intermediate gluons are kept on their mass shell. With this prescription $(k_1^2 = k_2^2 = 0)$ and with M_2^2 fixed the loop integral in the graph (3) reduces actually to an integration over the angle between the three-momenta $k_1 - k_2$ and $k_3 = K$ in the 2^{++} rest frame.

For the kinematically independent quantities one can choose the helicity amplitudes

$$1^{--}(1,0,-1) \to \gamma(1) + 2^{++}(0,1,2) \equiv (A^0, A^1, A^2)\,. \tag{7}$$

The result of my calculation along the steps (i)–(iii) is that the ratios of the helicity amplitudes depend on the mass ratio M_2/M_1 and are given by:

‡1 A closely related form has been given recently by Berends and Komen [8].

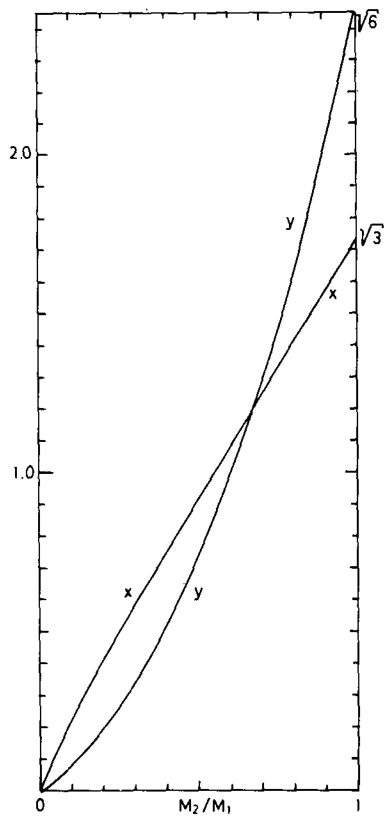

Fig. 1. Ratio of helicity amplitudes, $x = A_1/A_0$, $y = A_2/A_0$, for $^3S_1(Q\bar{Q}) \to \gamma + {}^3P_2(q\bar{q})$, as function of the mass ratio $M_{2^{++}}/M_{1^{--}}$.

Table 1
Ratios of the helicity amplitudes, obtained from eq. (8).

	x	y
$J/\psi \to \gamma f$	0.76	0.54
$J/\psi \to \gamma f'$	0.88	0.70

$$x \equiv A_1/A_0 = \frac{2}{\sqrt{3}}\left(\frac{z-1}{z+1}\right)^{1/2} \frac{-2 - 6z^2 + 3z^{-1}(1-z^4)\,[\ln(z-1) - \ln(z+1)]}{-10 + 6z^2 + 3z^{-1}(1-2z^2+z^4)\,[\ln(z-1) - \ln(z+1)]}\,,$$

$$y \equiv A_2/A_0 = \frac{1}{\sqrt{6}}\,\frac{z-1}{z+1}\,\frac{38 + 6z^2 + 3z^{-1}(1+6z^2+z^4)\,[\ln(z-1) - \ln(z+1)]}{-10 + 6z^2 + 3z^{-1}(1-2z^2+z^4)\,[\ln(z-1) - \ln(z+1)]}\,, \tag{8}$$

where $z = (M_1^2 + M_2^2)/(M_1^2 - M_2^2)$; x and y are shown graphically in fig. 1.

For the physical discussion of the result (8), I consider first two limiting cases.

(a) If $M_2/M_1 \to 1$ then $(x, y) \to (\sqrt{3}, \sqrt{6})$: it corresponds to an electric dipole (E1) radiation pattern. This limit would be physically realized if there were two different heavy quarks with nearly equal masses.

(b) If $M_2/M_1 \to 0$, then $(x,y) \to 0$: the spin-two meson is in the helicity-zero state. This limit should be satisfied rather well in the decays $\Upsilon(9.4) \to \gamma + f, f'$.

The application of eq. (8) to the observed [1,2] decays gives values for the ratios of the helicity amplitudes, as shown in table 1. The amplitudes, x, y, can be experimentally determined from the angular distribution of the reaction (4). It has the form [12][‡2,3]:

‡2 The decay distribution of a 2^{++} meson into a pair of pseudoscalar mesons with momenta p_1, p_2 follows uniquely from $\mathcal{M}_3 \propto E_{\mu\nu}^{(\lambda)}(p_1 - p_2)^\mu (p_1 - p_2)^\nu$.

‡3 If the z-axis is chosen opposite to the γ-direction ($\equiv 2^{++}$ helicity convention), the terms proportional to $\cos\phi_p$ change their sign.

$$W(\theta_\gamma;\theta_p,\phi_p) \propto (1+\cos^2\theta_\gamma)\,[(3\cos^2\theta_p-1)^2+3/2\,y^2\sin^4\theta_p]+3x^2\sin^2\theta_\gamma\sin^2 2\theta_p$$
$$+\sqrt{3}\cdot x\sin 2\theta_\gamma\sin 2\theta_p(3\cos^2\theta_p-1-(\sqrt{3}/\sqrt{2})y\sin^2\theta_p)\cos\phi_p+\sqrt{6}\cdot y\sin^2\theta_\gamma\sin^2\theta_p(3\cos^2\theta_p-1)\cos 2\phi_p\,, \tag{9}$$

where θ_γ is the angle between the photon and the e^+e^- beam axis, θ_p, ϕ_p are the angles of the pseudoscalar mesons, in the 2^{++} rest frame, relative to the photon with $\phi_p = 0$ defined by e^+e^-.

The predicted angular dependence does not seem to be inconsistent with a preliminary analysis [13] of experimental data on $J/\psi \to \gamma f$ [1]. A precise measurement would be an interesting check of applied QCD.

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