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MINIMAL MASS CORRECTIONS IN DEEP INELASTIC LEPTON SCATTERING

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We consider the problem of finite mass corrections in deep inelastic electron (muon) and neutrino scattering. Using a non-perturbative approach, we are able to predict the minimal mass corrections to scaling. It is found that they already account for most of the scaling violations displayed by the electron (muon) scattering data except, perhaps, at very large x where we cannot make any statement. The ratio σ_L/σ_T as well as the antineutrino cross section are also well reproduced while the neutrino cross section leaves some room for further scaling corrections. The significance of various scaling variables is critically discussed. From a theoretical point of view most of them are inferior to the plain variable x, and none of them correctly accommodate finite target and threshold mass effects. A brief discussion of the analytic structure of the moments is included.

1. Introduction

The violation of Bjorken scaling as being displayed by the recent data on deep inelastic e, μ, ν and $\bar{\nu}$ scattering [1–4] has attracted a lot of interest from the theoretical point of view. Within the framework of renormalizable field theories strict Bjorken scaling cannot hold. Violation of scaling arises as a consequence of the renormalization procedure. Asymptotically free gauge theories [5,6], which come nearest to scaling [7] and currently provide the most promising theoretical framework in hadron physics, display in leading-order perturbation theory a characteristic pattern of logarithmic deviations from scaling at large enough Q^2 [8–14]. Alternatively, it is quite possible that most of the violation of Bjorken scaling seen in the data is due to other causes [15]. The spectacular rise in F_2 with Q^2 at small x, for example, may well be attributed to the excitation of the new hadronic degrees of freedom [16].

So far no clear picture as to the origin of the scaling violations has arisen. Comparison of asymptotic freedom corrections to scaling with experiment is aggravated by finite m^2/Q^2 corrections which are inevitable in the present-day regime of experiments [17]. Here m is some characteristic hadron mass which may be as large as, e.g., the ψ mass. The importance of such corrections is underlined by the empirical fact that (already) at SLAC energies scaling is strongly improved if the data is plotted *versus* the Bloom-Gilman variable [18] $x' = Q^2/(2m_N\nu + M^2)$.

The question arises whether asymptotically free gauge theories or the more straightforward (but closely related) parton model can say anything about corrections at the level of m^2/Q^2. It has been argued that these corrections are adequately represented through the variable ★ [19]

$$\xi = x\,\frac{1 + (m_f^2 - m_i^2)/Q^2 + \sqrt{[1 + (m_f^2 - m_i^2)/Q^2]^2 + 4m_i^2/Q^2}}{1 + \sqrt{1 + 4m_N^2 x^2/Q^2}}\,, \tag{1.1}$$

which is based on perturbation theory applied to the twist-2 Wilson coefficients. However, as we shall see, the ξ variable does not correctly accommodate finite target and threshold mass effects. This is what we must demand the *true* scaling variable to arrange, in particular, considering the high charm threshold. For certain m_i, m_f we even find the variable x superior to ξ. This issue has also been criticized by other authors [17,20,21].

Consider, e.g., charm production off charmed sea quarks. The threshold for this process involves either the ψ mass or the D and the charmed baryon mass. Neither of them can be represented through the masses of the struck and produced partons alone, the only masses appearing in the twist-2 operators, but require twist greater than two operators [17]. Since the calculation of the relative weight of matrix elements for twist-2 to higher-twist operators involves knowledge of the target wave function, it seems impossible that finite mass corrections can be calculated by perturbation methods.

We shall offer a vastly model-independent discussion of non-perturbative mass corrections to Bjorken scaling. Our primary mathematical tool will be the DGS representation [22] which has proven itself very useful for investigating certain features of forward current-hadron amplitudes [23]. The DGS representation can be understood as a generalization of the light-cone representation [24,25] arising from the Wilson expansion [26], in the sense that it incorporates the full analytic structure of the forward Compton amplitude. Scaling is not automatically a property of the DGS representation but is provided for by requiring that asymptotically it merges into the light-cone representation.

In sect. 2 we shall outline what we understand by minimal mass corrections. In sect. 3 earlier attempts of casting the finite mass corrections into some scaling variable are critically discussed. Furthermore, we derive the analytic structure of the moments. In sect. 4 we give a comparison with experiment. Finally, some concluding remarks are presented in sect. 5.

★ Here m_i and m_f are the initial and final parton masses, respectively.

2. Preliminaries and the concept of minimal mass corrections

We begin with recapitulating the basic properties of the DGS representation of the forward Compton amplitude:

$$T_{\mu\nu} = i\int d^4x\, e^{-iqx}\langle p|T(j_\mu(x)j_\nu(0))|p\rangle$$
$$= \left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) T_2$$
$$+ (q_\mu q_\nu - q^2 g_{\mu\nu})\frac{m_N\nu}{q^2} T_1 \tag{2.1}$$

(m_N being the target mass). The DGS representation for T_2, and similarly for T_1, is given by

$$T_2(q,p) = \int_0^\infty d\sigma \int_{-1}^{+1} d\alpha \frac{G_2(\alpha,\sigma)}{q^2 + 2m_N\nu\alpha + m_N^2\alpha^2 - \sigma}\ . \tag{2.2}$$

The support of $G_2(\alpha, \sigma)$ is drawn in fig. 1 *. It is important to note that it does not extend down to $\sigma = 0$. If, despite this, all mass terms in the denominator of eq. (2.2) (i.e., σ and $m_N^2\alpha^2$) are disregarded and the dependence of $G_2(\alpha,\sigma)$ on σ is integrated out, eq. (2.2) reduces to the light-cone representation [24]

$$T_2(q,p) = \int_{-1}^{+1} d\alpha \frac{\tilde{G}_2(\alpha)}{q^2 + 2m_N\nu\alpha}\ , \qquad \tilde{G}_2(\alpha) = \int_0^\infty d\sigma\, G_2(\alpha,\sigma)\ . \tag{2.3}$$

The DGS representation has originally been derived on grounds of causality **. Later, Nakanishi [22] has shown from an independent point of view that it holds in every order in perturbation theory.

The structure functions are given by

$$F_2(x,q^2) = \frac{2m_N\nu}{\pi}\ \mathrm{Im}\ T_2(q,p)$$
$$= 2m_N\nu \int_0^\infty d\sigma \int_{-1}^{+1} d\alpha\, \delta(q^2 + 2m_N\nu\alpha + m_N^2\alpha^2 - \sigma)\, G_2(\alpha,\sigma)$$
$$= \int_0^\infty d\sigma\, G_2(\alpha_\sigma,\sigma)\frac{\nu}{\sqrt{\nu^2+Q^2+\sigma}}\ , \tag{2.4}$$

* See, e.g., ref. [27]. The support may be smaller, but G_2 is definitely zero outside the boundaries drawn in fig. 1.
** Ref. [22], first reference.

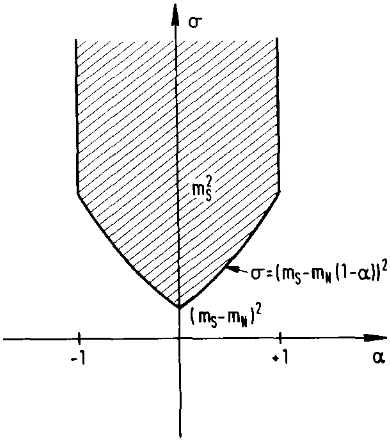

Fig. 1. Support of the spectral function $G_2(\alpha, \sigma)$, m_S being the lowest s-channel threshold. Here we have chosen $m_S = m_N + m_\rho$.

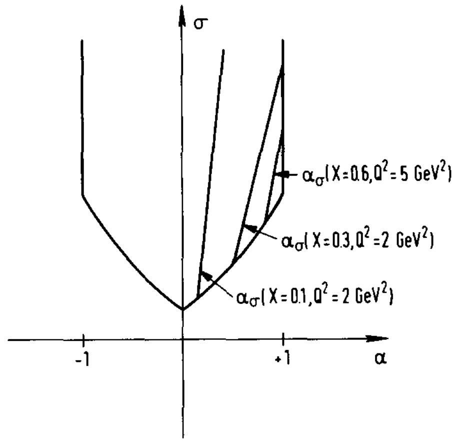

Fig. 2. Line of support of the scaling variable α_σ for various x and Q^2.

and similarly for F_1, where

$$\alpha_\sigma = x\left(1 + \frac{\sigma}{Q^2}\right) \frac{2}{1 + \sqrt{1 + 4m_N^2 x^2 (Q^2 + \sigma)/Q^4}} , \qquad Q^2 = -q^2 . \tag{2.5}$$

In order to guarantee Bjorken scaling (within logarithms) the spectral function $G_2(\alpha, \sigma)$ must essentially behave for large σ like $\sigma^{-\lambda}$ with $\lambda \geqslant 1$. In field theory we only can have $\lambda = 1$ (plus logarithms). The case $\lambda > 1$ which corresponds to "exact" asymptotic scaling is not possible in four dimensions within the framework of renormalizable field theory [5,6].

The integral (2.4) describes a path $\alpha = \alpha_\sigma$ in the (α, σ) plane. Clearly, the integrand is non-zero only on that section of the path which intersects with the support of $G_2(\alpha, \sigma)$ as has been indicated in fig. 2. At the upper end the integration is cut off at $\alpha_\sigma = 1$ which corresponds to

$$\sigma_{\max} = \frac{1-x}{x} Q^2 + m_N^2 , \tag{2.6}$$

while the lower limit of integration is implicitly given by the boundary

$$\sigma = (m_S - m_N(1 - \alpha_\sigma))^2 \tag{2.7}$$

(*cf.* fig. 1). For $x = 0$ the path is along the σ axis ($\alpha = 0$) and extends to infinity. For the upper (threshold) value of x, given by

$$x_{\max} = \frac{Q^2}{Q^2 + m_S^2 - m_N^2} , \tag{2.8}$$

the path shrinks to the single point

$$\sigma = m_S^2, \quad \alpha_\sigma = \alpha_{m_S^2} = 1 . \tag{2.9}$$

This is to say that the upper and lower limit of integration, i.e., (2.6) and (2.7), coincide for $x = x_{max}$ and, what is important to remember, that α_σ has the unique threshold *one*.

Let us now imagine that the spectral function $G_2(\alpha, \sigma)$ is smooth and positive * and sufficiently damped for large σ. More precisely, let us assume that $G_2(\alpha, \sigma) \sim \sigma^{-\lambda}$ with $\lambda > 1$. This brings us close to the parton model, inasmuch as it leads to asymptotic scaling. Field-theoretically $\lambda > 1$ cannot be realized. But remember that, primarily, we are interested in finite mass corrections rather than scaling violations of intrinsic field-theoretical origin **. Under these conditions the structure functions F_2 can then be rewritten in the form

$$F_2(x, Q^2) = \sum_i F_2^i(\alpha_{\sigma_i}) \frac{\nu}{\sqrt{\nu^2 + Q^2 + \sigma_i}} , \tag{2.10}$$

where we have divided the path of integration (eq. (2.4)) into small subsections and applied the mean-value theorem. The lowest and highest value of σ_i can be read off from eqs. (2.7) and (2.6) respectively. In practice, eq. (2.10) means that F_2 depends on x and Q^2 only through the various α_{σ_i} (being given by eq. (2.5) with σ replaced by σ_i) and through the boundaries being imposed on σ_i (eqs. (2.6) and (2.7)).

The latter dependence is rather annoying since our goal is, like that of many outriders, to case the finite mass corrections to Bjorken scaling into a single scaling variable. Therefore, let us make the change of variables

$$\sigma \to \bar{\sigma} = \sigma - m_N^2 \alpha^2 - 2m_N(m_S - m_N)|\alpha| \tag{2.11}$$

in eq. (2.2) and subsequent equations. In terms of $\bar{\sigma}$ and α, the support of G_2 has then the simple rectangular shape as shown in fig. 3, so that at least the x, Q^2 dependence *via* the lower boundary is eliminated.

Eq. (2.4) becomes, in terms of the new variable $\bar{\sigma}$,

$$F_2(x, Q^2) = \int_{(m_S - m_N)^2}^{\infty} d\bar{\sigma}\, \bar{G}_2(\eta_{\bar{\sigma}}, \bar{\sigma}) \frac{1}{1 - 2m_N(m_S - m_N)x/Q^2} , \tag{2.12}$$

where $\bar{G}_2(\alpha, \bar{\sigma}) = G_2(\alpha, \sigma)$ and

$$\eta_{\bar{\sigma}} = x \frac{1 + \bar{\sigma}/Q^2}{1 - 2m_N(m_S - m_N)x/Q^2} . \tag{2.13}$$

* This excludes non-scaling contributions of the sort being discussed later. There is also no first principle which forbids $G_2(\alpha, \sigma)$ to become negative.

** For $\lambda = 1$, the structure functions will display logarithmic deviations from scaling of the sort found in (perturbative) asymptotically free gauge theories due to the fact that (2.4) becomes logarithmically divergent and the upper limit of integration being given by (2.6).

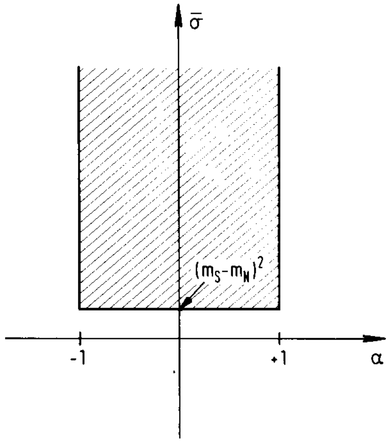

Fig. 3. Support of the spectral function $\bar{G}_2(\alpha, \bar{\sigma})$ for $m_S = m_N + m_\rho$.

The integral (2.12) is cut off at

$$\bar{\sigma}_{\max} = \frac{1-x}{x} Q^2 - 2m_N(m_S - m_N)\,. \tag{2.14}$$

The variable $\eta_{\bar{\sigma}}$ can also be directly obtained from (2.5) by inserting

$$\sigma = \bar{\sigma} + m_N^2\alpha_\sigma^2 + 2m_N(m_S - m_N)\,\alpha_\sigma$$

(and resolving the resulting expression with respect to α_σ). In other words

$$\eta_{\bar{\sigma}} = \alpha_{\sigma = \bar{\sigma} + m_N^2\alpha_\sigma^2 + 2m_N(m_S - m_N)\alpha_\sigma}\,, \tag{2.15}$$

or, what amounts to the same,

$$\alpha_\sigma = \eta_{\bar{\sigma} = \sigma - m_N^2\eta_{\bar{\sigma}}^2 - 2m_N(m_S - m_N)\eta_{\bar{\sigma}}}\,. \tag{2.16}$$

Let us now, starting from eq. (2.12), repeat the steps which led to (2.10). Similar to (2.10) we can write

$$F_2(x, Q^2) = \frac{1}{1 - 2m_N(m_S - m_N)x/Q^2} \sum_{i=1}^{N} F_2^i(\eta_{\sigma_i}) \tag{2.17}$$

where now

$$\sigma_1 \in ((m_S - m_N)^2, (m_S - m_N)^2 + \Delta m^2)\,,$$

$$\vdots$$

$$\sigma_i \in ((m_S - m_N)^2 + (i-1)\,\Delta m^2, (m_S - m_N)^2 + i\Delta m^2)\,. \tag{2.18}$$

The upper value of σ_i, i.e., σ_N (being given by (2.14)), is still x and Q^2 dependent. Since, by fiat, the series (2.17) is sufficiently convergent, the main contribution will, however, come from low and medium σ_i, so that the dependence on this boundary can be neglected at least for *smaller* x and/or *larger* Q^2. This brings us close to our goal. The finite mass corrections to scaling are now entirely cast into a functional dependence on the scaling variables η_{σ_i}.

The higher σ_i is, the larger are the scaling violations induced by η_{σ_i}. Since we do not know the various F_2^i, it is impossible to take full account of the mass corrections. But we can easily estimate the minimal corrections to Bjorken scaling by putting all σ_i in (2.17) equal to their lowest value $(m_S - m_N)^2$. This leads us to

$$F_2(x, Q^2) = F_2(\eta)\ \frac{1}{1 - 2m_N(m_S - m_N)x/Q^2}\ , \tag{2.19}$$

where

$$\eta = \eta_{(m_S - m_N)^2} = x\ \frac{1 + (m_S - m_N)^2/Q^2}{1 - 2m_N(m_S - m_N)x/Q^2}\ . \tag{2.20}$$

The variable η has the desired threshold properties, i.e., $\eta \to 1$ for $x \to x_{max}$ (*cf.* (2.8)). This is to be contrasted with the many other scaling variables being around whose threshold value depends on Q^2 and only asymptotically reaches 1 (see also the discussion of ref. [17]).

3. Significance of ancient scaling variables and the structure of moments

Before we now estimate the mass corrections numerically, let us discuss earlier attempts of incorporating hadronic mass corrections, i.e., other scaling variables, and the structure of moments in this new light.

Each scaling variable can be cast into the form (2.13) where $\bar\sigma$ is some function of $\eta_{\bar\sigma}$ and the mass parameters. To give some concrete examples, the variable x corresponds to

$$\bar\sigma = -2m_N(m_S - m_N)\eta_{\bar\sigma}\ . \tag{3.1}$$

The Bloom-Gilman variable,

$$x' = \frac{x}{1 + M^2x/Q^2}\ , \tag{3.2}$$

is equivalent to setting

$$\bar\sigma = -[2m_N(m_S - m_N) + M^2]\eta_{\bar\sigma}\ , \tag{3.3}$$

and the variable ξ (eq. (2.2)) with $m_i = m_f = 0$ (the original Nachtmann variable [28]) is recovered by *

* It should be noted that the Nachtmann variable follows (more straightforwardly) from (2.5) by setting $\sigma \equiv 0$, i.e. $\xi = \alpha_{\sigma\equiv 0}$ while x corresponds to $\sigma = m_N^2\alpha_\sigma^2$.

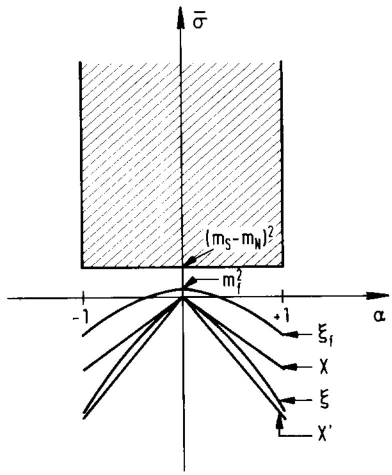

Fig. 4. Equivalent support of various scaling variables: x' as given by eq. (3.2) with $M^2 = 1$ GeV^2; ξ is the Nachtmann variable (eq. (1.1) with $m_i = m_f = 0$); $\xi_f = x(1 + (m_f^2 - m_N^2 x^2)/Q^2)$ as an approximation [20] to (1.1) with $m_i = m_f$ (here $m_f = 0.5\ (m_S - m_N)$).

$$\bar{\sigma} = -m_N^2 \eta_{\bar{\sigma}}^2 - 2m_N(m_S - m_N)\eta_{\bar{\sigma}} \,. \tag{3.4}$$

This means that the variables x, x', ξ, etc., correspond to a path in the $(\alpha, \bar{\sigma})$ plane which is to be interpreted as the support, or some approximation of it (following the discussion of the last paragraph), of the spectral function $\bar{G}_2$.

Clearly, this line of support should lie within the actual support of $\bar{G}_2$ as drawn in fig. 3. Any line of support falling outside this region would be in conflict with causality and stability of the target which are the essential ingredients going into the DGS representation and fig. 3. In fig. 4 we have drawn the (equivalent) support of various scaling variables being discussed in the literature together with the support of $\bar{G}_2$. It is seen that *all* scaling variables fall outside the support of $\bar{G}_2$ and, what is most surprising, that most of them are worse (in the sense described above) than the plain variable x ⋆. The variable ξ (eq. (1.1)) is for, e.g., $m_i = m_f = m_c$ ⋆⋆ somewhat closer to reality than x, but the improvement is very small as compared to the boundary of the support of $\bar{G}_2$ (i.e., the variable (2.20), *cf.* fig. 4) which is lying much higher up. The fact that the scaling variables being mentioned are in disagreement with the stability of the proton has also been pointed out by previous authors [17, 20,21].

The failure of especially the ξ variable underlines the importance of higher-twist operators and of non-perturbative elements as discussed in sect. 1. The finding that

⋆ *Cf.* eqs. (3.1), (3.3) and (3.4).
⋆⋆ Here m_c means the mass of the charmed quark.

also the phenomenologically motivated scaling variables like, e.g., the Bloom-Gilman variable, which fit the deviations from scaling in some limited x, Q^2 regime, have no theoretical basis signals a rather complex origin of scale braking at large x. We will come back to this point in sect. 5.

Let us now discuss the moments. We shall be interested in the Nachtmann moments [28]

$$\int \frac{dx}{x} \xi^n F_2(x, Q^2) = \mu_n \,, \tag{3.5}$$

where the quark masses are set equal to zero. For the conclusions we like to draw later on, the actual value of the quark masses is vastly irrelevant. We could have also taken the moments over x^n since ξ is, from the above point of view, not really favoured over x. If we insert (2.12) into (3.5) we obtain

$$\int_{(m_S-m_N)^2}^{\infty} d\bar{\sigma} \int_0^1 \frac{d\eta_{\bar{\sigma}}}{\eta_{\bar{\sigma}}} \eta_{\bar{\sigma}}^n \left\{ \frac{Q^2}{Q^2 + \bar{\sigma} + 2m_N(m_S - m_N)\eta_{\bar{\sigma}}} \right.$$
$$\left. \times \frac{2}{1 + \sqrt{1 + 4m_N^2 Q^2 \eta_{\bar{\sigma}}^2 / (Q^2 + \bar{\sigma} + 2m_N(m_S - m_N)\eta_{\bar{\sigma}})^2}} \right\}^n \bar{G}_2(\eta_{\bar{\sigma}}, \bar{\sigma}) = \mu_n \,. \tag{3.6}$$

For the "minimal" approximation (2.19) this reduces to

$$\int_0^1 \frac{d\eta}{\eta} \eta^n \left\{ \frac{Q^2}{Q^2 + (m_S - m_N)^2 + 2m_N(m_S - m_N)\eta} \right.$$
$$\left. \times \frac{2}{1 + \sqrt{1 + 4m_N^2 Q^2 \eta^2 / (Q^2 + (m_S - m_N)^2 + 2m_N(m_S - m_N)\eta)^2}} \right\}^n F_2(\eta) = \mu_n. \tag{3.7}$$

It is seen that $\mu_n(Q^2)$ has, except for $n = 0$, a zero at $Q^2 = 0$ and a cut * from $q^2 = -Q^2 = (m_S - m_N)^2$ to $+\infty$ (for (3.7) only up to $q^2 = m_S^2 - m_N^2$) while for large positive Q^2 it tends to a constant. This means that $\mu_n(Q^2)$ will be strongly Q^2 dependent in the region $|Q^2| = 0\,((m_S - m_N)^2)$. For the old physics $(m_S - m_N)^2 = m_\pi^2$ and $(m_S - m_N)^2 = (m_\Lambda + m_K - m_N)^2 \approx 0.45$ GeV^2, respectively which are small numbers. But for, e.g., charm production **, $(m_S - m_N)^2 \gtrsim 9$ GeV^2, which is well above the Q^2 of most of the existing data. Since charm production is substantial as we shall see, the Q^2 dependence of the moments (3.5) induced by the physical threshold cannot be neglected, especially, compared to the logarithmic dependences predicted by asymptotic freedom. After fig. 4 it is needless to say that this situation will

* This has also been found in ref. [29].
** Not necessarily from the sea.

not change very much if the quark masses are taken into account, i.e., if the full ξ (eq. (1.1); $m_i, m_f \neq 0$) is considered.

If the scaling variable (1.1) were to reproduce the finite mass corrections correctly, the right-hand side of eq. (3.5) should be *independent of* Q^2 (not mentioning asymptotic freedom corrections). But since ξ (eq. (1.1)) only reproduces the free-field approximation for the twist-2 Wilson coefficients, our result is not really surprising. As has been pointed out in sect. 1 and elsewhere [17], operators as high as twist-six are needed in order to accommodate the correct threshold behaviour.

The free field twist-2 approximation is particularly poor for production of heavy particles like charm. Since charm production causes strong scaling violations as we shall see, this source of finite mass corrections must be carefully considered before searching for logarithmic deviations from scaling.

Therefore, we suggest analyzing the data in terms of the moments

$$\int_0^1 \frac{d\eta}{\eta}\, \eta^n F_2(\eta) = \mu_n\,, \tag{3.8}$$

with asymptotic freedom corrections superimposed on the right-hand side of (3.8). This requires us, of course, to disentangle the various contributions to F_2 since, e.g., charm production from the sea needs a different η than production of strange particles.

The moments (3.8) reflect the minimal mass corrections needed on grounds of causality and stability of the target. We think that this should be the criterion for handling finite mass corrections. Again, we like to emphasize that this cannot be achieved by any kind of perturbation calculation (on the quark level).

4. Comparison with experiment

We shall now evaluate the minimal mass corrections as given by (2.19) for F_2^{ep}, σ_L/σ_T, σ^ν, $\sigma^{\bar\nu}$ and $\sigma^{\bar\nu}/\sigma^\nu$. The various structure functions will be phrased in terms of the quark-parton distribution functions as, e.g., listed in ref. [30]. There are six quark distributions (for us) for the proton: $V_u(x)$, $V_d(x)$, $S_u(x)$, $S_d(x)$, $S_s(x)$ and $S_c(x)$. In line with SU(4) we shall assume that the sea distributions are the same, i.e., $S_u(x) = S_d(x) = S_s(x) = S_c(x) = S(x)$.

4.1. F_2^{ep}

In the minimal mass correction approximation the structure function can then be written

$$F_2^{\mathrm{ep}}(x, Q^2) = \left(\tfrac{4}{9}\eta_v V_u(\eta_v) + \tfrac{1}{9}\eta_v V_d(\eta_v)\right) \frac{1}{1 - 2m_N m_\pi x/Q^2}$$

$$+ \tfrac{10}{9}\,\eta_s S(\eta_s)\,\frac{1}{1-4m_N m_\pi x/Q^2} + \tfrac{2}{9}\,\eta_{ss} S(\eta_{ss})\,\frac{1}{1-4m_N m_K x/Q^2}$$

$$+ \tfrac{8}{9}\,\eta_{sc} S(\eta_{sc})\,\frac{1}{1-2m_N m_\psi x/Q^2}\ , \tag{4.1}$$

where η_v, η_s, η_{ss} and η_{sc} are obtained from (2.20) by setting $m_S = m_N + m_\pi$, $m_N + 2m_\pi$, $m_N + 2m_K$ and $m_N + m_\psi$, respectively, corresponding to the lowest possible s-channel thresholds ⋆. Throughout this paper the valence-quark distribution will be parametrized according to ref. [13], while the sea contribution is parametrized in the form ⋆⋆

$$S(x) = 0.124(1-x)^7/x\ . \tag{4.2}$$

In fig. 5 we have drawn F_2^{ep} as a function of Q^2 for various x together with existing data. It is found that the scaling violations seen in the data are quantitatively accounted for already by the minimal mass corrections. As to the source of the scaling violations, at $\omega = 20$ and $Q^2 \lesssim 4\ \mathrm{GeV}^2$ they divide up into 60% u and d quarks and 40% s quarks whereas charm is not excited. Beyond $Q^2 \approx 4\ \mathrm{GeV}^2$ the scaling violations are mostly due to charm production. At $\omega = 60$ charm production becomes effective only at $Q^2 \gtrsim 2\ \mathrm{GeV}^2$ while for $\omega \geqslant 160$ the scaling violations due to u, d and s quarks are $\lesssim 20\%$. As was to be expected, charm production adds a great deal to the scaling violations. But there are finite mass corrections coming from other sources which are as important, depending on the ω, Q^2 range. For the ω, Q^2 regime considered there is no need for any further Q^2 dependence. Most of the authors attribute the Q^2 dependence entirely to the logarithmic corrections to scaling predicted by asymptotic freedom. In the light of our analysis this interpretation must, however, be rejected.

We cannot say very much for large x since here the upper boundary of the support of $\overline{G}_2$ (i.e., (2.14)) comes into play and, moreover, $\overline{G}_2$ cannot be assumed to be smooth. The reason is that this region is obscured by higher-twist effects (*cf.* ref. [17]), especially, quasi-elastic contributions which do not survive in the scaling limit and (eventually) correspond to $\overline{G}_2$ being singular ⋆⋆⋆ at $\alpha = 1$.

In ref. [33] the large-x scaling violations could be explained in terms of such (twist-6 and higher) contributions. This interpretation receives strong support from our analysis. Since, as we have seen, there is not much room for asymptotic freedom corrections at small x, they must also be small at large x. This means that the effect of asymptotic freedom corrections has certainly been overestimated.

We will continue the discussion on the large-x region in sect. 5.

⋆ On the level of planar quark diagrams.

⋆⋆ In agreement with dimensional counting [31].

⋆⋆⋆ Cornwall, Corrigan and Norton, ref. [23].

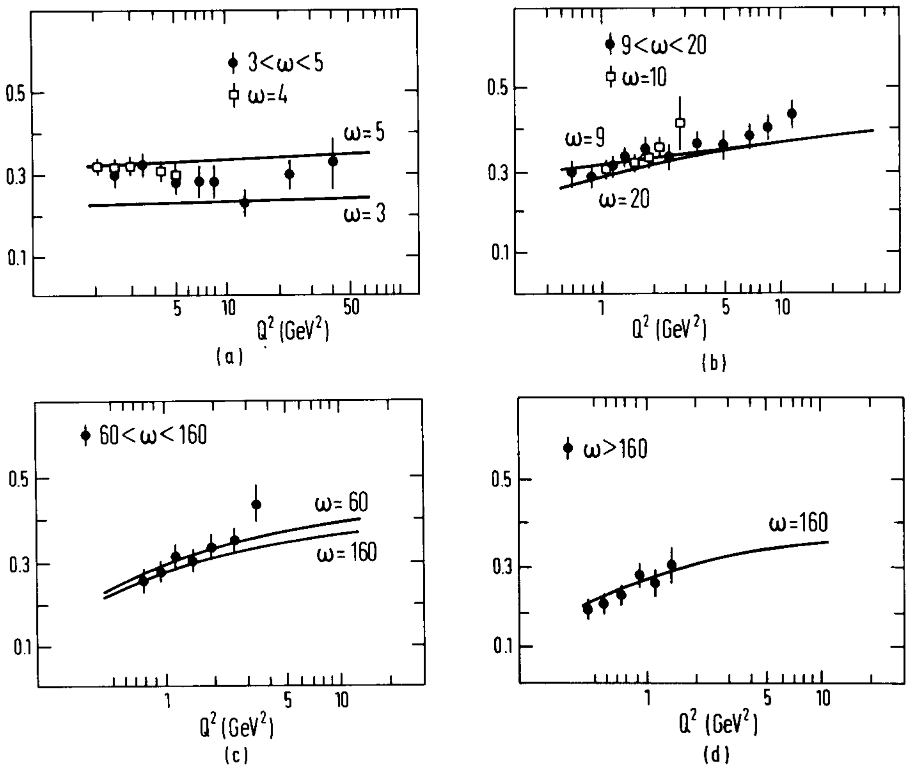

Fig. 5. The proton structure function $F_2(x, Q)$ as a function of Q^2 for various ω together with the minimal mass corrections predicted. The data are from refs. [1] (□) and [32] (•).

4.2. σ_L/σ_T

As we have the quark-parton model in the back of our mind, we will find for large Q^2

$$xF_1(x, Q^2) = F_2(x, Q^2) . \tag{4.3}$$

In our way of writing this extends down to smaller Q^2 in the form

$$\eta F_1(\eta) = F_2(\eta) , \tag{4.4}$$

which is obvious from (2.19) and the same expression for F_1 and (4.3). If there is only one threshold mass involved, this leads to *

* The θ function reflects the support properties of $F_{1,2}$.

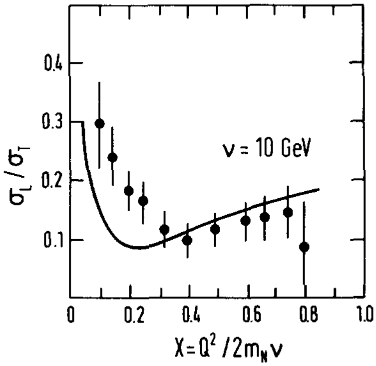

Fig. 6. The ratio σ_L/σ_T *versus* x. The data are from ref. [1]. Our prediction is shown for ν = 10 GeV. Other ν within the SLAC energy range give a similar picture.

$$\frac{\sigma_L}{\sigma_T}=\frac{2m_N}{\nu}\frac{F_2}{F_1}\left(1+\frac{\nu^2}{Q^2}\right)-1$$

$$=\frac{2m_N}{\nu}\eta\theta(\eta)\left(1+\frac{\nu^2}{Q^2}\right)-1\,. \tag{4.5}$$

As can easily be checked, (4.5) extends consistently down to $Q^2=0$ where it gives $\sigma_L/\sigma_T=0$.

In fig. 6 we have drawn our prediction for σ_L/σ_T together with existing data. Though the agreement is not perfect, the shape of the curve fits very well to the data. It should, however, be noted that σ_L/σ_T depends sensitively on the parametrization of the structure function * which we have not touched in this paper.

4.3. σ^ν, $\sigma^{\bar\nu}$ and $\sigma^{\bar\nu}/\sigma^\nu$

The neutrino and antineutrino structure functions are now straightforward. For example, we find

$$F_3^{\nu p}(x,Q^2)=\cos^2\theta_C V_d(\eta_v)\frac{1}{1-2m_N m_\pi x/Q^2}$$

$$+\sin^2\theta_C V_d(\eta_c)\frac{1}{1-2m_N m_D x/Q^2}\,,$$

where η_v and η_c are obtained from (2.20) by setting $m_S=m_N+m_\pi$ and $m_S=m_N+m_D$, respectively. The Cabibbo angle is taken to be $\cos^2\theta_C=0.94$. It is to be noted

* Note that in reality this does not drop out from (4.5).

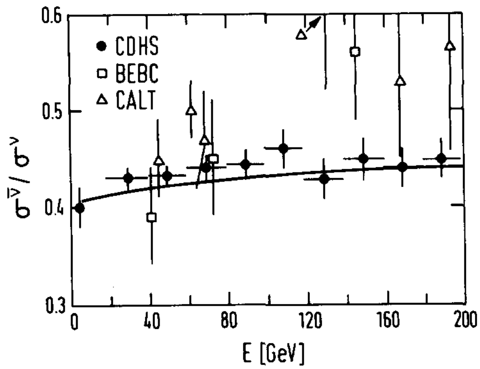

Fig. 7. The ratio $\sigma^{\bar{\nu}}/\sigma^{\nu}$. The data are from refs. [34–36].

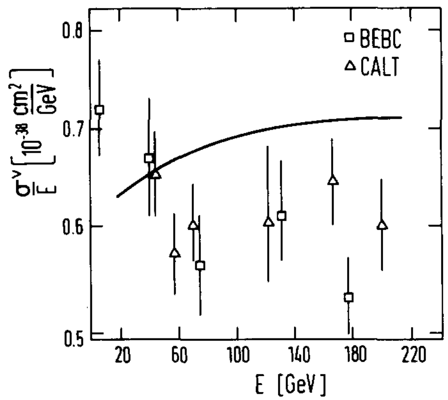

Fig. 8. The neutrino cross section σ^{ν}/E. The data are from refs. [34–36].

that the $\cos^2\theta_C$ and $\sin^2\theta_C$ factors do not simply add to one because of the different thresholds being involved. In figs. 7, 8 and 9 we have shown our predictions for

$$\sigma^{\bar{\nu}}/\sigma^{\nu}, \qquad \sigma^{\nu} = \tfrac{1}{2}(\sigma^{\nu p} + \sigma^{\nu n}), \qquad \sigma^{\bar{\nu}} = \tfrac{1}{2}(\sigma^{\bar{\nu}p} + \sigma^{\bar{\nu}n}),$$

respectively. Let us first look at the ratio $\sigma^{\bar{\nu}}/\sigma^{\nu}$ where we have the best experimental information. Our predictions are in very good agreement with the data and there is no need for any other corrections. The antineutrino cross section $\sigma^{\bar{\nu}}$ is also very well reproduced as is seen in fig. 9. The neutrino cross section is consistent with the data at lower energies while at higher energies it lies somewhat higher than the BEBC [35] and CALT [36] data points (fig. 8), but it should be said that there is some inconsistency in the data. The BEBC and CALT neutrino and/or antineutrino data are not consistent with $\sigma^{\bar{\nu}}/\sigma^{\nu}$ quoted by the CDHS [34] group. Furthermore, we like to

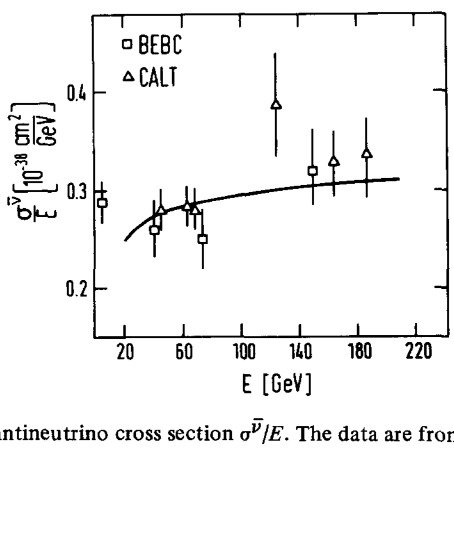

Fig. 9. The antineutrino cross section $\sigma^{\bar{\nu}}/E$. The data are from refs. [34–36].

remind the reader that the quark-parton model without any mass corrections gives $\sigma^{\nu}/E \approx 0.9 \cdot 10^{-38}\ \mathrm{cm}^2/\mathrm{GeV}$ at $E = 220$ GeV which lies 40% above the mass-corrected cross section (fig. 9). This is to say that the remaining discrepancy is small as compared to the free quark-parton model we started with, which is completely off. We believe that this discrepancy can be fitted by also incorporating large-x effects à la ref. [33] (see sect. 5); but the asymptotic freedom interpretation [14] is also welcome, only this effect must be much smaller than originally claimed.

5. Discussions

The fact that most of the scaling violations (at medium and smaller x) are already accounted for by the minimal mass corrections proves that $\overline{G}_2(\eta_{\overline{\sigma}}, \overline{\sigma})$ (*cf.* eq. (2.12)) is indeed dominated by $\overline{\sigma} \approx (m_{\mathrm{S}} - m_{\mathrm{N}})^2$ which means that scaling sets in as rapidly as possible. It should, however, be said that the picture might slightly change for still higher energy data. Here our calculations are also incomplete since we did not include the very recently discovered new hadronic degrees of freedom [37] Υ, Υ', etc.

We would like to stress that we have not attempted to fit the valence-quark distributions to the data but rely on the parametrization of ref. [13] which, together with our idea of scaling violation, is probably not the best fit. We have also done the calculations using Barger and Phillips' parametrization [30] of the valence-quark distribution. This gives a slightly poorer fit.

For larger x, the approximation (2.19) does not make much sense and, hence, should not be confronted with the data. To give an example, let us consider those type of diagrams where at least one vector meson (say a ρ) is produced. This comes close to reality as it is well-known that the multihadron final states are resonance mediated to a very large extent [38]. For e.g., $x = 0.7$ and $Q^2 = 5\ \mathrm{GeV}^2$ we then find $\overline{\sigma}_{\min} = 0.6\ \mathrm{GeV}^2$ and $\overline{\sigma}_{\max} = 0.7\ \mathrm{GeV}^2$ (*cf.* eqs. (2.12) and (2.14)) which is very close, and even for $Q^2 = 10\ \mathrm{GeV}^2$ we only get $\overline{\sigma}_{\max} = 2.85\ \mathrm{GeV}^2$. So, the x and Q^2 dependence *via* $\overline{\sigma}_{\max}$ can definitely not be neglected. In order to estimate the effect of this extra x, Q^2 dependence we need to know the spectral function $\overline{G}_2(\alpha, \overline{\sigma})$ explicitly for large α which is hopeless. But even if we know $\overline{G}_2(\alpha, \overline{\sigma})$ explicitly, we do not believe that the finite mass corrections near $x = 1$ can be cast into some *theoretical* scaling variable.

On top of this difficulty the large-x region suffers from higher-twist contributions as, e.g., discussed in ref. [33]. These contributions correspond to the spectral function *

$$\overline{G}_2(\alpha, \overline{\sigma}) \sim \int_0^1 \mathrm{d}\xi\, \frac{\xi^3(1-\xi)}{\xi - \alpha}\, \delta''\!\left(\frac{\xi\overline{\sigma} - \alpha\overline{\sigma}_{\min}}{\xi - \alpha} - d^2\right). \tag{5.1}$$

* Eqs. (2), (3) in ref. [33] with x replaced by η (eq. (2.20)).

which falls outside the class of spectral functions considered in our discussion. They are *a priori* restricted to large x and can at best be *parametrized* according to our ideas on the bound-state dynamics of the nucleon [31].

Ref. [33] fits the scaling violations at large x extremely well. By adding a piece like (5.1) to the spectral function in (2.12) it is obvious that we also can account for the large-x scaling violations.

As far as probing the field-theoretic structure is concerned, it is fair to say that the physics of the large-x region is too involved that it can be appropriately described by perturbation theory. So, if we want to trace out the kind of underlying field theory it must be done at small x. But here, we are afraid to say, there is no clear signal yet which points at scaling violations of the sort predicted by renormalizable field theory, much less can one distinguish between conventional renormalizable field theories and QCD.

It is well-known that the scaling violations at large x can be well-described (for SLAC energies) by several phenomenological scaling variables (like the Bloom-Gilman variable [18]). As we have seen, these variables fall outside the support of $\overline{G}_2$ so that they have really no theoretical basis. We believe that they reflect in some sense the effect of higher-twist operators/quasi-elastic contributions which drop out asymptotically. It is clear that any meaningful scaling variable must be a superposition of (2.13) (in $\overline{\sigma}$).

At energies far above SLAC energies the average σ_L/σ_T will be much smaller. For, e.g., $\nu = 150$ GeV we find $\sigma_L/\sigma_T \lesssim 0.1$ which has to be checked experimentally. We don't see any contradiction to the naive parton model [39] here. Only the naive parton model is not applicable for very small and large x since here the initial parton cannot be on (near) its mass-shell [20,21,40]. So the formula *

$$\sigma_L/\sigma_T = 4(k_\perp^2 + m_i^2)/Q^2 \tag{5.2}$$

should only be taken seriously near the quasi-elastic peak where it is well-consistent with $k_\perp \approx 500$ MeV (fig. 6).

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