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Evading the axion without massless quarks

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We point out a possible mechanism which avoids the presence of axions without having a massless quark. Some of the quarks must obtain their masses through the dynamical spontaneous breakdown of a chiral symmetry.

Recently it was pointed out by Peccei and Quinn[1] that the possible large CP violation due to the instanton[2] effects can be avoided by imposing a U(1) symmetry on strong, electromagnetic, and weak interactions (including the Higgs interactions). If this U(1) is to be violated spontaneously then we expect the existence of a Nambu-Goldstone[3] boson or a pseudo-Nambu-Goldstone[4] boson since the chiral symmetry is violated by the instanton anyway. The existence of such a particle was pointed out by Weinberg[5] and by Wilczek.[6] In their recent articles they call it the "axion." An alternative without an axion was suggested in which they must have at least one massless quark.

In this note we want to point out the existence of a natural model with chiral symmetry, and also a way to avoid the existence of axions without massless quarks. The model was already discussed by us.[7] The model Lagrangian is invariant under the group S_3 in addition to the ordinary $SU(2)\times U(1)$ of Weinberg and Salam.[8] The origin of the S_3 symmetry was not stated in the paper. We now have the following reasoning. If we want to violate CP through the vacuum expectation values of Higgs bosons which we take to be all SU(2) doublets we need at least three such doublets.

Nature does not seem to have any means to distinguish between these three doublets, which means that the Lagrangian is invariant under the arbitrary transformations among the three Higgs doublets. To avoid the appearance of Nambu-Goldstone bosons we restrict ourselves to discrete symmetries. Then follows the S_3 symmetry automatically. Note that this symmetry is defined through the Higgs bosons, not through quarks. In the case of a four-quark model the left-handed doublets

$$\left\{\begin{pmatrix}\mathcal{P}_1\\ \mathcal{N}_1\end{pmatrix}_L,\begin{pmatrix}\mathcal{P}_2\\ \mathcal{N}_2\end{pmatrix}_L\right\}$$

constitute an S_3 doublet and $\{\mathcal{N}_{1R},\mathcal{N}_{2R}\}$ also transforms like a doublet. But to obtain a result which is consistent with the observation we are forced to assign $\mathcal{P}_{1R}$ and $\mathcal{P}_{2R}$ to singlets separately. This results in a massless up quark providing the freedom $u_R\to e^{i\alpha}u_R$ necessary to avoid the large CP violation due to the instanton. This U(1) symmetry is not violated by the Higgs bosons. Thus the up-quark seems to stay massless which is not consistent with the current-algebra calculations.[9] Our aim here is to show that this may not be the case, due to the dynamical spontaneous breakdown[10] which presumably avoids the appearance of a Nambu-Goldstone boson[11] (i.e., the would-be axion). To be able to compare our results with the closely related results of massless QED[12] we work in the lepton sector. Here

$$\left\{\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L,\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L\right\}$$

is an S_3 doublet and e_R and μ_R are separately singlets in S_3. We have $m_e=0$ just as we have $m_u=0$ in the quark sector. The customary way to study the dynamical spontaneous breakdown is to write down the Dyson-Schwinger (DS) equation for the electron propagator and to look for a symmetry-breaking solution to the equations. If we do this, only the diagrams with electron propagators will be important. The reason is that to cancel a small coupling constant in front of the momentum integral in the DS equation, we need a very slow damping (when $p^2\to\infty$) propagator.[12] We expect only the electron propagator can have this property since this is the only one which corresponds to the dynamical breaking. To clarify the approximation involved we use a different approach, instead of directly referring to the DS equation.

Neglecting all the contributions other than the photon and the Z-boson exchange, the effective Lagrangian can be written as

$$\mathcal{L}_{\text{eff}} = \bar{e}(i\gamma^{\mu}\partial_{\mu})e + \tfrac{1}{2}e_0^2 \int T(J_{\mu}(x)D_{\gamma}^{\mu\nu}(x,y)J_{\nu}(y))\,d^4x\,d^4y + \frac{G_0^2}{8}\int T(J_{\mu}^5(x)D_Z^{\mu\nu}(x,y)J_{\nu}^5(y))\,d^4x\,d^4y\,, \tag{1}$$

where the path integrals over the photon field and the Z-boson field are already evaluated, neglecting non-Abelian terms. Here

$$J_{\mu} = \bar{e}\gamma_{\mu}e \text{ and } J_{\mu}^5 = \bar{e}\gamma_{\mu}\gamma_5 e \tag{2}$$

and D_{γ} and D_Z are the photon and the Z-boson propagators in a certain gauge. We assumed $\sin^2\theta_W = \frac{1}{4}$ to simplify the Z-boson coupling. For this value of the Weinberg angle the electron–Z-boson coupling is pure axial vector and there is no parity-violating effect in the electron propagator. After Fiertz-transforming the four-fermion part we set

$$\rho(x-y) = \langle 0|T(\bar{\psi}_e(x)\psi_e(y))|0\rangle$$

to be the order parameter and use the mean-field approximation or the Hartree-Fock-Bogoliubov approximation.[13] Let us work in the Feynman gauge and set $D_{(\gamma,Z)}^{\mu\nu} = g^{\mu\nu}D_{(\gamma,Z)}$. We then define

$$A(K^2) = \int \left[e_0^2 D_{\gamma}(x) + \frac{G_0^2}{2}D_Z(x)\right] \times \rho(-x)e^{iKx}d^4x\,. \tag{3}$$

The gap equation in terms of $A(K^2)$ takes the following form:

$$A(K^2) = -4i\rho_0^2\int \frac{d^4K'}{(2\pi)^4}\frac{1}{(K'-K)^2}\frac{A(K'^2)}{K'^2 - A^2(K'^2)} - iG_0^2\int\frac{d^4K'}{(2\pi)^4}\frac{1}{(K'-K)^2 - m_Z^2}\frac{A(K'^2)}{K'^2 - A^2(K'^2)}\,. \tag{4}$$

The mass of the electron is defined by the solution of

$$m_e + A(+m_e^2) = 0\,. \tag{5}$$

Equation (4) is exactly the same as the one we obtain from the DS equation using the rainbow-graph approximation as was done by Baker, Johnson, and Willey[12] in the case of massless QED. The point here is that we did not have to rely on the Gell-Mann–Low condition[14] to justify the approximation. We studied Eq. (4) in detail and found the following: (1) The equation certainly gives a symmetry-breaking solution with $m_e \neq 0$. (2) Although Eq. (4) contains the scale parameter m_Z unlike in the case of massless QED, we still have a set of solutions with one free parameter, and the value of m_e in terms of m_Z cannot be determined. This situation was already pointed out by Maskawa and Nakajima[15] some time ago. Whether this is due to the approximation we took or of a more general nature we do not know.

A natural continuation of the discussion given above is to look for a finite unified theory as an extension of finite QED.[12] We expect that some of the masses are of a Higgs origin and some are of a dynamical origin. We are of course far from achieving this goal.

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