HADRONIC DECAY OF τ

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We present predictions for invariant mass distributions of the pions in $\tau \to \nu + 2\pi$ and $\tau \to \nu + 4\pi$ as well as improved predictions for hadronic decay rates of τ. We discuss a number of predictions which, if contradicted by τ decays, will imply the existence of a new weak interaction.

There is now substantial experimental evidence for the existence of a new charged heavy lepton τ; some information on detailed properties of the heavy lepton is becoming available [1]. In addition to purely leptonic decay modes

$$\tau \to \nu_\tau e \nu_e, \quad \nu_\tau \mu \nu_\mu , \tag{1}$$

hadronic decay modes

$$\tau \to \nu_\tau + \text{hadrons} , \tag{2}$$

have now become targets of experimental scrutiny. The latter decays are particularly interesting because they reveal different hadronic components of the weak current which couples to τ. At present, observed hadronic decays are being compared with a naive theory. The decay of τ is described by a hamiltonian

$$\mathcal{H} = (G_F/\sqrt{2}) J_\nu J^\nu ,$$
$$J_\nu = \bar{\nu}_e \gamma_\nu (1-\gamma_5) e + \bar{\nu}_\mu \gamma_\nu (1-\gamma_5)$$
$$+ \bar{\nu}_\tau \gamma_\nu (r_+(1+\gamma_5) + r_-(1-\gamma_5))\tau + J_\nu^{\text{hadron}} , \tag{3}$$

where J_ν^{hadron} is the *conventional* hadronic current which consists of vector and axial vector components. The τ neutrino is taken to be massless. There are a number of rigorous predictions of this theory which, if contradicted by experiments, imply the existence of a new weak interaction.

Theoretical predictions for hadronic decay modes of a heavy lepton using the above Hamiltonian have been given by Thacker and Sakurai, Tsai, and Bjorken and Llwellyn Smith [2], with much fore sight long before the discovery of τ. In the meantime, much data on

$$e^+e^- \to \gamma \to n\pi , \quad n = 2, 4, 6 , \tag{4}$$

for center of mass energies up to 2 GeV has become available [3,4]; information necessary to estimate the relative decay rate

$$\gamma(n\pi) \equiv \frac{\Gamma(\tau \to \nu_\tau + n\pi)}{\Gamma(\tau \to \nu_\tau e \nu_e)} . \tag{5}$$

The new data for process (4) shows that $R = \sigma(e^+e^- \to \text{had})/\sigma(e^+e^- \to \mu^+\mu^-)$ can be as large as 4; this value is considerably larger than 2, the value assumed in previous calculations [2,5].

Recognizing the importance of understanding the hadronic component of the τ-current [5] and seeing the possibility of improving the existing calculations, we have thus reexamined the hadronic decays of τ. In this note we shall present our analysis with particular emphasis on detecting any possible deviation between the theory and experiments.

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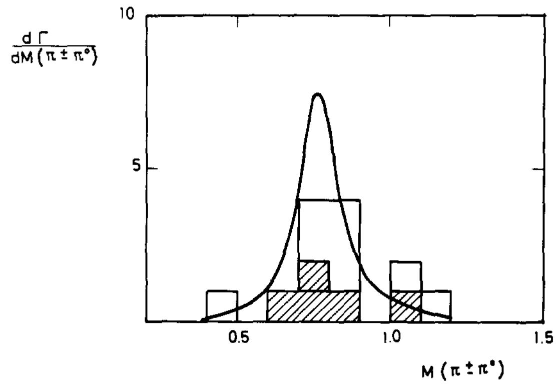

Fig. 1. A prediction for invariant mass distribution of two pions in a decay $\tau^{\pm} \to \nu\pi^{\pm}\pi^{0}$. The Gounalis–Sakurai formula for $F_{\pi}(Q^2)$ with Γ_{ρ} = 152 MeV and m_{ρ} = 773 MeV was used. The predicted distribution is normalized to the experimental data which are given in ref. [11].

In discussing the decay of τ into a neutrino and pions, it is natural to distinguish between decays with an even and an odd number of pions. Only the $I = 1$ component of J_{ν}^{had} can contribute to τ decay (since the current must be charged). This implies a relation

$$\text{charge conjugation} = -G \text{ parity} . \qquad (6)$$

The vector (axial vector) component of J_{ν}^{had}, therefore, must be responsible for the even (odd)-pion decays.

For even-pion decays, and using the conserved-vector-current property of J_{ν}^{had}, the matrix element $\langle 0|J_{\nu}^{\text{had}}|2n\pi\rangle$ is related to that of the electromagnetic current $\langle 0|J_{\nu}^{\text{e.m.}}|2n\pi\rangle$. This leads to a prediction for relative decay rates [2,5]

$$\gamma(2n\pi) = \frac{2}{M^8}\cos^2\theta_c \int d\theta^2 (M^2 - Q^2)^2$$

$$\times (M^2 + 2Q^2) R(2n\pi) , \quad n = 1, 2, \ldots , \qquad (7)$$

where M is the mass of τ,

$$R(2n\pi) = \sigma(e^+e^- \to 2n\pi)/(4\pi\alpha^2/3Q^2) ,$$

and $\sqrt{Q^2}$ is the invariant mass of $2n$ pions. In this paper, we have used the mass value of M = 1.81 GeV which is the latest result of the DASP group [6]. Note that r is independent of r_+ and r_-, the nature of the $\tau\nu_{\tau}$ current.

An analysis of odd-pion decays requires more care. The decay $\tau \to \nu + \pi$ can be calculated since $\langle 0|J_{\nu}^{\text{had}}|\pi\rangle$ is known from the $\pi^{\pm}$ decay rate. The decay $\tau \to \nu + 3\pi$ can be estimated if it is dominated by $\tau \to \nu + A_1$, since the matrix element $\langle 0|J_{\nu}^{\text{had}}|A_1\rangle$ can be estimated from Weinberg's sum rules [7]. For $\tau \to \nu + 5\pi$, and $\nu + 7\pi$, we assume

$$\Gamma(\tau \to \nu + 5\pi) \simeq \Gamma(\tau \to \nu + 4\pi) ,$$
$$\Gamma(\tau \to \nu + 7\pi) \simeq \Gamma(\tau \to \nu + 6\pi) , \qquad (8)$$

based on asymptotic chiral symmetry arguments.

We first discuss even-pion decays.

(a) $\underline{\tau \to \nu\pi\pi}$. This decay mode can be predicted from the pion electromagnetic form factor in the time like region. Noting that

$$R(2\pi) = \sigma(e^+e^- \to 2\pi) \Big/ \left(\frac{4\pi\alpha^2}{3Q^2}\right)$$

$$= \frac{1}{4}\left(1 - \frac{4m_{\pi}^2}{Q^2}\right) |F_{\pi}(\theta^2)|^2 , \qquad (9)$$

eq. (7) becomes

$$\frac{d\gamma(2\pi)}{dx} = \tfrac{1}{2}(1-x)^2(1+2x)\left(1 - \frac{4m_{\pi}^2}{M^2 x}\right)^{3/2}$$

$$\times |F_{\pi}(M^2 x)|^2 \cos^2\theta_c , \qquad (10)$$

where $x = Q^2/M^2$. The experimental data for $F_{\pi}(Q^2)$ is in excellent agreement with the Gounalis and Sakurai formula [8,9]. Near the ρ resonance where it gives the dominant contribution to r. Using the latest values of the parameters [10], m_{ρ} = 773 ± 3 MeV, Γ_{ρ} = 152 ± 3 MeV, we show, in fig. 1, our prediction for the invariant mass distribution of the two pions, including recent DASP data [11]. Integrating this distribution, we obtain the relative decay rate $r(2\pi)$ shown in table 1. We have used a measured pion form factor in our prediction. This enabled us to avoid an intrinsic ambiguity associated with a narrow width approximation; an approximation used in all previous estimates of the relative branching ratio [2,5]. The ambiguity can be seen quantitatively from the following: Γ_{ρ} = 152 ± 3 MeV leads to the $\rho\pi\pi$ coupling constant $f_{\rho\pi\pi}^2/4\pi$ = 2.9±0.1. $\Gamma(\rho \to e^+e^-)$ = (4.0 ± 0.5)$10^{-5}\Gamma_{\rho}$ leads [9] to the γ–ρ coupling constant $f_{\rho}^2/4\pi$ = 2.25 ±0.3. Two coupling constants must be equal if the narrow width approximation were exact. Clearly, 30% ambiguity present in this approximation can not be tolerated for a quantitative experimental check of the theory.

Table 1
Summary of results for the relative decay rate $r(x) = \Gamma(\tau \to \nu x)/\Gamma(\tau \to \nu e \nu)$. For comparison purposes, we have also listed experimental numbers which were summarized in ref. [1]. Results from the DASP and PLUTO groups can also be found in refs. [11] and [16] respectively.

Decays	r $M = 1.81$ GeV	Branching ratio	Experiments	Comments
$\tau \to \nu e \nu_e$	1	0.18	$B_e = 0.17 \pm 0.03$ (DASP) $B_e = 0.16 \pm 0.03$ (PLUTO) $B_e = 0.186 \pm 0.038$ (SLAC-LBL) $B_e = 0.224 \pm 0.076$ (LEAD-GL)	 assume $B_e = B_\mu$ assume $B_e = B_\mu$
$\tau \to \nu \mu \nu_\mu$	0.973	0.18	$r_\mu = 0.92 \pm 0.03$ (DASP) $B_\mu = 0.14 \pm 0.034$ (PLUTO)	
$\tau \to \nu \pi$	0.595	0.1	$B_\pi = 0.022 \pm 0.028$ (DASP)	used $B_e = 0.18$
$\tau \to \nu 2\pi$	1.24	0.22	$B_{2\pi} = 0.24 \pm 0.09$ (DASP) $r(2\pi) = 1.41 \pm 0.6$ (PLUTO)	
$\tau \to \nu \mathrm{A1}$	0.41 0.54	~0.1	$B_{\mathrm{A}_1} = 0.11 \pm 0.04$ (PLUTO)	
$\tau \to \nu 4\pi$ $\tau \to \nu 5\pi$ $\tau \to \nu 6\pi$ $\tau \to \nu 7\pi$	0.44 ± 0.10 ~0.44 ~0.11 ~0.11	0.20	$B(\geqslant 3ch + n\gamma)$ $= 0.35 \pm 0.11$ (DASP)	$\frac{\Gamma(\tau^+ \to \nu \pi^+ 3\pi^0)}{\Gamma(\tau^+ \to \nu \pi^0 \pi^- 2\pi^+)} \approx \frac{1}{4}$
$\tau \to \nu \mathrm{K}$	0.03	<0.01		
$\tau \to \nu \mathrm{K}^*$	0.05	0.01		
$\tau \to \nu Q_1$	0.02	<0.01		
$\tau \to \nu \mathrm{K} n\pi$ $(n \geqslant 3)$	0.07	0.01		

(b) $\tau \to \nu + 4\pi$. The four pion cross sections $\sigma(e^+e^- \to 2\pi^+2\pi^-)$ and $\sigma(e^+e^- \to \pi^+\pi^-2\pi^0)$ have been measured by the DCI and the Novosibirsk groups [3,4]. These have large contributions to e^+e^- total cross section in the energy region $1\,\mathrm{GeV} < \sqrt{s} < 2$ GeV. Inserting

$$R(4\pi) = [\sigma(e^+e^- \to 2\pi^+2\pi^-) + \sigma(e^+e^- \to \pi^+\pi^-2\pi^0)] \Big/ \frac{4\pi\alpha^2}{3Q^2}\,, \tag{11}$$

in (7), we obtain the four pion invariant mass distribution shown in fig. 2. Integrating this distribution, we obtain the relative decay rate shown in table 1.

There are two different classes for the $I = 1$ four pion system [12]. If the reduced matrix elements for these two classes are approximately equal, we have

$$\Gamma(\tau^+ \to \nu + \pi^+ + 3\pi^0) : \Gamma(\tau^+ \to \nu\pi^-2\pi^+\pi^0) \approx 1:4\,. \tag{12}$$

An examination of $\sigma(e^+e^- \to 2\pi^+2\pi^-)$ and $\sigma(e^+e^- \to \pi^+\pi^-2\pi^0)$ data indicate approximate equality of the two reduced matrix element for $\sqrt{s} \lesssim 1.6$ GeV. The dominant contribution to the integral in (7) comes from $\sqrt{s} \leqslant 1.6$ GeV; we thus expect (12) to hold reasonably well.

(c) $\tau \to \nu + 6\pi$. There is very little $e^+e^- \to 6\pi$ data. In the following, we estimate the relative decay rate, finding a small value. There are four classes of $I = 1$ six pion system [12]. Since all four reduced matrix elements can not be obtained from the data, we assume their equality. This approximation leads to a relation:

$$\sigma(e^+e^- \to 3\pi^+3\pi^-) : \sigma(e^+e^- \to 2\pi^+2\pi^-2\pi^0) : \sigma(e^+e^- \to \pi^+\pi_4\pi^0) = 45 : 162 : 45\,. \tag{13}$$

We further approximate the "effective cross section" by [13]

$$\sigma(e^+e^- \to 3\pi^+3\pi^-) = 2.2 \text{ nb for } \sqrt{s} \geqslant 1.2 \text{ GeV}\,. \tag{14}$$

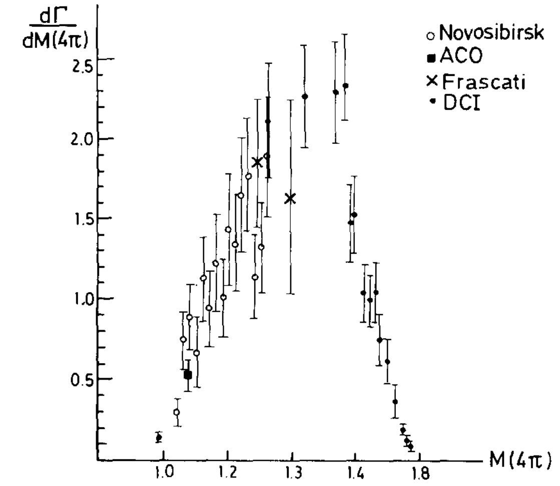

Fig. 2. A prediction for normalized invariant mass distribution of four pions in a decay $\tau \to \nu + 4\pi$. The uncertainty in the prediction shown by the band is due to the error associated with measurements of $\sigma(e^+e^- \to 2\pi^+2\pi^-)$ and $\sigma(e^+e^- \to \pi^+\pi^-2\pi^0)$.

We then obtain the result given in the table. Since the relative decay rate is small, our approximation will not significantly affect total branching ratios.

We now treat the odd pion decays.

(d) $\tau \to \nu + \pi$. It is well known that [2]

$$r(\pi) = 12\pi^2 \left(\frac{m_\pi}{M}\right)^2 C_A^2 , \tag{15}$$

where we have ignored m_π, m_{ν_τ}, m_e relative to M, C_A = pion decay constant $x \cos\theta_c = 0.94 \times 0.974 = 0.92$. Since this is a very solid prediction, we have very little to add. We only stress that this result is valid for any linear combination of V+A and V−A leptonic τ current [14]. If this result is contradicted by experiments, the axial current component of J_ν^{had} is not the conventional one.

(e) $\tau \to \nu + 3\pi$. In order to estimate this decay, the matrix element $\langle 0|J_\nu^{\text{had}}|3\pi\rangle$ is necessary. If the three pion state is dominated by A_1, the matrix element is

$$\langle 0|J_\nu^{\text{had}}|A_1^0\rangle = \frac{m_A^2}{f_A} \cos\theta_c \epsilon_\nu , \tag{16}$$

where m_A is the mass of A_1, θ_c is the Cabibbo angle, ϵ_ν is the polarization vector of A_1 and f_A is the A_1-axial vector current coupling constant, which can be estimated by saturating Weinberg's first and second sum rules with vector and axial vector mesons. The following relations are obtained [15]

$$\left(\sqrt{2}\frac{m_A}{f_A}\cos\theta_c\right)^2 + (C_A m_\pi)^2 = \left(\frac{\sqrt{2}m_\rho}{f_\rho}\cos\theta_c\right)^2 \quad \text{which gives } \frac{f_A^2}{4\pi} = 8.4 ,$$

$$\frac{m_\rho^2}{f_\rho} = \frac{m_A^2}{f_A} \quad \text{which gives } \frac{f_A^2}{4\pi} = 11.0 , \tag{17}$$

(we have used $f_\rho^2/4\pi = 2.25 m_A = 1.15$ GeV) from the first and second sum rules respectively. Then we find [2,5]

$$r(A_1) = 6\pi x_{A_1}(1-x_A)^2(1-2x_A)\frac{4\pi}{f_A^2}\cos^2\theta_c , \tag{18}$$

where $x_A = (m_A/M)^2$, we obtain two results stated in table 1 for two values of $f_{A_1}^2/4\pi$ given in (17). The difference between these two results give a minimum ambiguity in our prediction.

(f) $\tau \to \nu + K, \nu + K^*, \nu + Q_1$. We have nothing new to add these decays. For completeness and for computing branching ratios, we have included results of previous calculations [2,5] for these decays in table 1.

(g) $\tau \to \nu + K + n\pi\ (n \geqslant 3), \nu + K + \bar{K} + n\pi$. In estimating $\tau \to \nu + K + n\pi$, we have multiplied the Cabibbo suppression factor by the rate for $\tau \to \nu + n\pi\ (n \geqslant 4)$.

The decay $\tau \to \nu + K + \bar{K} + n\pi$ requires that a massive $K\bar{K}$ pair be created out of the vacuum. This is much suppressed by the small phase space in τ decay, and we neglect it.

We have increased the reliability of estimates for two relative decay rates $\tau \to \nu 2\pi$ and $\tau \to \nu 4\pi$. These together with $\tau \to \nu\pi$ and $\tau \to \nu\mu\nu$ are boxed in table 1 to stress a point that any disagreement between experiments and these predictions is fatal for the theory. As it can be seen in table 1, experiments are in good agreement with predictions except those for $\tau \to \nu\pi$. The importance of further confirmation of this disagreement in $\tau \to \nu\pi$ decay can not be over emphasized. If this is confirmed, the axial component of the τ current is not the conventional one. Then further studies of $\tau \to \nu 2\pi$ and $\tau \to \nu 4\pi$ are badly needed to check the vector component of the τ current. Since the "theoretically well known decays", $\tau \to \nu e \nu, \nu\mu\nu, \nu\pi, \nu 2\pi, \nu 4\pi$ are estimated to constitute ~ 75% of the τ decay, we are hopeful that detailed properties of the τ current should be available in the near future.

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Note added: After this work was completed, we have received a paper by F.J. Gilman and D.H. Miller, SLAC-PUB-2046 (1977), which deals with the same subject. Their results are in agreement with ours. We thank Dr. F.J. Gilman for discussions and constructive critisism of our work.

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