

COLOUR SCREENING AND QUARK CONFINEMENT

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It is proposed that in quantum chromodynamics the colour charge of gluons and of anything with zero triality is screened by a dynamical Higgs mechanism with Higgs scalars made out of gluons, but the center Z_3 of the gauge group $SU(3)$ is left unbroken, and single quarks, which have nonzero triality, are not screened. Long range forces between them persist therefore. Given that the Higgs mechanism produces a mass gap, the most favorable configuration of field lines between, e.g., quark and antiquark will be in strings analogous to magnetic field lines in a superconductor. The strings confine the quarks. The screening mechanism, on the other hand, produces not only the mass gap (which leads to string formation) but is also responsible for saturation of forces, i.e. absence of bound states of six quarks, etc.

It seems attractive to believe [1] that strong interactions are described by quantum chromodynamics (QCD), a nonabelian gauge theory of quarks and gluons with gauge group $G = SU(3)$, called the colour group. It extends to a gauge group of the 2nd kind due to minimal coupling of an octet of gluons to the quarks. There is an unspecified number of quark fields ψ^i , $i = 1, 2, \dots$, all of them are colour triplets. The lagrangian density,

$$\begin{aligned} \mathcal{L} &= -(1/2g^2) \text{Tr} F_{\mu\nu} F_{\mu\nu} + \sum_i \bar{\psi}^i (iD_\mu \gamma_\mu - m_i) \psi^i, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu], \\ D_\mu &= \partial_\mu - iA_\mu, \quad A_\mu = \frac{1}{2} \sum_{c=1}^8 \lambda^c A_\mu^c. \end{aligned} \quad (1)$$

λ^c are Gell-Mann's 3×3 matrices.

Quantum chromodynamics is supposed to form the quantum field theoretic basis of the quark model. The quark model is successful in the classification of hadrons if two basic assumptions are made.

(1) *Quark confinement*: All physical states have zero colour triality. I.e., a hadron may contain 3 quarks or a quark and an antiquark, but not one quark or two quarks, etc.

(2) *Saturation of forces*: Elementary particles are

made of no more than three constituent quarks or a quark and an antiquark.

Colour triality refers to the transformation law under the center Z_3 of the colour group $SU(3)$. A quark has triality 1, an antiquark -1 , and gluons have 0. Triality is added modulo 3. The elements γ of the center Z_3 are

$$\gamma = e^{2\pi i n/3} \quad 1 = \exp 2\pi i n \lambda_8 / \sqrt{3} \quad (n = 0, 1, 2).$$

A main problem is then to derive validity of these two assumptions from QCD – or rather (since the lagrangian (1) is only a formal expression to begin with) to establish existence of a quantum field theory which deserves to be called QCD and for which assumptions (1) and (2) are true.

We feel that before one can hope to obtain any rigorous results in this direction it is necessary to gain an intuitive understanding of the physical mechanisms which are responsible for quark confinement and saturation of forces. The present paper is an attempt to do so.

The mechanism which we shall outline has to the best of our knowledge not been proposed in this form before. But it is in the spirit of a number of theoretical ideas that have been developed during the past years, notably in studies of the strong coupling limit of lattice gauge theories [2,3], strings and bag models [4,5] and

“the world as a superconductor” [7]. It explains quark confinement and saturation of forces as the combined effect of mechanisms that can be seen at work in nature elsewhere, in particular in superconductivity. We hope that this will make it appear less miraculous.

We propose that a dynamical Higgs mechanism [6] takes place in QCD with Higgs scalars composed of gluons.

The Higgs mechanism is often presented as a kind of spontaneous symmetry breaking. However, local gauge invariance is not and cannot be spontaneously broken. The Higgs mechanism is best understood as a screening mechanism. This has repeatedly been pointed out in the literature [8–10]. The simplest example is a complete Higgs mechanism in which all colour charges are completely screened locally (cf. later) by Higgs scalars, and all physical states have therefore zero charge. We speak of a complete Higgs mechanism only when there is no nontrivial subgroup of G which remains unbroken, not even a discrete one.

In QCD the gluons, and therefore also Higgs scalars composed of them, transform trivially under Z_3 , therefore triality charge cannot be screened (at least not locally)^{‡1}. We propose that a dynamical Higgs mechanism takes place in QCD in which only the center Z_3 of G remains unbroken. Since Z_3 is a discrete subgroup and not a Lie group, there will be no surviving massless vector mesons associated with it. The Higgs mechanism will therefore produce a mass gap. This is its first achievement.

Let us show that a dynamical Higgs mechanism with only Z_3 unbroken is possible in principle. We need a possibly reducible multiplet of (composite) scalar fields $\phi(x)$ such that

$$V\phi = \phi, V \in G \quad \text{implies} \quad V \in H \subset G, H \simeq Z_3, \quad (2)$$

for generic ϕ (in particular $\phi \neq 0$). Consider the trace-

^{‡1} We are not prepared to discuss here whether a phase of QCD with broken Z_3 could also exist. In a pure Z_3 Yang–Mills theory on a lattice, without any charged fields, such a phase exists in 3 and 4 dimensions [15]. Our theory is (continuum limit of) a Z_3 gauge theory in which the coupling constant at each plaquette is itself a fluctuating variable. We do not know whether these fluctuations can suppress the possibility of Z_3 breaking altogether. Instanton effects, etc., would have to enter here.

less symmetric 8×8 matrix of fields

$$\phi^{ab}(x) = F_{\mu\nu}^a(x) F_{\mu\nu}^b(x) - \frac{1}{8} \delta^{ab} F_{\mu\nu}^c(x) F_{\mu\nu}^c(x) \quad (3)$$

($a, b = 1 \dots 8$).

It decomposes into an octet and a 27-plet of hermitean fields. The octet part can also be written as a traceless hermitean 3×3 matrix which can be diagonalized, with eigenvalues arranged in increasing order, by a gauge transformation $V \in SU(3)$. In the generic case this leaves a stability group $U(1) \times U(1)$ (diagonal matrices in $SU(3)$). This remaining gauge freedom is removed with the help of the 27-plet, except for the center. Thus (2) is fulfilled. We remark that the same construction would fail for $SU(2)/Z_2$; in this case there would remain a stability group of four elements (rotations by π around principal axes of ϕ). It also fails in two space–time dimensions because there is then only one independent component of $F_{\mu\nu}$; its stability group is always nontrivial.

Because of the unbroken Z_3 subgroup, complete screening of all colour charges will not take place in QCD. The Higgs scalars are colour octets or 27-plets, they will therefore screen octets and decuplets and, more generally, anything with colour triality zero. Single quarks or antiquarks however are triplets and their colour charge will therefore not be screened by the Higgs scalars. In other words, long range colour electric forces between the quarks will persist; there will be electric flux from one quark to another antiquark or pair of quarks, etc.

This idea can be made precise on a lattice. Given any gauge group of the 2nd kind with nontrivial center, there is a Gauss theorem for that center; it involves only observable, i.e. gauge invariant quantities. If the group is abelian, its center is the whole group^{‡2} and we have the well known version of Gauss’ theorem which says that the charge in a bounded domain Λ of space is equal to the electric flux through its boundary $\partial\Lambda$. An analog exists also for a discrete center. In particular the number of quarks in a bounded domain Λ in space can be determined by measuring (a certain

^{‡2} The center of a nonabelian gauge group is the analog of the whole group in the abelian case in several respects, cf ref. [18]. E.g., pure Yang–Mills theories admit no local gauge if the center is not trivial (cf. later).

gauge invariant function of) the colour electric flux through $\partial\Lambda$. This result is already implicit in the work of Kogut and Susskind [3] on hamiltonian formulation of lattice gauge theories. They consider $G = SU(2)$ but the result carries over to $G = SU(N)$, in particular to $SU(3)$.

Their theory lives on a cubic space lattice with lattice sites r , and links between nearest neighbors. Link $b = (r, m)$ leaves lattice site r in direction $m = \pm 1, \pm 2, \pm 3$. Time is continuous. Quark fields $\psi_a(r)$ are associated with lattice sites and parallel transporters $U(r, m) \approx \exp iaA_m$ are associated with links. They are 3×3 matrices $U = (U^a_b) \in SU(3)$. The full space of states, including unphysical ones, is obtained by acting with any product of components of $U(r, m)$ and $\psi(r')$ on a gauge invariant "bare vacuum" $|0\rangle$ (= exact vacuum in the limit $g \rightarrow \infty$),

$$\pi \psi_c(r') U(r, m)^a_b |0\rangle, \tag{4}$$

where the product is over all r', r, m belonging to some set $\{s\}$. Any link (r, m) may be included an arbitrary number of times. Spinor indices are omitted.

The physical states are gauge invariant. This requires that the colour indices at each point are contracted to form a local singlet. As a result, a complete set of physical states may be graphically represented by a set of continuous (possibly branching^{‡3}) flux lines which may begin or terminate only at quark sites. At every lattice site the difference of the number of flux lines entering and leaving is equal modulo three to the number of quarks at the site. Examples are shown in fig. 1. As a consequence, the number of flux lines leaving a bounded area Λ of space minus the number of flux lines entering it is equal modulo three to the number of quarks inside. Antiquarks are counted as -1 quark.

A formal proof of this result may also be obtained by considering the action of a gauge transformation which is equal to a fixed nontrivial element of the center of G inside Λ and unity outside, and using the fact that $\gamma U(r, m) \gamma^{-1} = U(r, m)$ for every γ in the center [13]. If $|\psi\rangle$ is a state with n quarks in Λ one finds

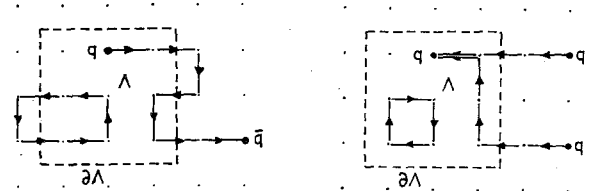


Fig. 1. Gauge invariant states. The number of flux lines leaving a bounded area Λ through its boundary $\partial\Lambda$ minus the number of flux lines entering it is equal modulo three to the number of quarks inside Λ .

$$\exp 2\pi i Y_\Lambda |\psi\rangle = e^{2\pi i n/3} |\psi\rangle$$

$$\text{for } Y_\Lambda = 2g^{-2} \int_{\partial\Lambda} d\sigma E^8 / \sqrt{3}.$$

$E_i^a(r) = F_{0i}^a(r) = ia^{-1} \text{Tr } \lambda^a \dot{U}(r, i) U(r, i)^{-1}$ is the electric field including a factor g (cf. eq. (1)). $\int d\sigma$ is a^2 times the sum over bonds (r, i) leaving Λ (on a lattice).

The conclusion of this analysis is also confirmed by a result of Callan et al. [14], who showed for the abelian Higgs model in continuous 2-dimensional space-time that long range forces between external charges persist in the presence of the Higgs mechanism, for external charges that are *not* integral multiples of the Higgs charge.

For our further discussion we make the customary assumption that virtual quark-antiquark creation processes may be neglected in the discussion of the quark confinement problem. Thus we may consider the forces between static quarks.

Given that the Higgs mechanism produces a mass gap, and that a quantum of total colour electric flux nevertheless must persist between, e.g., a quark and an antiquark, however large their distance, what will be the distribution of flux? In his 1976 Cargèse lectures [20] Wilson has presented a general argument that the flux will be constrained to a flux tube of diameter 2ξ if there is a mass gap m , i.e. a finite correlation length $\xi = 1/m$ for the gluons. Thus there will be a string. Energy in the string is proportional to its length, and the quarks are confined by the linearly rising potential due to the string^{‡4}.

^{‡3} The graphical representation of a state is nonunique. This can be used to avoid branching flux lines for $SU(N)$. This can be proven by using Young symmetrizers [12].

^{‡4} It is essential here that the center Z_3 is not broken. Otherwise the fluctuations of the electric field in the vacuum would be so large that they would drown the flux due to the external charges.

Wilson's argument is general and does not depend on the origin of the mass gap. For a mass gap produced by a Higgs mechanism one can give a further argument based on analogy with a superconductor. It is taken from Parisi's work [5]. In a superconductor one also has a (dynamical) Higgs mechanism. Suppose for the sake of the argument, that there are magnetic monopoles, so that we may bring a monopole and an anti-monopole into a superconductor. The monopoles cannot be screened by the Higgs scalars, since these carry no magnetic charge. There is a Gauss theorem for the magnetic flux in the presence of magnetic charges. It implies that there must be magnetic flux lines between the monopole and antimonopole. All this is analogous to the situation for quarks. The magnetic flux will be constrained to a flux tube due to the Meissner effect, i.e. because the photon has acquired a mass. As a result there will be a string confining the monopole.

In conclusion then, the Higgs screening mechanism is responsible for the formation of strings because it produces a mass gap.

There is also phenomenological evidence for colour screening. It is responsible for saturation of forces. Imagine a colour singlet object composed of two subsystems of three quarks, each of them in a colour octet state. The three quark systems have triality zero and so they can screen their colour charge with the help of (Higgs scalars made from) gluons. As a result there will be no long range colour electric forces between the two subsystems and they will not be confined to each other.

Theoretical evidence for colour screening comes from the so called "strong coupling limit" (large g) in lattice gauge theories. Kogut and Susskind [3] have not only confirmed that colour triplets of quarks are confined in this limit. They have also shown that the same would *not* be true for colour octet quarks, or decuplets, etc. Such quarks can screen their colour with the help of gluons, as a result there are no long range forces between them and particles with the flavor quantum numbers of a single quark would exist. (These results only hold in more than two space-time dimensions. In two dimensions total screening is not possible in QCD, cf. the discussion after eq. (3).) In conclusion, confinement is only possible for quarks which transform nontrivially under the center of the gauge group. This is in agreement with our earlier discussion.

The same conclusion could also be drawn from Wilson's criterium [2] for confinement in gauge theories on a euclidean space-time lattice. Let C a connected closed path consisting of lattice links b_1, b_2, \dots, b_n . The parallel transporter $U[C]$ around the path C is

$$U[C] = U(b_n) \dots U(b_2) U(b_1). \tag{5}$$

Given any character χ of the gauge group G , $\chi(U[C])$ is a gauge invariant observable which does not depend on the choice of initial point on C [15]. Consider its expectation value $\langle \chi(U[C]) \rangle$. Typically one finds in the strong coupling limit, for C a large rectangle,

$$\langle \chi(U[C]) \rangle \propto \exp[-\text{const.} \times \text{area enclosed by } C], \tag{6a}$$

if χ is a character of a representation of G which is non-trivial on the center Γ of G . This indicates confinement. In contrast

$$\langle \chi(U[C]) \rangle \propto \exp[-\text{const.} \times \text{length of } C], \tag{6b}$$

if χ is a character of a one-valued representation of G/Γ . In particular there is no confinement by strings when G has trivial center. An example of the first kind is the character $\chi_3(U) = \text{Tr } U$ of the fundamental 3-dimensional representation of $SU(3)$. An example of the second kind is the character $\chi_8(U)$ of the octet representation of $SU(3)$ or $SU(3)/Z_3$. The importance of the gauge group's center for confinement was pointed out by the author in ref. [10]. Recently it was also emphasized by 't Hooft [16].

On a lattice, pure Yang-Mills theories with gauge group $SU(3)$ and $SU(3)/Z_3$ are different, and the relation between them is not trivial.

Suppose the euclidean lagrangian for the $SU(3)$ theory is chosen as

$$\mathcal{L} = \sum_P \lambda_8 \chi_8(U[P]) + \lambda_3 [\chi_3(U[P]) + \text{c.c.}]. \tag{7}$$

$U[P]$ is the parallel transporter around a plaquette P (closed path of four links). The two terms in (7) are essentially different even though $\chi_8(U) = \chi_3(U) \times \bar{\chi}_3(U) - 1 \approx 3[\chi_3(U) + \bar{\chi}_3(U)] - 10$ for $U = U[P] \approx 1$. The point is that the χ_3 -term suppresses the contribution of configurations $\{U(b)\}$ with $U[P] \approx \gamma \in Z_3$, $\gamma \neq 1$ to the path integral, whereas the χ_8 -term does not since $\chi_8(\gamma) = \chi_8(1) = \text{max}$.

Lagrangian (7) can be used for an SU(3)/Z₃ theory as well if and only if λ₃ = 0. In this case obviously

$$\langle \chi^1(C_1) \dots \chi^n(C_n) \rangle_{\text{SU}(3)/\text{Z}_3} = \langle \chi^1(C^1) \dots \chi^n(C^n) \rangle_{\text{SU}(3)}, \tag{8}$$

for χⁱ any characters of one-valued representations of SU(3)/Z₃, and C_i closed paths. We wrote χ(C) for χ(U[C]). Eq. (8) is consistent with eqs. (6). In fact, for λ₃ = 0 one has “superconfinement”,

$$\langle \chi(U[C]) \rangle = \chi(1) \lim_{\alpha \rightarrow \infty} \exp[-\alpha \times \text{area enclosed by } C], \tag{6a'}$$

in place of eq. (6a). For small λ₃, eq. (8) remains approximately true, and also confinement (6a) is true if power series expansion in λ₃ is permissible. However, to have a continuum limit which admits incorporation of dynamical quark fields, λ₃ must approach a critical value. In conclusion, SU(3) and SU(3)/Z₃ theory have not only different gauge groups but also different lagrangians.

Despite of this complication, it helps conceptually to consider the pure SU(3)/Z₃ Yang–Mills theory first in an attempt to study the dynamical Higgs mechanism. This theory is simpler. It has no confinement by strings, and it admits a *complete* Higgs mechanism in the sense defined earlier.

Existence of a covariant local gauge appears to be a sufficient condition for occurrence of a complete Higgs mechanism. A covariant local gauge is a gauge such that all the fields in the lagrangian become Lorentz covariant field operators in the Hilbert space of physical states in that gauge. This indicates charge screening.

To see this, note that the electrical charge superselection sectors in quantum electrodynamics (QED) are characterized by the nonexistence of a local field (as an operator in the Hilbert space of *physical* states) which can make transitions between them. This is in contrast with superselection sectors associated with global symmetries. There, observables cannot make such transitions but local fields exist which can [21]. We shall therefore speak of charge sectors of the 2nd kind. In the case of QED, note that locality of the electron field ψ(x, t) would imply

$$[Q, \psi(x, t)] = 0, \tag{9a}$$

since [ψ(x, t), F_{0i}(y, t)] = 0 for y ∈ S,

S a sphere around x, and the electric charge is

$$Q = \int_j^0(x, t) d^3x = \int_S F_{0i}(y, t) d\sigma^i(y), \tag{9b}$$

by Gauss’s law. Intuitively this feature comes about because a physical electron is infinitely big since it cannot exist without its Coulomb field. There is an operator which creates a physical electron, for instance the electron field in the Coulomb gauge. However, it is not local.

The defining feature of a Higgs screening mechanism as we understand it is a breakdown of the superselection structure of 2nd kind coming about because the operators which are supposed to make transitions between sectors do actually not lead out of the vacuum sector. (The same thing happens when global symmetries are spontaneously broken [11].) It seems reasonable to expect that this situation will prevail when there exists a covariant local gauge.

Roughly speaking, a local gauge converts a gauge theory into an “ordinary theory” without any gauge freedom, without ruining the fundamental properties of a quantum field theory: locality, Lorentz covariance, etc. For “ordinary theories” cyclicity of the vacuum is the usual situation.

Local gauges can also be characterized in a purely geometric way. This will be discussed elsewhere [18]. They are supposed not to leave any gauge freedom, not even global transformations. An example of a non-covariant local gauge for a pure SO(3) Yang–Mills theory was recently discussed by Goldstone and Jackiw [19].

An example is provided by the exactly soluble Stückelberg model, which is the simplest example of a Higgs model. It is a U(1)-gauge theory with one complex field φ(x) of unit length, φ(x) = e^{iφ(x)}, and describes one free massive vector particle which is its own antiparticle. Clearly there is no charge of any kind in this theory. The local vector potential is B_μ(x) = φ̄(∂_μ - ieA_μ)φ(x). The field strength F_{μν} = ∂_μA_ν - ∂_νA_μ = ∂_μB_ν - ∂_νB_μ, so B_μ really is a vector potential.

A candidate for a local gauge in a SU(3)/Z₃ theory is defined with the help of the scalar field φ(x) introduced in eq. (3), by requiring that its octet part be a diagonal 3 × 3 matrix, etc., cf. after eq. (3).

The fields in the local gauge can be considered as

gauge invariants (just as mass = energy in the rest frame is Lorentz invariant). One would have to show that they are well defined random variables in the euclidean theory. On a lattice this is possible (for all g , $0 < g \leq \infty$).

With charge screened there is no "reason" for massless vector mesons any more [9]. However, one should show that the screening mechanism actually does produce a mass gap, and one would have to control the continuum limit.

Results for the $SU(3)/Z_3$ theory are relevant to the $SU(3)$ theory if eq. (8) is approximately true (in the continuum limit). In particular a mass gap will then carry over from one to the other theory. More generally, the $SU(3)$ theory should admit a local partial gauge. It will still leave the freedom of Z_3 gauge transformations. The Higgs screening mechanism will then again break down the superselection structure, except that now there may a priori still appear triality superselection sectors. Given that the Higgs screening mechanism produces a mass gap and that the Z_3 symmetry is not broken, triality superselection sectors are prevented by the mechanism for confinement outlined earlier.

Lastly we point out that it is not really essential for our purpose to have a *dynamical* Higgs mechanism. We could just as well introduce fundamental Higgs scalars. They would have to be purely neutral except for colour, and in particular transform trivially under the center of $SU(3)$. Given sufficiently many such scalars, one can always enforce the Higgs mechanism by adding a suitable self-interaction of the Higgs scalars to the lagrangian \mathcal{L} . For instance with three octets ϕ_1, ϕ_2, ϕ_3 ,

$$v(\phi) = \lambda_1 \sum_i (\phi_i^2 - \alpha)^2 + \lambda_2 \sum_{i < j} (\phi_i \cdot \phi_j)^2,$$

λ_1, λ_2 large,

would do. The $SU(3)/Z_3$ -theory would then be manageable by perturbative methods. The main objection to this would be that one would not be dealing with QCD in the form (1) any more, but would have to add extra fields whose couplings are not fixed by the principle of gauge invariance. Also one would not expect to have asymptotic freedom. Nevertheless it may be interesting to consider such models also, with \mathcal{L} interpreted as effective lagrangian.

In any case, unlike quarks, gluons are screened but not confined by strings. It would be important to investigate the physical properties of the colour screened gluons, including sources of $q\bar{q}$ -admixture.

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References

- [1] H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B (1973) 365.
- [2] K. Wilson, Phys. Rev. D10 (1974) 2445.
- [3] J. Kogut and L. Susskind, Phys. Rev. D11 (1975) 395.
- [4] For references on bags and strings see: H. Joos, Quark confinement, DESY-report 76/36 (1976).
- [5] G. Parisi, Phys. Rev. D11 (1975) 970.
- [6] The original literature on the Higgs mechanism may be traced from: J. Bernstein, Rev. Mod. Phys. 46 (1974) 7.
- [7] Y. Nambu and G. Jona Lasinio, Phys. Rev. 122 (1961) 345; Y. Nambu, Phys. Rev. D10 (1974) 4262; Phys. Rep. 23C (1976) 250; H. Pagels, Phys. Rev. D14 (1976) 2747; H. Kleinert, Phys. Lett. 59B (1975) 163; 62B (1976) 77.
- [8] B. Zumino, Cargèse lectures (1972); I. Sucher and C.H. Woo, Phys. Rev. D8 (1973) 2721; G.F. De Angelis, D. de Falco and F. Guerra, A note on the abelian Higgs Kibble model on a lattice: absence of spontaneous magnetization, Salerno preprint (1977).
- [9] J.A. Swieca, Phys. Rev. D13 (1976) 312.
- [10] G. Mack, Quark and colour confinement through dynamical Higgs mechanism, DESY report 77/58 (1977), esp. Secs. 2, 3, 5.
- [11] This was pointed out by R. Haag.
- [12] M. Hamermesh, Group theory (Addison Wesley, Reading, MA, 1964).
- [13] M. Lüscher, private communication.
- [14] C.G. Callan, R. Dashen and D.J. Gross, Phys. Rev. D16 (1977) 2526.
- [15] B. Balian, J.M. Drouffe and C. Itzykson, Phys. Rev. D10 (1974) 3376; D11 (1975) 2098.
- [16] G.'t Hooft, On the phase transition towards permanent quark confinement, preprint Utrecht (1977).
- [17] A.M. Polyakov, Nucl. Phys. B120 (1977) 429.
- [18] G. Mack, Physical principles and geometric aspects of gauge theories, in preparation.
- [19] J. Goldstone and R. Jackiw, Phys. Lett. 74B (1978) 81.
- [20] K. Wilson, in: New developments in quantum field theory and statistical mechanics, eds. M. Levy and P. Mitter (Plenum, New York, 1977).
- [21] S. Doplicher, R. Haag and J.E. Roberts, Commun. Math. Phys. 13 (1969) 1; 15 (1969) 173.