# Light-quark mass spectrum in quantum chromodynamics

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Quantum chromodynamics has placed the problem of hadronic symmetry breaking on a rational basis. The current-quark mass ratios can be shown to be renormalization-group invariants up to small and controllable corrections from flavor interactions. We calculate the mass ratios of the light $u$, $d$, and $s$ quarks using the pseudoscalar-meson mass spectrum, the baryon mass spectrum, and the $\eta\to 3\pi$ decay. The main theoretical assumptions are that low-lying-resonance and Born terms correctly estimate the photonic contribution to isotopic mass splitting and that chiral perturbation theory—equivalently kaon partial convervation of axial-vector current—correctly estimates chiral-symmetry breaking. Taking account of all leading-order chiral corrections to the meson spectrum and from the baryon spectrum and $\eta\to 3\pi$ decay we obtain $m_u/m_d = 0.38 \pm 0.13$ and $m_d/m_s = 0.045 \pm 0.011$. We conclude that while a vanishing up-quark mass is not rigorously ruled out it is unattractive from the standpoint of the presently consistent phenomenology of hadronic symmetry breaking.

## I. CURRENT-QUARK MASSES AND QUANTUM CHROMODYNAMICS

In any discussion of the quark mass spectrum it is important to distinguish between current-quark masses and constituent-quark masses. To provide a framework for our discussion, we will adopt quantum chromodynamics (QCD) as our strong-interaction theory.[1] The Lagrangian for the $SU_c(3)$ gauge theory with $N$ quark flavors is

$$\mathcal{L}^{QCD} = -\tfrac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + i\bar{q}_i \not{D} q_i + \Delta\mathcal{L}^{QCD} + \text{gauge terms}, \qquad (1.1)$$
$$\Delta\mathcal{L}^{QCD} = -\sum_{i=1}^{N} m_i^0 \bar{q}_i q_i ,$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

is the Yang-Mills field strength and $D_\mu$ is the covariant derivative. Here $m_i^0$ is the bare current-quark mass of the $i$th flavored quark. An important feature of QCD in the absence of weak (flavored) interactions is that the current-quark mass term $\Delta\mathcal{L}^{QCD}$ is the only term that can break the chiral-$SU(N)\times SU(N)$ symmetry of $\mathcal{L}^{QCD}$ and maintain renormalizability.

Constituent-quark masses are the physical masses of the quarks as they make their appearance in hadrons. This constituent-quark mass is not the same as the current-quark mass since QCD presumably undergoes a PCAC (Partial conservation of axial-vector current) phase transition. This implies that even in the absence of explicit flavor breaking ($m_i^0=0$), the chiral $\gamma_5$ symmetry of the vacuum is broken. All the quarks then can acquire a common mass $M$ by the Nambu–Jona Lasinio mechanism.[2] When explicit flavor breaking is turned on ($m_i^0 \neq 0$), then the constituent masses are shifted and split.

The problem with discussions of constituent-quark masses is that the constituent-quark mass, to the author's knowledge, has never been given a precise definition independent of a specific application and its dynamical details. If quarks could exist as free physical particles then the constituent-quark mass could be precisely defined as the mass of that particle. However, if quarks are permanently confined inside hadrons, the constituent-quark mass depends on the details of the dynamics, and a precise definition has not yet been given. In spite of this difficulty, the use of the concept of a constituent, physical quark mass in specific phenomenological applications, especially nonrelativistic potential models of permanently bound quarks, is quite useful.

By contrast, the current-quark masses can be given a precise phenomenological meaning. The reason is that the divergences of the currents that are measured in electroproduction and neutrino-production experiments are given by $m_i^0 \bar{q}_i q_i$ and $m_i^0 \bar{q}_i \gamma_5 q_i$. These operators make their appearance in the operator-product expansion for the current correlation function near the light cone. For example, the $W_4$ and $W_5$ structure functions measured in neutrino production from nucleons vanish in the chiral limit as $m_i^0 \to 0$.[3] Collins has shown that moments of these structure functions give direct information about current-quark masses.[4] While in principle it is possible to determine the current-quark masses in this way, it is nearly impossible in practice since the distributions are multiplied by a lepton mass, and hence are very small for the light leptons.

Actually, what one obtains from such experiments that probe the light cone is not the bare mass $m_i^0(\Lambda)$ which depends on the cutoff $\Lambda$ but the cutoff-independent quantity $m_i(\mu)$ where $\mu$ is the renormalization mass. The relation of $m_i(\mu)$ to the bare mass $m_i^0(\Lambda)$ is

$$m_i(\mu)=Z(\Lambda/\mu)m_i^0(\Lambda)\,. \tag{1.2}$$

In (1.2), $Z$ is defined using the Weinberg renormalization prescription.[5] $Z^{-1}$, which renormalizes the operator $\bar{q}_i q_i$, is to be calculated in the theory with $\Delta\mathcal{L}^{\rm QCD}=0$. Hence, $Z$ is flavor independent. Equation (2) implies

$$R_{ij}=\frac{m_i(\mu)}{m_j(\mu)}=\frac{m_i^0(\Lambda)}{m_j^0(\Lambda)}\,, \tag{1.3}$$

so that the ratios $R_{ij}$ of the current-quark masses are cutoff and $\mu$ is independent. The renormalization-group-invariant quantities $R_{ij}$ are unambiguous pure numbers that can, in principle, be extracted from experiment. It is the determination of these ratios that will be the subject of this article.

The $\mu$ dependence of $m_i(\mu)$ is established[5] from the renormalization-group equation

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma(g)\right)m_i(\mu)=0\,, \tag{1.4}$$

where $\beta(g)=-(b_0/2)g^3+\cdots$ is the Callan-Symanzik function and $\gamma(g)=\gamma_0 g^2+\cdots$ the anomalous dimension of the operator $m_i^0\bar{q}_i q_i$ which is independent of the specific flavor, $i$. Equation (1.4) can be solved for $m_i(\mu)$ up to an integration constant $m_i$,

$$m_i(\mu)=m_i F(\mu)\,. \tag{1.5}$$

Clearly $R_{ij}=m_i/m_j$, and for large $\mu$ asymptotic freedom implies

$$F(\mu)\underset{\mu\to\infty}{\longrightarrow}(\ln\mu)^{-\gamma_0/b_0}\,. \tag{1.6}$$

We can define $m_i$ to be the renormalized current-quark mass.

The above discussion shows that current-quark mass ratios $R_{ij}$ are unambiguous in QCD if we ignore flavor-breaking interactions due to weak-interaction gluons [quantum flavor dynamics (QFD)]. However, if we examine isospin breaking the photon interaction must be taken into account. We will, however, ignore the usual weak interaction for $Q^2 \ll m_W{}^2 \sim (60\ \text{GeV})^2$. As a consequence of the electromagnetic flavor-breaking interactions, the operator $m_i^0\bar{q}_i q_i$ is no longer universally renormalized and there are ambiguities in the quark mass ratios as emphasized by Collins.[4] However, for quarks of the same electric charge, since the electromagnetic interaction couples only to the charge, the renormalization is still universal. Hence, the quark mass ratios

$$R_{ij}=\frac{m_i^0(\Lambda)}{m_j^0(\Lambda)}\,,\quad q_i=q_j=\pm\tfrac{1}{3},\pm\tfrac{2}{3} \tag{1.7}$$

for quarks of the same charge are unambiguous.

For quarks with different charges the mass ratios $m_i(\mu)/m_j(\mu)$ are not renormalization-group-invariant. The renormalization-group equation in the presence of electromagnetism is

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_g(g,e)\frac{\partial}{\partial g}+\beta_e^i(e,g)\frac{\partial}{\partial e}+\gamma_i(g,e)\right)m_i(\mu)=0\,, \tag{1.8}$$

where

$$\beta_g(g,e)=-(b_0/2)g^3+O(g^3e^2)\,,$$
$$\beta_e^i(e,g)=(e^2/12\pi^2)q_i^2+O(e^2g^2)\,,$$
$$\gamma_i(g,e)=\gamma_0 g^2+\gamma_1 q_i^2 e^2+\cdots\,.$$

Although $\beta_g$ is flavor independent $\beta_e^i$ and $\gamma_i$ are not. Expanding in powers of $\alpha=e^2/4\pi$ and defining $m_i=m_i(\mu_0)$, where $\mu_0$ is arbitrary, one finds[4] using Eq. (1.8),

$$R_{ij}(\mu)=\frac{m_i(\mu)}{m_j(\mu)}=\frac{m_i}{m_j}\,[1+\alpha A(q_i{}^2-q_j{}^2)\ln(\mu/\mu_0)]\,, \tag{1.9}$$

where $A$ is a numerical constant of order unity. This equation exhibits the ambiguity if $q_i{}^2\neq q_j^2$ since $R_{ij}(\mu)$ is $\mu$ dependent.

Although this ambiguity exists, it is numerically unimportant. If we compare the mass ratio $R_{ij}(\mu)$ at two different values of $\mu$ ($\mu_1$ and $\mu_2$) the difference is small compared to the mass ratio itself,

$$\frac{R_{ij}(\mu_1)-R_{ij}(\mu_2)}{R_{ij}}=\alpha A(q_i{}^2-q_j{}^2)\ln(\mu_1/\mu_2) \ll 1 \tag{1.10}$$

providing $\mu_1/\mu_2$ is not of $O(e^{-1/\alpha})$. This is certainly the case for experimentally available energies. Equivalently, the cutoff dependence in $m_i^0(\Lambda)/m_j^0(\Lambda)$ is small compared to the ratio itself and we can ignore it. We conclude that quark mass ratios in QCD, even in the presence of electromagnetism, are unambiguous up to very small and controllable corrections of $O(\alpha)$.

Having reached this conclusion, there remains the problem of actually extracting the values of the quark mass ratios from experimental data. The remainder of this article will be devoted to this task. The technique we will use is the chiral perturbation theory previously developed by us.[6,7]

The basic observation of chiral perturbation theory is that many strong-interaction amplitudes approach their chiral $SU(N)\times SU(N)$ values as $m_i \to 0$ in a nonanalytic fashion such as $m_i \ln m_i$ or $m_i^{1/2}$. Chiral perturbation theory is a systematic calculational technique for calculating the coefficients of all such nonanalytic terms which are usually the leading order terms in chiral-symmetry breaking. The main assumptions are that these nonanalytic terms are the dominant contribution to symmetry breaking, and that one can do perturbation theory in the chiral-symmetry-breaking parameters, the renormalized-quark masses $m_i$. The current-quark mass $m_i$ is not a dimensionless parameter. A typical dimensionless parameter that makes its appearance in chiral perturbation theory is $\gamma_i = \mu_i^2/16\pi^2 F_\pi^2$, where $\mu_i$ is a pseudoscalar mass that vanishes as $m_i \to 0$ and $F_\pi = 93$ MeV is the pion decay constant. As long as this parameter is small, we have confidence that chiral perturbation theory is applicable.

For the lightest quarks, $u$ and $d$, perturbation theory about the $SU(2)\times SU(2)$ limit is very good since $\gamma_u \approx \gamma_d \approx 0.014$. For the $s$ quark $\gamma_s \approx 0.1$, so there is reason for less confidence in perturbation theory about chiral $SU(3)\times SU(3)$. However, we remind the reader that in the Gell-Mann–Oakes–Renner[8] scheme, which is consistent with all available information on chiral-symmetry breaking,[7] the breakings of chiral $SU(3)\times SU(3)$ and ordinary SU(3) are comparable, both breakings being generated by the strange-quark mass $m_s$. Consequently, the corrections to kaon PCAC, equivalent to chiral perturbation theory, and ordinary SU(3) are comparable. Conversely, any attempt to give up kaon PCAC will have to account for the success of SU(3) symmetry in a new way. This seems to us to be an unattractive and unnecessary option, although it is not strictly ruled out.

For heavy quarks such as the $c$, we have no reason to believe that chiral perturbation theory is applicable, since $\gamma_c \approx 1$. It is difficult to suppose that the $D$ mesons are approximate Nambu-Goldstone bosons such as the $\pi$ and $K$. In fact, it is possible that as one increases a current-quark mass there is a phase transition at a critical value $m = m_{\text{crit}}$ beyond which there is no remnant of the Nambu-Goldstone states for that flavor. In any case, we have no reliable way of determining the ratio of light $(u,d,s)$ quark masses to heavy $(c,t,b\ldots)$ quark masses with the methods available at this time. For this reason we restrict ourselves to examining the light-quark mass ratios.

The origin of the quark mass spectrum is a deep problem. This problem, evidently, is connected with the problem of establishing mixing angles such as Cabibbo angle and other mixing angles[9] which arise upon diagonalizing the general quark mass matrix $m_{ij}$. A solution to this problem may come from the interplay of QCD and QFD in a unified theory and from the systematics of the quark vs lepton spectrum. In the present state of the art of unified gauge theories such connections remain speculative.

## II. QUARK MASS RATIOS AND CHIRAL PERTURBATION THEORY

What are the motivations for examining the current-quark mass spectrum? There are at least two reasons: (a) In most weak-interaction models (QFD) the current-quark mass term is generated by spontaneous symmetry breaking of a local flavor group. A knowledge of mass ratios is useful in providing constraints on model building. (b) The vacuum of QCD will in general violate $P$ and $T$ invariance since one can add to $\mathcal{L}^{\text{QCD}}$ the term $(\theta/32\pi^2)F_{\mu\nu}\tilde{F}_{\mu\nu}$ without altering renormalizability.[10] This term gives a nontrivial contribution to the action from instantons if $\theta \neq 2\pi n$ and will violate $P$ and $T$ symmetry. But this interaction term is not invariant under a chiral transformation on a single quark field, such as $u_R \to e^{i\alpha}u_R$, and could be rotated away if $\mathcal{L}^{\text{QCD}} + \Delta\mathcal{L}^{\text{QCD}}$ were invariant under this chiral transformation. This is possible providing $m_u^0 = 0$. While a massless up quark is not the only solution to the problem of potentially strong $P$ and $T$ violations,[11] it is of interest to see if this solution is phenomenologically viable.

Our plan is as follows. First we describe the problem of determining the $\Delta I = 1$ quark mass differences and then give the formulas for determining the quark mass ratios in terms of meson and baryon masses. We give a careful treatment of symmetry breaking which is controlled by chiral perturbation theory. We will discuss separately the isosinglet-isosinglet mass ratio $(m_d + m_u)/2m_s$ and the isovector-isosinglet mass ratio $(m_d - m_u)/2m_s$.

The ratio $(m_d + m_u)/2m_s$ of $SU(2)\times SU(2)$ to $SU(3)\times SU(3)$ chiral breaking can be determined in chiral perturbation theory to lowest order to be given by $\mu_\pi^2/2\mu_K^2$. Including the complete leading-order corrections from chiral perturbation theory we find

$$\frac{m_d + m_u}{2m_s} = 0.031\,. \tag{2.1}$$

The ratio $(m_d - m_u)/2m_s$ of isospin to SU(3) breaking is far more difficult to determine in a reliable way. The reason for this is that electromagnetism is known to violate isospin and require divergent counterterms in the quark mass matrix. Because these counterterms are divergent (in most QFD theories), in the presence of electromagnetism there is a $m_u$-$m_d$ mass difference which is a free parameter. In practice, the $m_u$-$m_d$ mass difference is determined from: (i) the ground-state pseudoscalar meson mass difference $K^+$-$K^0$, (ii) the $\Delta I=1$ baryon mass differences, (iii) the $\eta\to 3\pi$ decay. Below, we will review the theoretical problem of defining the $m_u$-$m_d$ mass difference and the phenomenological problem of extracting it from the data.

From the meson mass spectrum we obtain

$$\frac{m_d-m_u}{2m_s}=0.0175\,, \tag{2.2}$$

and from the baryon spectrum the consistent value,

$$\frac{m_d-m_u}{2m_s}=0.0105\pm 0.003\,. \tag{2.3}$$

The extraction of this ratio from the $\eta\to 3\pi$ decay is sensitive to the details of SU(3) × SU(3) symmetry breaking (an extrapolation on the order of the $\eta$ mass is required). So the $\eta\to 3\pi$ decay is inherently less reliable. If one ignores SU(3) × SU(3) symmetry breaking, one obtains from the measured rate a ratio which is consistent with (2.2) and (2.3) obtained from the hadron mass spectrum. Depending on the treatment of symmetry breaking, one can obtain results that improve or spoil the agreement with (2.2) and (2.3).

From (2.1) and (2.2) or (2.3) we have

$$\frac{m_u}{m_d}\approx 0.38\,,\qquad \frac{m_d}{m_s}\approx 0.045\,, \tag{2.4}$$

which rules out a vanishing up-quark mass. The main ways to avoid this conclusion are: (i) The contribution of the low-lying states to the Cottingham formula for the electromagnetic mass shifts of the hadrons is underestimated by a factor ≈4. (ii) Kaon PCAC or equivalently chiral SU(3) × SU(3) perturbation theory fails. If the latter is the case, then we also lose the rationale for an approximate SU(3) symmetry. Further, it is not clear that if an alternative approach is adopted it will lead to a consistent account of *all* symmetry breaking. *We conclude that while a vanishing up-quark mass is not rigorously ruled out, it is unattractive from the standpoint of the currently consistent phenomenology of hadronic symmetry breaking.*

### A. Calculational procedure for $\Delta I=1$ mass terms

The symmetry-breaking part of QCD that we will consider is

$$\Delta\mathcal{L}^{\rm QCD}=m_u^0\bar{u}u+m_d^0\bar{d}d+m_s^0\bar{s}s\,, \tag{2.5}$$

to which is to be added the conventional electromagnetic effective action

$$\mathcal{L}^{\rm EM}=\frac{\alpha}{(2\pi)^3}\int d^4x\,D^{\mu\nu}(x)\,T^*\left(V_\mu^Q\left(\frac{x}{2}\right)V_\nu^Q\left(-\frac{x}{2}\right)\right)\,, \tag{2.6}$$

where $V_\mu^Q$ is the electromagnetic current and

$$D_{\mu\nu}^{(x)}=-\int\frac{d^4q}{q^2+i\epsilon}e^{iq\cdot x}(g_{\mu\nu}+\text{gauge terms})\,. \tag{2.7}$$

Before proceeding with calculations, it is necessary to discuss the extraction of the electromagnetic mass shifts of quarks (tadpole) from the conventional electromagnetic interaction. This has been discussed in detail by Gasser and Leutwyler,[12] Gunion,[13] and more recently, from the viewpoint of the renormalization group, by Collins.[14] We will simply review the conclusion of these articles.

First we note that $\Delta\mathcal{L}^{\rm QCD}$ has no $\Delta I=2$ part. On the other hand, $\mathcal{L}^{\rm em}$, if one examines the operator-product expansion of the $T^*$ product, has finite matrix elements for its $\Delta I=2$ part. Therefore, the Cottingham integral is convergent for $\Delta I=2$ mass shifts such as $\mu_{\pi^+}{}^2-\mu_{\pi^0}{}^2$ and $m_{\Sigma^+}+m_{\Sigma^-}-2m_{\Sigma^0}$. As is well known, saturation of the Cottingham formula with low-lying states for these combinations yields excellent agreement with experiment.[15] The $\Delta I=2$ mass shifts are evidently well understood, but they give no information on the quark mass spectrum.

By contrast, the $\Delta I=0$, 1 parts of $\mathcal{L}^{\rm em}$ can be shown from the operator-product expansion to give divergent contributions. These divergences are canceled by the mass counterterms in $\Delta\mathcal{L}^{\rm QCD}$ to yield finite results for symmetry breaking. While the subtraction of the logarithmically divergent part is unambiguous in a renormalizable theory such as QCD, there can remain a finite term after the subtraction. This finite term in the quark masses is ambiguous because it depends on the subtraction procedure used; different subtraction procedures treat finite parts differently. However, this set of finite rescalings is precisely that which is controlled by the renormalization group, and the ambiguity is due to the fact that the renormalization subtraction can be carried out at different renormalization masses. This has been discussed in detail by Collins.[14]

To see how this works in practice, let us regulate $\mathcal{L}^{\rm em}$ by the introduction of a cutoff $\Lambda$ in (2.7) according to the replacement $1/q^2\to 1/q^2-1/(q^2-\Lambda^2)$. Then, if we consider the contribution of $\Delta\mathcal{L}^{\rm QCD}+\mathcal{L}^{\rm em}$ to a physical mass shift $\Delta M$, it can be written as

$$\Delta M=(\Delta M)_\Lambda^{\rm QCD}+(\Delta M)_\Lambda^{\rm em}\,, \tag{2.8}$$

where the two terms correspond to the matrix elements of $\Delta\mathcal{L}^{QCD}$ and $\mathcal{L}^{em}$, respectively. Now as $\Lambda\to\infty$, we have

$$(\Delta M)^{em}_{\Lambda} \underset{\Lambda\to\infty}{\longrightarrow} \alpha C \ln(\Lambda/\mu)\,, \tag{2.9}$$

with $C$ a constant and $\mu$ the renormalization mass. By the renormalizability of QCD and QFD, $\Delta M$ must be $\Lambda$ independent, so that

$$(\Delta M)^{QCD}_{\Lambda} \underset{\Lambda\to\infty}{\longrightarrow} -\alpha C \ln(\Lambda/\mu)\,. \tag{2.10}$$

Defining the finite quantities

$$\begin{aligned}(\Delta M)^{em} &= (\Delta M)^{em}_{\Lambda} - \alpha C \ln(\Lambda/\mu)\,,\\ (\Delta M)^{tad} &= (\Delta M)^{QCD}_{\Lambda} + \alpha C \ln(\Lambda/\mu)\,,\end{aligned} \tag{2.11}$$

we can split up the contributions to the physical mass shifts as

$$\Delta M = (\Delta M)^{em} + (\Delta M)^{tad}\,, \tag{2.12}$$

where tad means tadpole contribution. Here $(\Delta M)^{em}$ represents the contribution of the low-lying states, Born terms, and resonances, before the onset of scaling behavior in the structure functions in the Cottingham integral. The scaling part of the structure functions which gives the divergence in the Cottingham formula[16] has been explicitly subtracted out of the definition of $(\Delta M)^{em}$ and is not to be included in numerical estimates.

The finite ambiguity mentioned above is simply reflected in the fact that we can change the numerical value of $(\Delta M)^{em}$ or $(\Delta M)^{tad}$ by a different choice of cutoff $\Lambda$, or equivalently, a different choice of renormalization point $\mu$. This ambiguity we see from (2.12) is given by

$$\delta(\Delta M)^{tad} = \alpha C \ln(\Lambda_1/\Lambda_2)\,, \tag{2.13}$$

where $\Lambda_1$ and $\Lambda_2$ are two choices of cutoff. The constant $C$ has been computed by Collins[14] with the result

$$C = \frac{(\Delta M)^{tad}}{12\pi}\left(\frac{4m_u - m_d}{m_u - m_d}\right)\,, \tag{2.14}$$

where $m_{u,d}$ are the renormalized quark masses. To estimate the ambiguity, we choose $m_u/m_d \approx \frac{1}{3}$, $\Lambda_1/\Lambda_2 = 100$, so that $\delta(\Delta M)^{tad} = 4.4\times 10^{-4}(\Delta M)^{tad}$. The ambiguity due to change in cutoff by a factor of 100 is $10^{-4}$, compared to the tadpole term we are computing. Our conclusion is that we can extract the tadpole contribution from physical $\Delta I = 1$ mass differences if we subtract just the nonscaling Born and resonance terms in the Cottingham integral from the physical mass difference up to a small and negligible ambiguity.

### B. Pseudoscalar mesons

The ground-state pseudoscalar mesons are a good source of information about the current-quark mass ratios. The $\eta$ and $\eta'$ can mix, and this introduces an additional parameter rendering this system useless for our problem of determining quark mass ratios. This leaves the $\pi$ and $K$ system.

We use the standard method invoking the PCAC relation for the axial-vector currents $A^a_\mu(x)$, $a = 1, \ldots, 8$,

$$\partial_\mu A^a_\mu(x) = i[{}^5Q^a, \mathcal{L}]\,, \tag{2.15}$$

where $\mathcal{L} = \mathcal{L}^{QCD} + \Delta\mathcal{L}^{QCD} + \mathcal{L}^{em}$ with $\Delta\mathcal{L}^{QCD}$ and $\mathcal{L}^{em}$ given by (2.5) and (2.6). $\mathcal{L}^{QCD}$ is chiral invariant, so $[{}^5Q^a, \mathcal{L}^{QCD}] = 0$. We will take the matrix element of (2.15) between the vacuum and the physical $\pi$ and $K$ states. With the definitions

$$\begin{aligned}\langle 0\,|A^{\pi^-}_\mu(0)\,|\pi^+(K)\rangle &= i\sqrt{2}\,F_{\pi^+}K_\mu\,,\\ \langle 0\,|\bar d i\gamma_5 u\,|\pi^+\rangle &= \sqrt{2}\,Z_{\pi^+}{}^{1/2},\ \text{etc}\end{aligned} \tag{2.16}$$

[in the chiral limit the $Z$'s are proportional to $Z(\Lambda/\mu)$ of (1.2)] we obtain the equations

$$\begin{aligned}F_{\pi^+}\mu_{\pi^+}{}^2 &= Z_{\pi^+}{}^{1/2}(m^0_d + m^0_u) + F_{\pi^+}(\delta\mu_{\pi^+}{}^2)^{em}\,,\\ F_{\pi^0}\mu_{\pi^0}{}^2 &= Z_{\pi^0}{}^{1/2}(m^0_d + m^0_u) + F_{\pi^0}(\delta\mu_{\pi^0}{}^2)^{em}\,,\\ F_{K^+}\mu_{K^+}{}^2 &= Z_{K^+}{}^{1/2}(m^0_s + m^0_u) + F_{K^+}(\delta\mu_{K^+}{}^2)^{em}\,,\\ F_{K^0}\mu_{K^0}{}^2 &= Z_{K^0}{}^{1/2}(m^0_s + m^0_d) + F_{K^0}(\delta\mu_{K^0}{}^2)^{em}\,,\end{aligned} \tag{2.17}$$

where $(\delta\mu^2)^{em}$ is the contribution of $\mathcal{L}^{em}$ and is explicitly of order $\alpha$. We have dropped the $\pi^0$-$\eta$ mixing terms in the $\pi^0$ equation, which are second order in isospin breaking. Otherwise, (2.17) is exact. The quantities $Z^{1/2}$, $m^0$, $(\delta\mu^2)^{em}$ are all divergent. After the renormalizations and ratios are taken, everything will be finite as we have previously discussed. To extract information from (2.17), we will make the safe assumption of (i) dropping terms of second order in isospin breaking relative to terms of first order in isospin breaking, (ii) dropping terms first order in isospin breaking relative to terms of first order in SU(3) breaking, and (iii) using $\delta_{\pi^+}{}^2/\mu_{\pi^+}{}^2 = 3\alpha\ln 2m_\rho{}^2/4\pi\mu^2_\pi + O(\alpha\ln\mu_\pi{}^2)$ and $\delta\mu_{\pi^0}{}^2/\mu_{\pi^0}{}^2 = O(\alpha\ln\mu_\pi{}^2)$ we find we can drop $\delta\mu_\pi{}^2$ relative to $\mu_\pi{}^2$. For the quark mass ratios we can replace the bare values with the renormalized values and obtain

$$\begin{aligned}\frac{m_d + m_u}{2m_s} &= \frac{\mu_\pi{}^2(1-\epsilon)}{2\mu_K{}^2 - \mu_\pi{}^2(1-\epsilon)}\,,\\ \frac{m_u - m_d}{2m_s} &= \frac{\mu_{K^+}{}^2 - \mu_{K^0}{}^2 - (\Delta\mu_K{}^2)^{em}}{2\mu_K{}^2 - \mu_\pi{}^2(1-\epsilon)} - \frac{\delta_K}{2}\,,\end{aligned} \tag{2.18}$$

and

$$\mu_{\pi^+}{}^2 - \mu_{\pi^0}{}^2 = (\Delta\mu_\pi{}^2)^{em} + \delta_\pi\mu_\pi{}^2\,, \tag{2.19}$$

where

$$\delta_\pi = 1 - \frac{Z_{\pi^0}^{1/2}}{Z_{\pi^+}^{1/2}} \frac{F_{\pi^+}}{F_{\pi^0}},$$

$$\delta_K = 1 - \frac{Z_{K^0}^{1/2}}{Z_{K^+}^{1/2}} \frac{F_{K^+}}{F_{K^0}}, \tag{2.20}$$

$$\epsilon = 1 - \frac{Z_K^{1/2}}{Z_\pi^{1/2}} \frac{F_\pi}{F_K},$$

and

$$(\Delta\mu_K^2)^{em} = (\delta\mu_{K^+}^2)^{em} - (\delta\mu_{K^0}^2)^{em},$$
$$(\Delta\mu_\pi^2)^{em} = (\delta\mu_{\pi^+}^2)^{em} - (\delta\mu_{\pi^0}^2)^{em} \tag{2.21}$$

are the conventional photonic contribution to the meson mass differences from just the low-lying states in the Cottingham formula. In Eq. (2.20), $\delta_{\pi,K}$ are of the order of isospin violation while $\epsilon$ is of the order of SU(3) violations. If we also drop terms of the order of isospin violation times chiral SU(2) × SU(2) violation relative to isospin violations (2.19) implies the well known result

$$\mu_{\pi^+}^2 - \mu_{\pi^0}^2 = (\Delta\mu_\pi^2)^{em}. \tag{2.22}$$

To determine the quark mass ratios from (2.18) we need to know $(\Delta\mu_K^2)^{em}$, $\delta_K$, and $\epsilon$. These quantities cannot be obtained directly from experiment, and one is required to make theoretical assumptions.

The contribution from the low-lying states to the kaon mass difference $(\Delta\mu_K^2)^{em} \approx 2\mu_K(\Delta\mu_K)^{em}$ was estimated by Socolow[17] in 1965 with the result

$$(\Delta\mu_K)^{em} \approx 3 \text{ MeV (Socolow)}. \tag{2.23}$$

Recently, use has been made of the Dashen sum rule[18] which implies

$$(\Delta\mu_K^2)^{em} = (\Delta\mu_\pi^2)^{em}, \tag{2.24}$$

which upon using (2.22) implies

$$(\Delta\mu_K)^{em} \approx 1.5 \text{ MeV (Dashen sum rule)}.$$

However, it has been emphasized by us[19] that the Dashen sum rule is obtained in perturbation theory. The corrections to the Dashen sum rule are as large as the terms one is retaining, and hence, the assumption upon which it was obtained is false. This could be the explanation for the discrepancy between the Socolow calculation and the Dashen sum rule. In view of this, we will use Socolow's estimate (2.23) for $(\Delta\mu_K)^{em}$. It should be noted, however, that the final results obtained for the quark mass ratios are insensitive to which value is used.

Since the quantities $\delta_K$ and $\epsilon$ involve the ratio $Z_K^{1/2}/Z_\pi^{1/2}$, etc. they cannot be obtained from any experiment. However, if we assume the validity of chiral perturbation theory, then these quantities can be calculated *exactly*, independent of any model, to leading order in the chiral-symmetry breaking. The results of these calculations are[6,7]

$$\frac{F_K}{F_\pi} = 1 + \frac{3(\mu_K^2 - \mu_\pi^2)}{64\pi^2 F_\pi^2} \ln(M^2/\mu^2),$$

$$\frac{Z_K^{1/2}}{Z_\pi^{1/2}} = 1 + \frac{(\mu_K^2 - \mu_\pi^2)}{192\pi^2 F_\pi^2} \ln(M^2/\mu^2),$$

$$\frac{Z_{K^+}^{1/2}}{Z_{K^0}^{1/2}} = 1 + \frac{(\Delta\mu_K^2)^{tad}}{192\pi^2 F_\pi^2} \ln(M^2/\mu^2), \tag{2.25}$$

$$\frac{F_{K^+}}{F_{K^0}} = 1 + \frac{3(\Delta\mu_K^2)^{tad}}{64\pi^2 F_\pi^2} \ln(M^2/\mu^2),$$

so

$$\delta_K = -\frac{(\Delta\mu_K^2)^{tad}}{24\pi^2 F_\pi^2} \ln(M^2/\mu^2),$$

$$\epsilon = \frac{(\mu_K^2 - \mu_\pi^2)}{24\pi^2 F_\pi^2} \ln(M^2/\mu^2),$$

where

$$(\Delta\mu_K^2)^{tad} = \mu_{K^+}^2 - \mu_{K^0}^2 - (\Delta\mu_K^2)^{em}.$$

Here, $F_\pi = 93$ MeV, $M$ is a cutoff we will take to be the nucleon mass, and $\mu^2$ = mean (mass)$^2$ of the ground-state pseudoscalars $\approx 0.17$ GeV$^2$. Using those numbers and (2.23) we obtain from (2.25)

$$\frac{F_K}{F_\pi} = 1.21, \quad \frac{Z_K^{1/2}}{Z_\pi^{1/2}} = 1.023, \quad \epsilon = 0.18$$

$$\frac{F_{K^+}}{F_{K^0}} - 1 = -6.3 \times 10^{-3}, \quad \frac{Z_{K^+}^{1/2}}{Z_{K^0}^{1/2}} - 1 = -7.0 \times 10^{-4},$$

$$\delta_K = 5.6 \times 10^{-3}. \tag{2.26}$$

The above calculation in chiral perturbation theory can be checked against experiment for the quantity $F_K/F_\pi$. From $K_{l3}$ decay one finds

$$\frac{F_K}{f_+(0) F_\pi} = 1.26 \pm 0.02 \tag{2.27}$$

with $f_+(0)$ the $K_{l3}$ decay vector form factor. From the nonrenormalization theorem[20] we know that $f_+(0)$ is near 1.0. An explicit calculation of the exact first-order correction in chiral-symmetry breaking gives[20] $f_+(0) = 0.97$, so that we obtain

$$\frac{F_K}{F_\pi} = 1.22 \pm 0.02, \tag{2.28}$$

in excellent agreement with the calculated ratio (2.26). This gives us confidence that our other estimates in (2.26) are roughly correct even though they cannot be directly obtained from experiment.

Using the results of chiral perturbation theory (2.26) and Socolow's estimate of the kaon electromagnetic mass difference (2.23), the quark mass ratios can be gotten from (2.18) with the result

$$\frac{m_d+m_u}{2m_s}=0.031\ (0.038)\,,\qquad \frac{m_d-m_u}{2m_s}=0.0175\ (0.015)\,, \tag{2.29}$$

where the numbers in parentheses are what would have been obtained if we had ignored the corrections of chiral perturbation theory and set $\epsilon=0$, $\delta_K=0$. Equation (2.29) implies

$$\frac{m_u}{m_d}=0.28\ (0.44)\,, \tag{2.30}$$

so the up quark has nonvanishing mass.

Some attempts have been made to obtain $m_u=0$. Since $Z_K^{1/2}/Z_\pi^{1/2}$ is not obtainable from experiment, some authors[21] have supposed $Z_K^{1/2}/Z_\pi^{1/2}\approx 0.36$, which is drastically different from the result of chiral perturbation theory. Such an assumption implies huge violations of ordinary SU(3) symmetry. Dominguez[22] has pointed out that the ratio $Z_K^{1/2}/Z_\pi^{1/2}$ makes its appearance in the nonrenormalization theorem for $f_+(0)$, and a value very different from 1.0 would wreck the agreements of Cabibbo theory. Further, if $Z_K^{1/2}/Z_\pi^{1/2}$ were wildly different from unity, there would be no good reason to suppose that a similar ratio of wave-function renormalization constants that makes its appearance in the baryon mass-difference calculations should be near unity. Then one loses the Gell-Mann–Okubo formula. It seems to us unlikely that if $Z_K^{1/2}/Z_\pi^{1/2}$ differs greatly from the result of chiral perturbation theory than the impressive agreement of broken SU(3) with experiment could be maintained.

Soper and Deshpande[23] have obtained $m_u=0$ by assuming $\delta_K=0.036$. This assumption represents a huge violation of isospin. The value of $\delta_K$ is almost an order of magnitude greater than what we have calculated from chiral perturbation theory $\delta_K=0.0056$. We think that the successes of chiral perturbation theory do not warrant such a large value of $\delta_K$, although it is not impossible that there is some special dynamical enhancement of $\delta_K$ to which chiral perturbation theory is blind. In the absence of a demonstration of such enhancement it is not possible to justify the large value of $\delta_K$.

We finally remark that the quark mass ratios we obtain are not very sensitive to the precise values of the chiral-perturbation-theory corrections $\epsilon$, $\delta_K$, providing they are within 50% of the values we obtain and not different by orders of magnitude.

### C. Baryons

From the baryon mass spectrum it is possible to establish an estimate of the ratio $(m_d-m_u)/2m_s$. To do so one must extract the tadpole contribution from the observed isotopic splittings according to

$$(\Delta M)_{ij}=(\Delta M)_{ij}^{\text{em}}+(\Delta M)_{ij}^{\text{tad}}\,,\qquad (\Delta M)_{ij}=M_i-M_j\,.$$

The low-lying-resonance and Born-term contributions to $(\Delta M)_{ij}^{\text{em}}$ from the Cottingham formula were calculated by Coleman and Schnitzer[24] for all the baryons. There have been many calculations of $(\Delta M)_{pn}^{\text{em}}$, and these generally agree with each other.[15] The results of these calculations are listed in Table I along with the experimental mass differences and the calculated tadpole terms $(\Delta M)_{ij}^{\text{tad}}$.

In order to calculate the ratio $(m_d-m_u)/2m_s$ we will (i) drop second-order SU(3) violations relative to first-order violations, (ii) drop SU(2) $\times$ SU(2) violations relative to SU(3) violations so $m_s\gg m_{d,u}$. Then, by considering the matrix elements of the divergence of the vector currents $\partial_\mu V_\mu^a=i[Q^a,\mathcal{L}]$ between baryon states, one obtains for the $\Delta I=1$ mass differences

$$(\Delta M)_{pn}^{\text{tad}}=M\left(1+\frac{F}{D}\right)\,,\qquad (\Delta M)_{\Sigma^+\Sigma^-}^{\text{tad}}=2M\frac{F}{D}\,,\qquad (\Delta M)_{\Xi^0\Xi^-}^{\text{tad}}=M\left(\frac{F}{D}-1\right)\,, \tag{2.31}$$

where $F/D$ is the $F/D$ ratio for the baryon mass matrix.

The three $\Delta I=1$ mass differences in (2.31) are

TABLE I. Baryon mass differences in MeV (to nearest 0.1 MeV).

| Mass difference | $(\Delta M)$ | $(\Delta M)^{\text{em}}$ | $(\Delta M)^{\text{tad}}=(\Delta M)-(\Delta M)^{\text{em}}$ |
|---|---|---|---|
| $p-n$ | −1.3 | 1.1 | −2.4 |
| $\Sigma^+-\Sigma^-$ | −8.0 | −0.7 | −7.3 |
| $\Xi^0-\Xi^-$ | −6.4 | −1.3 | −5.1 |
| $\Sigma^++\Sigma^--2\Sigma^0$ | 1.8 | 2.1 | 0 |

not all independent in this model of symmetry breaking. First, one finds from (2.31) the Coleman-Glashow formula

$$(\Delta M)^{\text{tad}}_{pn} + (\Delta M)^{\text{tad}}_{\Xi^0\Xi^-} + (\Delta M)^{\text{tad}}_{\Sigma^-\Sigma^+} = 0 , \qquad (2.32)$$

which is satisfied within experimental errors. Second, one obtains the $F/D$ ratio from the tadpole splittings as

$$\frac{F}{D} = \frac{(\Delta M)^{\text{tad}}_{\Sigma^+\Sigma^-}}{(\Delta M)^{\text{tad}}_{\Xi^-\Xi^0} + (\Delta M)^{\text{tad}}_{pn}} = -2.7 , \qquad (2.33)$$

which should be equal to the $F/D$ from the medium strong splittings

$$\frac{F}{D} = \frac{2}{3}\frac{M_\Xi - M_N}{M_\Lambda - M_\Sigma} = -3.3 \qquad (2.34)$$

since it is the same octet that is responsible for both splittings. These ratios differ by 18% in (2.33) and (2.34), and this gives us an idea of the uncertainties in our calculations of the tadpole terms.

Finally, we can compute the constant $M$ in terms of the ratio of quark masses with the result

$$M = \frac{3}{2}\left(\frac{m_d - m_u}{2m_s}\right)(M_\Sigma - M_\Lambda) . \qquad (2.35)$$

Using this and the Gell-Mann–Okubo formula, $2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma$, we obtain from (2.35) and (2.31) the result

$$\frac{m_d - m_u}{2m_s} = \frac{1}{2}\,\frac{(\Delta M)^{\text{tad}}_{np} + (\Delta M)^{\text{tad}}_{\Xi^0\Xi^-}}{m_\Xi + m_N - 2m_\Sigma} . \qquad (2.36)$$

From our previous analysis of the pseudoscalar mesons we recognize the left-hand side of (2.36) as $(\Delta\mu_{K^0K^+}{}^2)^{\text{tad}}/2\mu_K{}^2$ to leading order in chiral breaking so that

$$\frac{(\Delta\mu_{K^0K^+}{}^2)^{\text{tad}}}{\mu_K{}^2} = \frac{(\Delta M)^{\text{tad}}_{np} + (\Delta M)^{\text{tad}}_{\Xi^0\Xi^-}}{m_\Xi + m_N - 2m_\Sigma} , \qquad (2.37)$$

which is just the hybrid sum rule first obtained by Coleman and Glashow.[25]

The point of these remarks is that if the quark-mass-matrix model of symmetry breaking is correct it is equivalent to the old tadpole model, and so no really independent estimate of $(m_d - m_u)/2m_s$ is obtained from the baryons. However, we get an idea of the uncertainties in our calculations by seeing if the baryon mass spectrum gives a consistent answer for the ratio $(m_d - m_u)/m_s$. From (2.36) we find

$$\frac{m_d - m_u}{2m_s} = 0.0105 \pm 0.003 \qquad (2.38)$$

with an allowed 30% uncertainty since the $F/D$ ratio comparison is good only to 18% and SU(3) is good to only ≈15%. This result is not inconsistent with our analysis of the mesons which gave $(m_d - m_u)/2m_s = 0.0175$.

Allowing for experimental and calculational uncertainties, we conclude from our analysis of the mesons and baryons that

$$\frac{m_d - m_u}{2m_s} = 0.014 \pm 0.003 , \quad \frac{m_d + m_u}{2m_s} = 0.031 \pm 0.007 . \qquad (2.39)$$

From this we obtain

$$\frac{m_u}{m_d} = 0.38 \pm 0.13 , \quad \frac{m_d}{m_s} = 0.045 \pm 0.011 , \qquad (2.40)$$

as our estimate of the quark mass ratios.

To improve this analysis, we suggest that new estimates of the photonic contributions to kaon and baryon isotopic splittings be undertaken to check the calculations done over ten years ago.[17,24] Although chiral perturbation theory corrections to the pseudoscalar mesons have been done and provide important ≈20% corrections to the symmetric estimates, a similar calculation taking into account second-order SU(3) breaking in the baryon spectrum has not been done. This might improve the agreement between the meson and baryon estimate of $(m_d - m_u)/2m_s$.

### D. The decay $\eta \to 3\pi$

The purely photonic contributions to the $\eta \to 3\pi$ width vanishes in the chiral SU(2) × SU(2) limit (Sutherland's theorem[26]) and for broken SU(2) × SU(2) has been estimated[27] to be two orders of magnitude smaller than the experimental width. The decay can, however, be induced by the isospin-violating quark mass terms (which also induce $\pi^0$-$\eta$ mixing), and these are required in any case as counterterms to the usual electromagnetic interaction in QCD.[28]

The amplitude[29] for the $\eta \to \pi^+\pi^-\pi^0$ decay is parametrized as

$$T_{+-0} = A + BE_0 . \qquad (2.41)$$

From the central value of the decay width[30] $\Gamma_{+-0} \approx 203$ eV and various experimental values for $B/A$ one finds

$$0.57 \leq |A| \leq 0.70 . \qquad (2.42)$$

Assuming the decay is due to the term $(m_u^0 - m_d^0)(\bar{u}u - \bar{d}d)$, and using current algebra in the SU(2) × SU(2) limit, one finds $B/A = -2/\mu_\eta$ in good agreement with the observed slope $B/A \approx -2.1/\mu_\eta$. Using current algebra in the SU(3) × SU(3) limit, one obtain[31]

$$A=\left(\frac{m_u-m_d}{2m_s}\right)\frac{8\mu_K^2}{3\sqrt{3}F_\pi^2}. \qquad (2.43)$$

Assuming the sign of $A$ is negative, we obtain from the experimental range (2.42) the result

$$\frac{m_d-m_u}{2m_s}=0.0145\pm0.0015, \qquad (2.44)$$

in good agreement with our previous estimate (2.39) based on the meson and baryon spectrum.

The range represented in (2.44) only takes into account the experimental range (2.42) and not uncertainties associated with SU(3)×SU(3) symmetry breaking. The effects of symmetry breaking in this amplitude can be considerable since extrapolations on the order of $\mu_\eta^2$ are required. In a previous publication[29] we have found a chiral-symmetry correction factor ≈34% of $O(\mu_K^2\ln\mu_K^2)$ in the $\eta$ decay amplitude and a 20% correction in the relation between $m_s$ and $\mu_K^2$ suggesting that corrections of $O(\mu_K^2)$ can be large also. So it is not inconceivable that symmetry-breaking effects could dramatically alter the estimate (2.44) which is consistent with the meson and baryon spectrum. Another analysis of symmetry breaking in $\eta\to3\pi$ based on extended PCAC has been done by Dominguez and Zepeda.[32] They find that quark mass ratios so obtained are compatible with those obtained from the meson and baryon spectrum and have results consistent with ours.

We conclude that the estimate of quark mass ratios from the $\eta\to3\pi$ decay is not inconsistent with that obtained from the meson and baryon spectrum, but that the estimate is sensitive to potentially large symmetry-breaking corrections which are difficult to take into account reliably.

## III. CONCLUSIONS

The advent of QCD as a theory of the strong interactions has placed the problem of strong-interaction symmetry breaking on a rational basis. Current-quark mass ratios are renormalization-group invariants up to small and controllable terms coming from QFD interactions. Further, we see that the old wisdom of subtracting just the lowest-lying Born and resonance (nonscaling) contributions to the Cottingham formula from the observed isotopic mass difference and identifying the remainder as the tadpole can be justified using the renormalization group.

To actually extract the quark mass ratios from experimental data we have used (i) the meson spectrum, (ii) the baryon spectrum, (iii) the $\eta\to3\pi$ decay. Beyond this, one must make theoretical assumptions. The main theoretical assumptions are that (i) the low-lying states in the Cottingham formula contributing to $(\Delta M)^{\rm em}$ have been correctly estimated, (ii) chiral perturbation theory—equivalently kaon PCAC—correctly estimates the leading-order corrections to the symmetric results. These assumptions, we emphasize, lead to a completely consistent and coherent account of all hadronic symmetry breaking.

We have done a careful analysis of the ground-state pseudoscalar mass spectrum taking into account all leading-order chiral corrections to the quark mass ratios $(m_d+m_u)/2m_s$ and $(m_d-m_u)/2m_s$. The baryon mass spectrum and $\eta\to3\pi$ yielded values for these ratios consistent with those from the meson spectrum and we concluded

$$\frac{m_u}{m_d}=0.38\pm0.13,$$

$$\frac{m_d}{m_s}=0.045\pm0.011.$$

These ratios are consistent with those recently obtained by Weinberg,[33] Dominguez, and Zepeda,[32] and earlier by Gasser and Leutwyler,[12] and inconsistent with those estimates which obtain a zero-mass up quark.

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$$\epsilon_3=[(\tfrac{2}{3})^{1/2}\epsilon_0-(2/\sqrt{3})\epsilon_8](m_u-m_d)/2m_s,$$

$\epsilon_0=-(\frac{2}{3})^{1/2}F_K\mu_K^2$, $\epsilon_8/\epsilon_0=-\sqrt{2}$, valid to lowest order in SU(3) × SU(3) breaking.
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