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The 3 Gluon Decay of Quarkonium

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Abstract. We discuss the process

$e^+e^- \to Q\bar{Q} \to 3$ gluons $\to 3$ jets

with attention to the kinematics and observability of the jets. We also show how to check the gluon spin through jet or hadron angular distributions. Gluon flavor can be checked by looking for quantum number correlations between opposite jets. We predict that correlations exist off resonance for $e^+e^- \to q\bar{q}$, but not on resonance for $Q\bar{Q} \to 3g$.

1. Introduction

Quantum chromodynamics (QCD) is the local color gauge theory of the strong interactions [1]. It has two sorts of elementary colored quanta. These are spin 1/2 quarks with mass and flavor and vector gluons without mass and flavor. These quanta are bound in colorless physical hadrons. Such is the dogma. While there is evidence for confined quarks, the same cannot be said for gluons. We need to know if gluons really exist, have spin 1 and have no mass and flavor. The quarkonium decay [2, 3]

$$e^+e^- \to Q\bar{Q} \to 3 \text{ gluons} \to 3 \text{ jets} \tag{1}$$

is a good way to find this out. (Q is a heavy quark and $Q\bar{Q}$ the lowest ${}^3S_1$ bound state.)

The $Y(9.46)$ resonance found in pN collisions [4] has now been seen in e^+e^- collisions at DORIS [5]. Its properties are consistent with quarkonium expectations if $|e_Q|=1/3$ [5]. Off resonance there is clear evidence for $e^+e^- \to 2$ jet structure. The mean sphericity is small. On the Y resonance events are not two-jet like. The mean sphericity is large [6]. This is qualitatively what is expected for the 3 jet decay (1). However no dramatic evidence for three jets is seen (with rather limited statistics) [6]. We have been motivated by the Y discovery to work out the present $Q\bar{Q} \to 3g$ guide for experimentalists.

We expect 3 jets to be found at Y or the next heavier resonance. Once this happens, process (1) becomes a laboratory to study QCD. This paper contains Born approximation [7] phenomenology for (1) (see Fig. 1). Once it is clear that our lowest order predictions describe quarkonium decay in broad outline, it will be interesting to look for corrections. These should be small, $O(\alpha_S/\pi)$, except possibly near the boundary of the $3g$ phase space in (1).

We do not think that the lowest order $Q\bar{Q} \to 3g$ decay mechanism will be disturbed dramatically by the emission of soft gluons. (These could in principle have large couplings, unlike hard gluons whose coupling to quarks is $\sim g_S = \sqrt{4\pi\alpha_S(Q^2)}$, $\alpha_S/\pi \lesssim 0.1$, at distances $\sim M_Q^{-1}$.) This is because a quarkonium state is small and colorless. For large Q mass the $Q\bar{Q}$ S state radius approaches the chromodynamic Bohr radius, $r_0 = (\frac{2}{3}\alpha_S M_Q)^{-1}$. Since color cannot be smeared over radii much larger than r_0, there is no virtual emission and absorption of long wavelength gluons. We thus expect no important soft gluon contribution to the $Q\bar{Q} \to 3g$ annihilation amplitude. (The initial state is a colorless $Q\bar{Q}$ and not $Q\bar{Q}$ + soft gluon.) However this intuitive argument suggests that a gluon of wavelength near r_0 in (1) could either come from the initial state or from the decay. The two cannot really be separated so far as we can see. This is what we meant in saying that the Born approximation may be corrected by $>\alpha_S/\pi$ in some regions of phase space [in this case for $p(\text{gluon}) \lesssim O(1/r_0)$]. A caveat also applies to the region where two of the three gluons in (1) are separated by small transverse momenta. They interact, changing distributions compared to the lowest order expectations. (Note that we expect these corrections near the boundary of the $3g$ Dalitz plot. This is where momenta become parallel or small.)

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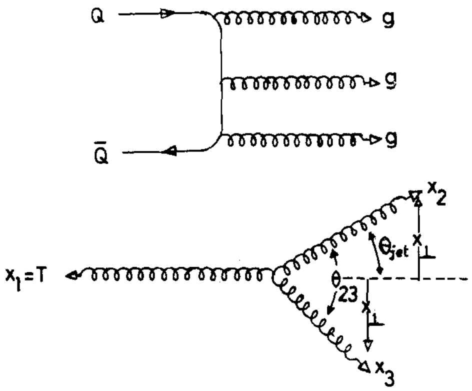

Fig. 1. $Q\bar{Q}\to 3g$ annihilation and the $3g$ kinematics

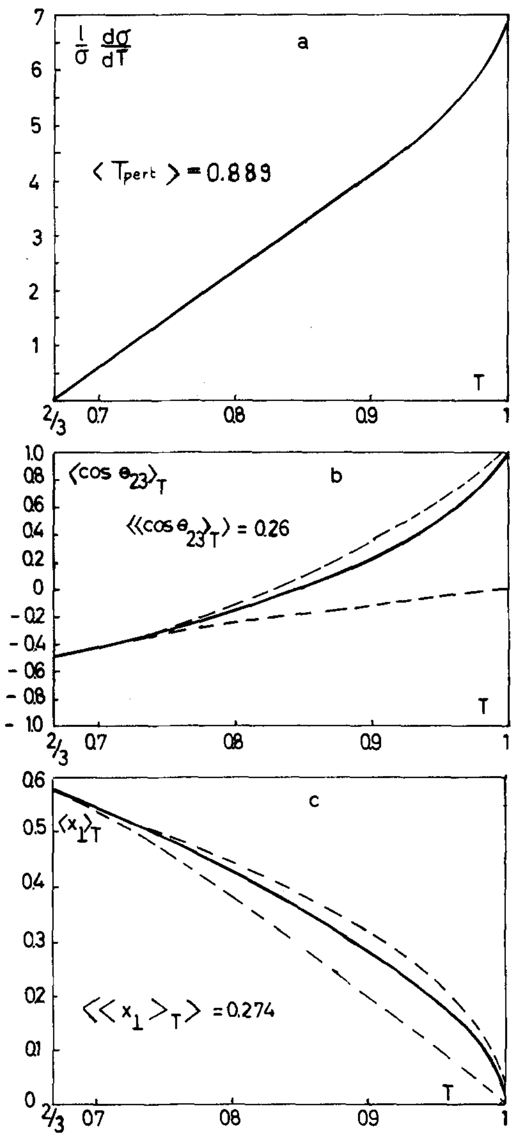

Fig. 2. **a** The normalized distribution of $3g$ events versus thrust, as described in the text. **b** The mean $\langle \cos\theta_{23} \rangle_T$ of the two less energetic gluons as a function of thrust. Dashed lines show the kinematic limits. **c** The mean $\langle x \rangle_T$ in Fig. 1 as a function of thrust. Dashed lines show the kinematic limits

We also believe in gluon jets of bounded $p_\perp$ (up to corrections of order α_S/π). A simple minded argument follows. Imagine a single colored quantum (quark or gluon) with large momentum. This quantum has an associated color current. The probability for such a current to create a hadron with very large $p_\perp^{had}$ (relative to the current direction) is negligible. This is because a hadrons' color is compensated over a distance $R \sim O(m_\pi^{-1})$ in space or $\langle p_\perp \rangle \sim 1/R$ in transverse momentum. If $p_\perp^{had} \gg \langle p_\perp \rangle \sim 1/R$ then the color current flux will not overlap the region in momentum space where the color density of the high $p_\perp$ hadron is nonzero. No high $p_\perp$ hadrons will emerge. A hadron with large $p_\perp$ *can* be produced, but only if the original colored quantum does not remain intact but fragments (e.g. $q\to gq$), giving a gluon or quark which itself has large $p_\perp$ relative to the original quantum's momentum. One of these quanta can have a flux of color overlapping the wavefunction of a high $p_\perp$ hadron (if $\mathbf{p}^{had}$ is parallel to $\mathbf{p}^q$ or $\mathbf{p}^g$). In QCD this is a multijet process and is suppressed by the small QCD coupling $\alpha_S(p_\perp)/\pi$.

From this discussion it is clear that we consider any broadening of a jet distribution as due to the nascent birth of a new jet (or fission of an old one)*. We see no reason why this argument does not apply to the hard hadrons in (1), which then lie in 3 jets. It is less clear how low momentum hadrons are made in a multijet process like (1)**. We will ignore ≥ 4 jet processes in $^3S_1$ quarkonium decay since they are $O(\alpha_S/\pi)$ of the lowest order rate.

In our discussion of $Q\bar{Q}\to 3g$, it will be useful to have a global variable describing the hadron final state and calculable in QCD. The most useful variable seems to be "thrust" [10, 11] (other variables appear less useful [12]). For $Q\bar{Q}\to 3g$ the definition of thrust for the 3 jets and its perturbative value are [11]

$$T_{had} = \max \frac{\sum |\mathbf{p}_{||}^i|}{\sum |\mathbf{p}^i|}$$
$$T_{pert} \max(x_1, x_2, x_3). \tag{2}$$

Where $\mathbf{p}_{||}^i$ is defined as the projection of $\mathbf{p}^i$ of hadron i on the thrust axis, and the scaled gluon momenta in (1) are

$$\mathbf{x}_n = 2\mathbf{p}_n^{gluon}/M_{QQ}, \qquad x_n = |\mathbf{x}^n|.$$

Note that the value of T is x_1 if we order $x_1 \geq x_2$ or x_3, and the T axis is parallel to $\mathbf{x}_1$. In the limit where all hadrons in a jet are parallel, $T_{had} = T_{pert}$. At low energies T_{had} is smeared by finite $p_\perp$ in the jets***. The

* We view recent QCD calculations as suggesting that there is no important perturbative jet broadening apart from the hard multijet processes [8]

** For this reason it is important to look at simpler processes like [9] $Q\bar{Q}(2^3S_1)\to\gamma + Q\bar{Q}(^3P_l)\to\gamma + 2$ gluon jets

*** Compare footnote ** on p. 4

exact smearing depends on the quarkonium mass, on the jet multiplicity and on the question whether all particles or only charged particles are seen. We leave this smearing to the reader and present here distributions in $T=T_{\text{pert}}$. In our discussion of single particle distributions we will, however, include $p_\perp$ smearing.

We first discuss $Q\bar{Q}\to 3g$ kinematics and the resolvability of jets, turning then to angular distributions as a test of gluon spin. Finally we show how quantum number correlations can check that gluons have no flavour.

II. Jet Kinematics; Resolvability of Jets

To judge the resolvability of 3 jets in $^3S_1$ or the quarkonium decay we need to know the distribution of the 3 quanta and also the angular size of a gluon jet. The former question can be answered by calculation of the perturbation diagram in Fig. 1. The latter question cannot yet be answered theoretically.

Kinematics and notation are shown on Fig. 1. We order the scaled gluon energies by $x_1 \geqq x_2 \geqq x_3$ (sometimes for convenience we also allow $x_1 \geqq x_3 \geqq x_2$). Note that $x_1+x_2+x_3=2$. Thus $T_{\text{pert}}=x_1$ for the $3g$ state. It is easy to calculate the perturbative thrust distribution. x_1, x_2, x_3 label one of six sectors of a Dalitz plot. The density on this Dalitz plot is

$$W(x_1,x_2,x_3) = \frac{x_1^2(1-x_1)^2+x_2^2(1-x_2)^2+x_3^2(1-x_3)^2}{x_1^2x_2^2x_3^2} \tag{3}$$

and we integrate over x_2, x_3 at fixed $x_1=T$ to get [11]

$$\begin{aligned}\frac{1}{\sigma}\frac{d\sigma}{dT} &= \frac{3}{\pi^2-9}\int_{2(1-T)}^{T} dx_2\, W(T,x_2)\\ &= \frac{3}{\pi^2-9}\left[2\frac{(3T-2)(2-T^2)}{T^3(2-T)^2}+\frac{4(1-T)}{T^2(2-T)^3}\right.\\ &\quad \left.\cdot(5T^2-12T+8)\ln\frac{2-2T}{T}\right]\end{aligned} \tag{4}$$

shown on Fig. 2a.

It is instructive to calculate the opening angle between gluons 2 and 3 in the hemisphere opposite $\mathbf{x}_1$. We will also calculate the momentum of gluons 2 and 3 transverse to the perturbative thrust axis $\mathbf{x}_1$. Define the thrust of the second most energetic quantum by $T_2=x_2$ and the opening angle of 2 and 3 by $\cos\theta_{23}(T_2)$, where for $x_2 \geqq x_3$,

$$\cos\theta_{23}(T_2)=1-2\frac{1-T}{T_2(2-T-T_2)}. \tag{5}$$

The range of (5) is fixed by $2(1-T)\leqq T_2 \leqq T$.

Its average as a function of T, $\langle\cos\theta_{23}\rangle_T$, is given by

$$\langle\cos\theta_{23}\rangle_T = 1+\frac{\int_{2(1-T)}^{T} dx_2\, W(T,x_2)\frac{2(1-T)}{x_2(x_2+T-2)}}{\int_{2(1-T)}^{T} dx_2\, W(T,x_2)} \tag{6}$$

where we integrate over $2(1-T)\leqq x_2 \leqq T$, including $x_2 \leqq x_3$ ($T_2=x_3$) and $x_2 \geqq x_3$ ($T_2=x_2$).

The result for $\langle\cos\theta_{23}\rangle_T$ is shown on Fig. 2b.

In the same way we calculate $\langle x_\perp\rangle$ in Fig. 1 by noting that

$$x_\perp \equiv x_2\sin\theta_{\text{jet}} = \frac{2}{x_1}[(1-x_1)(1-x_2)(1-x_3)]^{1/2} \tag{7}$$

and weighting as for $\langle\cos\theta_{23}\rangle_T$. This is shown in Fig. 2c.

From Fig. 2 we see that the "collinear" configuration $T\approx 1$, $\cos\theta_{23}=1$ is the most probable, and that the "star" configuration $T=2/3$ ($\theta_{12}=\theta_{23}=\theta_{31}=120°$) is the least probable. However, it is important to realize that many events have a large opening angle between gluons 2 and 3. For $T\leqq 0.85$ (about 30% of all events),

$$\left.\begin{aligned}\langle\theta_{23}\rangle &\geqq 90°\\ \langle x_\perp\rangle &\geqq 0.35\end{aligned}\right\}\ T\leqq 0.85. \tag{8}$$

Note that $\langle p_\perp^{\text{jet}}\rangle=\langle x_\perp\rangle\cdot M_{Q\bar{Q}}/2$ (measured in the $3g$ plane).

It is clear that as M_Q increases a larger and larger fraction of all events show 3 recognizible jets. For low quarkonium mass the fraction of $Q\bar{Q}\to 3g$ events with 3 clear jets will be small. This is because jets with small opening angle will not be resolved due to the finite $p_\perp$ of the jet fragments. In order to estimate the fraction of resolved 3 jet events at a resonance like $\Upsilon(9.46)$ we need to know the opening angle of a gluon jet. A simple criterion for resolvability can be adapted from optics. Two jets nearby in angle are at the border of resolvability if half the energy of one jet lies inside a cone of half angle δ^{jet} (it depends on the jet energy of course) and the angle between the two jets is at least $\delta_1^{\text{jet}}+\delta_2^{\text{jet}}$ (the two half angles). Clearly resolved jets require $\theta_{23} \gg \delta_1^{\text{jet}}+\delta_2^{\text{jet}}$*. (Our criterion is optimistic in that it ignores fluctuations in θ.)

We will assume that for 2.5–3 GeV jets $\delta^{\text{jet}}\sim 30°$ for both quarks and gluons. Then for the Υ "star" configuration, $\theta_{ij}\sim 120°$, each jet axis is $\sim 4\delta^{\text{jet}}$ from another. Thus a subset of $\Upsilon(9.46)$ decays around $T=2/3$ should show 3 jets. Note that the average

* We urge experimentalists to plot the fraction of hadronic energy outside a cone of variable angle δ around the thrust axis. This fixes $\delta_{\text{quark}}^{\text{jet}}$ from $e^+e^-\to q\bar{q}$. $\delta_{\text{gluon}}^{\text{jet}}$ depends on $\langle n\rangle_g^{\text{jet}}$ (the number of particles in a gluon jet) and on $\langle p_\perp\rangle_g$ in the jet. $\langle p_\perp\rangle_g$ can be found by fitting a plane through $Q\bar{Q}\to 3g$ events. The mean momentum perpendicular to the plane determines $\langle p_\perp\rangle_g$. Data (5) indicates that $\langle n\rangle_g^{\text{jet}}$ is not very much larger than $\langle n\rangle_q^{\text{jet}}$ for $\Upsilon(9.46)$.

$\theta_{23} = 75° \sim 2\delta^{\text{jet}}$ (Fig. 2), so that an "average" event at Y will not show a resolved 3 jet structure**. At a resonance with at least twice the Y mass δ^{jet} will be smaller (because $\delta^{\text{jet}} \sim \langle n \rangle^{\text{jet}}/p_{\text{jet}}$) so that even an average event will show 3 jets.

We emphasize that it is important to look for the energy pattern of jets rather than at the event as a whole. This is because low momentum particles are poorly correlated with jet axes. Weighting particle tracks with their energies will make jet structure clearer.

It may prove useful to study nearly planar events with $T = 2/3$ (stars), as these show the most dramatic jet structure. A cut selecting low multiplicity ($1 < n \leqq 3$ particles per jet) should also be effective in producing a clean jet structure. This is because mean momenta are high and the correlation of particles with the jet axes is better than for large n_{had}.

From this discussion, we do not expect obvious 3 jet structure is the average $Y \to 3g$ decay. Can anything be done to distinguish Y decays from 2 jet or phase space structure? We have already mentioned the search for "star" events. We give two further examples:

(i) Two particle correlations. Suppose we look at events where the two leading particles have momenta satisfying $p_1 \geqq p_2 \geqq p_{\min}$. Take the larger momentum as z axis and plot the angular distribution of p_2 for different values of the thrust,

$$\frac{1}{\sigma}\frac{d\sigma(T)}{d\cos\vartheta_{12}} \tag{9}$$

where T characterizes the event as a whole, and $\cos\vartheta_{12} = (\mathbf{p}_1 \cdot \mathbf{p}_2/|\mathbf{p}_1| \cdot |\mathbf{p}_2|)$. Provided $p_{\min} \gg \langle p_\perp \rangle \sim 300$ MeV this distribution will show striking changes with T. For large T (2 jets) it peaks at $\cos\vartheta_{12} = +1$ and $\cos\vartheta_{12} = -1$. For $T = 2/3$ (star events) (9) will peak at $\cos\vartheta_{12} = +1$ and $\cos\vartheta_{12} = 1/2$; (9) now has a minimum at $\cos\vartheta_{12} = -1$ between the two jets opposite $\mathbf{p}_1$. The width of the peaks in ϑ_{12} is of order $\Delta\vartheta \approx \langle p_\perp \rangle / p_2$ and decreases rapidly as p_2 increases. We do not expect this behavior from two jets off resonance nor do we expect it from a phase space model.

(ii) Front-back asymmetry. Suppose we divide each event into 2 hemispheres by a plane perpendicular to the jet or thrust axis. Gluon 1 defines the thrust axis by $x_1 = T$ and gives fragments with mean transverse momentum $\langle p_\perp \rangle_g$ relative to the thrust axis. The mean $p_\perp$ of hadrons in the opposite hemisphere will be larger. This is because we must now add the $p_\perp$ of gluons 2 and 3 (recall $x_1 \geqq x_2 \geqq x_3$) to the $p_\perp$ of their fragments. A bit of spherical trigonometry shows that we must use $\langle p_\perp^2 \rangle$. Defining $\langle p_\perp^2 \rangle_g$ as the mean $p_\perp^2$ in a gluon jet and $\langle p_\perp^2 \rangle_{\text{back}}$ as the average $\langle p_\perp^2 \rangle$ in the hemisphere opposite gluon 1 we have

$$\langle p_\perp^2 \rangle_{\text{front}} = \langle p_\perp^2 \rangle_g \quad \text{jet} \parallel \text{to } \mathbf{x}_1$$

$$\begin{aligned} \langle p_\perp^2 \rangle_{\text{back}} &= \langle p_\perp^2 \rangle_g + \langle p^2 \sin^2\theta_{\text{jet}} - \tfrac{3}{2} p_{\perp g}^2 \sin^2\theta_{\text{jet}} \rangle \\ &\approx \langle p_\perp^2 \rangle_g + \langle \sin^2\theta_{\text{jet}} \rangle [\langle p^2 \rangle - \tfrac{3}{2}\langle p_\perp^2 \rangle_g] \end{aligned} \tag{10}$$

where $\langle p^2 \rangle$ is the average particle momentum squared in a jet, and $\langle \sin^2\theta_{\text{jet}} \rangle$ is the average angle of jet 2 or 3 relative to the thrust axis. Note that $\langle p_\perp^2 \rangle_{\text{front}}$ is constant while $\langle p_\perp^2 \rangle_{\text{back}}$ decreases with T. We estimate for Y (9.46) that $\langle p^2 \rangle \approx 0.8\,\text{GeV}^2$, $\langle p_\perp^2 \rangle_{\text{front}} = \langle p_\perp^2 \rangle = 0.2\,\text{GeV}^2$, and $\langle \sin^2\theta_{\text{jet}} \rangle = 0.38$*, giving

$$\langle p_\perp^2 \rangle_{\text{front}} \approx 0.2\,GeV^2, \quad \langle p_\perp^2 \rangle_{\text{back}} \approx 0.4\,\text{GeV}^2.$$

It might be worth pointing out that the overall $\langle p_\perp^2 \rangle$ relative to the thrust axis is roughly

$$\langle p_\perp^2 \rangle_{\text{tot}} \approx \langle p_\perp^2 \rangle_g + \tfrac{2}{3}\langle \sin^2\theta_{\text{jet}} \rangle [\langle p^2 \rangle - \tfrac{3}{2}\langle p_\perp^2 \rangle_g] \tag{11}$$

where we weighted with the expected back to front 2:1 ratio of particles. Since (11) is easily measured, it can give a first indication of $\langle p_\perp^2 \rangle_g$ relative to $\langle p_\perp^2 \rangle_q$. This is done by comparing (11) for $Y \to 3g$ decays and off resonance for $e^+e^- \to q\bar{q}$. (11) indicates that they should not differ much if

$$\langle p_\perp^2 \rangle_g \approx \langle p_\perp^2 \rangle_q.$$

It is also possible to look for a difference in the two jet hemispheres by defining the thrust in the front and back hemispheres,

$$T_{\text{front}} = \max \frac{\tilde{\Sigma} |p_{\|}^i|}{\tilde{\Sigma} |\mathbf{p}^i|} \tag{12}$$

where the sum in numerator and denumerator is over particles in one hemisphere. T_{front} chooses the hemisphere with the larger sum of parallel momenta. T_{back} is defined the same way using particles in the hemisphere opposite that which maximizes (12). In perturbation theory $T = x_1$ and $T_{\text{front}} = 1$ whereas $T_{\text{back}} = (x_2 \cos\theta_{12} + x_3 \cos\theta_{13})/(x_2 + x_3) < 1$.

Observation of an asymmetry of the kind we have discussed here is a signal for unresolved 3 jet structure. It is probably easiest to compare $\langle p_\perp^2 \rangle_{\text{front}}$, $\langle p_\perp^2 \rangle_{\text{back}}$, and T_{front}, T_{back} for $Q\bar{Q} \to 3g$ and off resonance for $e^+e^- \to q\bar{q}$. Unfortunately our estimates for $Y(9.46)$ do not indicate that a large effect is to be expected.

The examples we have given are not meant to be exhaustive. They indicate a methodology for checking gauge theory predictions for $Q\bar{Q}$ decay. Many other ways can be imagined of looking for multi jet structure [11, 12].

** We can exploit this estimate of δ^{jet} to estimate the mean $\langle T_{\text{had}} \rangle$ for $Y \to 3g \to$ hadrons including the smearing of the gluon jets. We find $\langle T_{\text{had}} \rangle \cong \langle T_{\text{pert}} \rangle \cos\delta \cong 0.77$

* $\langle \sin^2\theta_{\text{jet}} \rangle$, is computed in the same way as $\langle x_\perp \rangle$ [*Eq.* (7)].

III. Angular Distributions; Quantum Numbers

In this section we take up gluon jet angular distributions in

$$e^+e^- \to Q\bar{Q} \to g(\mathbf{x}_1)+g(\mathbf{x}_2)+g(\mathbf{x}_2) \tag{13}$$

This will provide evidence that gluons are massless vector particles. We also comment on quantum number correlations as a test for gluon flavor. Our calculations are, as before, in Born approximation.
The cross section for (13) is [3]

$$\begin{aligned}\frac{1}{\sigma}\frac{d\sigma}{dx_1 dx_2 d\cos\theta d\chi} = \frac{27}{32\pi(\pi^2-9)}\{&(1+\cos^2\theta)\sigma_U(x_1,x_2)\\ &+2\sin^2\theta\sigma_L(x_1,x_2)\\ &+2\sin^2\theta\cos 2\chi\sigma_T(x_1,x_2)\\ &-4\sqrt{2}\cos\theta\sin\theta\cos\chi\sigma_I(x_1,x_2)\}\end{aligned} \tag{14}$$

The notation is as follows. We choose x_1 to be the most energetic gluon (jet); θ is the polar angle of $\mathbf{x}_1 = 2\mathbf{p}_1/M_{Q\bar{Q}}$ and χ is an azimuthal angle measured around $\mathbf{x}_1$ as axis; it is the angle between the plane of $\mathbf{x}_1$ abd $\mathbf{x}_2$ and the plane containing $\mathbf{x}_1$ and the e^+e^- beam in (13). We have zero beam polarization, so there is no azimuthal dependence of $\mathbf{x}_1$ around the e^+e^- beam axis.
We now identify $x_1 = T$ as before and integrate over x_2, x_3 to compute the angular distribution of the thrust axis and the azimuthal distribution of the $3g$ plane about the thrust axis from [3].

$$\begin{aligned}\sigma_U(T) = \int_{2-2T}^{T} dx_2 \sigma_U(T,x_2) = &\frac{8}{3}\frac{(3T-2)}{T^5(2-T)^2}\\ &\cdot[2+4T-11T^2+9T^3-3T^4]\\ &+\frac{32}{3}\frac{(T-1)}{T^4(2-T)^3}\cdot[-6+12T-13T^2+9T^3\\ &-3T^4]\,ln\frac{2-2T}{T}\\ \sigma_L(T) = \int_{2-2T}^{T} dx_2 \sigma_L(T,x_2) = &\frac{8(3T-2)(T-1)}{3T^5(2-T)^2}\\ &\cdot[2+6T-7T^2+2T^3]\\ &+\frac{16}{3}\frac{(T-1)}{T^4(2-T)^3}[12-24T+18T^2-6T^3+T^4]\\ &\cdot\ln\frac{2-2T}{T}\\ \sigma_T(T) = 2\sigma_L(T)\,; \quad \sigma_I(T) = 0\,.\end{aligned} \tag{15}$$

The meaning of σ_U and σ_L is clear. σ_T describes the tendency of the 3 gluon plane to lie near the plane defined by the e^+e^- axis and the thrust axis (because $\sigma_T > 0$). We find $\sigma_I(T)=0$ because we integrated over $x_2 \geqq x_3$ and $x_2 \leqq x_3$. $\sigma_I(x_1, x_2, x_3)$ is antisymmetric under the interchange $x_2 \rightleftharpoons x_3$ [3]. Had we ordered $x_1 \geqq x_2 \geqq x_3$, identifying x_2 as the second most energetic jet (or the second largest thrust T_2 as in Sect. I) we would find just the $\sigma_I(x_1, x_2)$ of [3]*. σ_I then describes the tendency of the second most energetic jet to lie on one side or the other of $\mathbf{x}_1$ relative to the beam axis**.
On Fig. 3 we have plotted the perturbative thrust axis angular distribution. We define

$$\frac{d\sigma(T)}{d\cos\theta} \sim 1+\alpha(T)\cos^2\theta$$

$$a(T) = \frac{\sigma_U(T)-2\sigma_L(T)}{\sigma_U(T)+2\sigma_L(T)}. \tag{16}$$

Because of its possible experimental interest we have also calculated a somewhat different quantity,

$$\langle\alpha\rangle_{T_{\min}} = \frac{\int_{T_{\min}}^{1} dT(\sigma_U(T)-2\sigma_L(T))}{\int_{T_{\min}}^{1} dT(\sigma_U(T)+2\sigma_L(T))} \tag{17}$$

which is useful if there is a cut on data with $T \geqq T_{\min}$. $\langle\alpha\rangle_{T_{\min}}$ is then the angular distribution of the thrust axis for all events with $T \geqq T_{\min}$. ($\sigma(T \geqq T_{\min})/\sigma$ is also plotted in Fig. 3).
Note that $\alpha(T=1)=1$. This is because gluons are massless vector particles. For $T=1$ all gluon momenta are collinear and helicity requires $\alpha=1$. This gives an easy way to exclude spinless gluons, for which $\alpha(1) = -1$ by helicity.
We note in passing that the average $\langle\alpha\rangle = \langle\alpha\rangle_{T_{\min}=2/3} = 0.39$ (Fig. 3c) is not affected by $p_\perp$ smearing, but only by the uncertainty with which the thrust axis direction is determined. $d\sigma/dT$ is affected by $p_\perp$ smearing (it even vanishes at $T \to 1$ due to this).
An interesting quantity in perturbation theory is the angular energy pattern [13, 14]. We consider the total energy deposited by gluons in the polar angle interval from θ_g to $\theta_g + d\theta_g$ relative to the e^+e^- beam

$$\frac{dE}{d\cos\theta_g} \sim (1+\alpha_E\cos^2\theta_g) \tag{18}$$

α_E is a number, calculated as follows. First we find the inclusive energy and angle distribution of a gluon,

* With the ordering $x_1 \geqq x_2 \geqq x_3$ the σ's defined in (14) are to be taken over 1/6 of the $3g$ Dalitz plot. In [3] we allowed for all orderings; The transcription to $x_1 \geqq x_2 \geqq x_3$ is trivial.
** Integrating *this* over $x_2 \geqq x_3$ will lead to a nonvanishing $\sigma_I(T)$. Of course, the half plane containing the second most energetic jet has to be identified experimentally in order to use this.

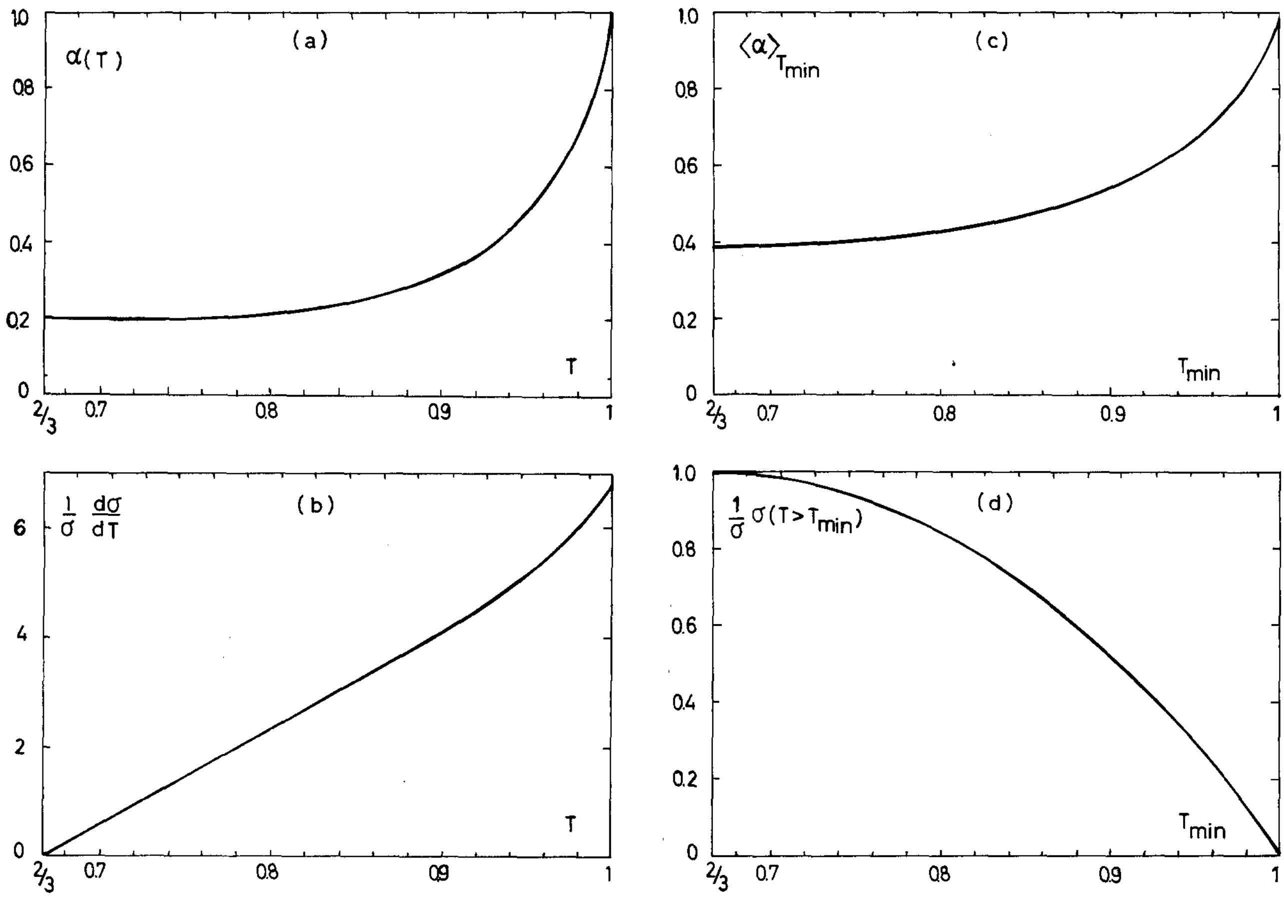

Fig. 3. a The angular distribution parameter $\alpha(T)$, defined in Eq. (16) of the text. **b** The distribution of $3g$ events versus thrust. **c** The parameter $\langle\alpha\rangle_{T_{min}}$, describing the polar angle distribution of the thrust axis for event with $T \geqq T_{min}$. **d** The fraction of all $3g$ events having $T \geqq T$

$x = 2p_g/M_{Q\bar{Q}}$*

$$\frac{1}{\sigma}\frac{d\sigma}{dx\cos\theta_g} = \frac{3}{4(\pi^2-9)}[\sigma_0(x)+\sigma_1(x)\cos^2\theta_g].$$

This was done in [3]. The result is

$$\sigma_0(x) = F(x) + 2G(x)$$

$$\sigma_1(x) = F(x) - 6G(x)$$

$$F(x) = \frac{x(1-x)}{(2-x)^2} + \frac{2-x}{x} + 2\frac{(1-x)^2}{(2-x)^3}\ln\frac{1}{1-x} - 2\frac{1-x}{x^2}\ln\frac{1}{1-x}.$$

$$G(x) = \frac{1-x}{x^4}\left\{\frac{2x(1-x)}{(2-x)^2} - 2x - \frac{4(1-x)}{(2-x)^3}\ln\frac{1}{1-x} + 2\frac{4-3x}{(2-x)^2}\ln\frac{1}{1-x} - x\ln\frac{1}{1-x}\right\}.$$

* Notice that we no longer require that x be the most energetic gluon. As a result, σ_0 and σ_1 are unrelated to $\sigma_U(T)$, $\sigma_L(T)$.

[In Fig. 4 we show $\sigma_0(x)$ and $\sigma_1(x)$.] The quantity α_E is just the ratio of the energy weighted integrals of the coefficients of $\cos^2\theta_g$ and unity

$$\alpha_E = \frac{\int_0^1 dx\, x\sigma_1(x)}{\int_0^1 dx\, x\sigma_0(x)} = 0.35. \tag{19}$$

Note that α_E does not take the $p_\perp$ smearing of gluon jets into account. Measurements of the polar angle dependence of the energy deposition may be a useful way to test the $3g$ decay mechanism of quarkonium. So far we have calculated quantities in perturbation theory, unsmeared by the finite $p_\perp$ of gluon jet fragments. We will not expatiate here on the details of this smearing. It depends on the details of gluon jets and on the quarkonium mass. It turns out that we can calculate a quantity which depends in a minimal way on these details. This is the angular distribution of a single detected hadron in the inclusive process

$$e^+e^- \to Q\bar{Q} \to 3g \to h(\mathbf{p}) + \ldots \tag{20}$$

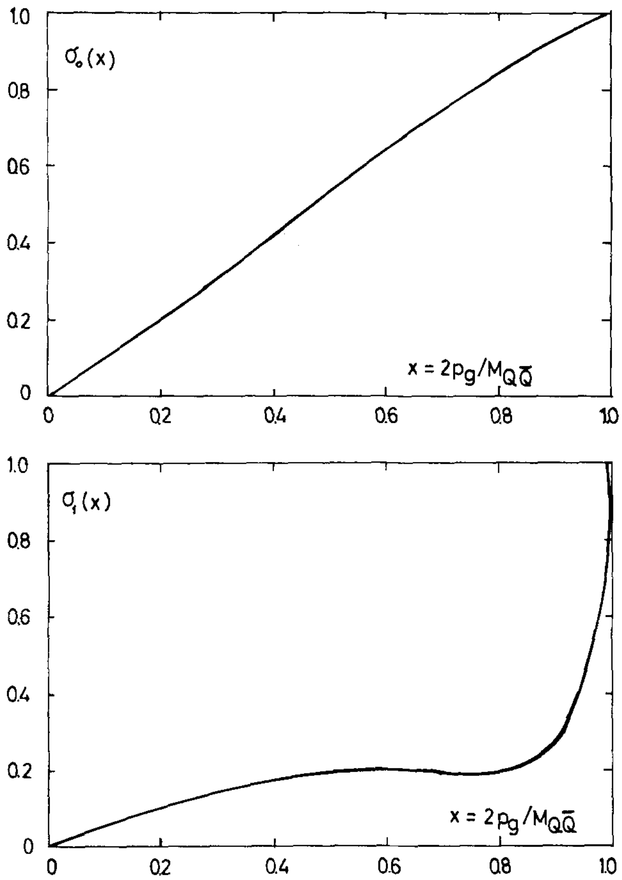

Fig. 4. The energy distribution functions of a single gluon in $Q\bar{Q}\to 3g$. The joint energy and angular distribution is proportional to $\sigma_0(x)+\sigma_1(x)\cos^2\theta_g$

and the deposited energy, analogous to (19). The polar angle distribution in (20) is

$$\frac{d\sigma}{dz\,d\cos\theta_h}\sim(1+\alpha(z)\cos^2\theta_h) \tag{21}$$

where $z=2|\mathbf{p}|/M_{Q\bar{Q}}$ and θ_h is the angle of $\mathbf{p}$ relative to the beam axis. In order to calculate $\alpha(z)$, we assume that the probability for a gluon to yield a hadron with transverse momentum $p_\perp$ and fraction z' of the gluon momentum is $D(z')\exp(-bp_\perp^2)$. We then fold the angular and energy distribution of the gluon with the z' and $p_\perp$ distribution of the detected hadron relative to the gluon jet. This has already been done for quark jets [15]. The procedure is as follows.

The inclusive $\cos\theta$ and energy dependence of a single hadron with momentum and energy p, E and fractional momentum $0\leqq z\leqq 1$ is

$$\frac{d\sigma}{d^3p/E}\sim\int d\Omega_g\cdot\left\{\int_z^1\frac{dx}{x}\sigma_0(x)D\left(\frac{z}{x}\right)e^{-bp_\perp^2}+\int_z^1\frac{dx}{x}\sigma_1(x)D\left(\frac{z}{x}\right)e^{-bp_\perp^2}\cos^2\theta_g\right\} \tag{22}$$

where we have integrated over the energy and angle of the jet (x and Ω_g). Now $p_\perp$ relative to the jet axis is $p\sin\tilde{\theta}$. Using the law of cosines we can convert the integral over $d\Omega_g$ to one over $\tilde{\theta}$. The result is

$$\frac{d\sigma}{d^3p/E}\sim\int_z^1\frac{dx}{x}\sigma_0(x)D\left(\frac{z}{x}\right)I_0(p)+\int_z^1\frac{dx}{x}\sigma_1(x)D\left(\frac{z}{x}\right)\{\tfrac{1}{2}I_2(p)+[I_0(p)-\tfrac{3}{2}I_2(p)]\cos^2\theta_h\}. \tag{23}$$

$$I_n(p)=\int_0^1 d\cos\tilde{\theta}\,e^{-bp^2\sin^2\tilde{\theta}}\cdot\sin^n\tilde{\theta}. \tag{24}$$

We find for $\alpha(z)$ in (21), $z=\dfrac{2|\mathbf{p}|}{M_{Q\bar{Q}}}$,

$$\alpha(z)=\varrho(z)\frac{I_0(p)-\frac{3}{2}I_2(p)}{I_0(p)+\frac{1}{2}\varrho(z)I_2(p)}=\varrho(z)\bar{\alpha}(p)\left[\frac{3+\varrho(z)}{4}+\frac{1-\varrho(z)}{4}\bar{\alpha}(p)\right]^{-1} \tag{25}$$

where $\bar{\alpha}(p)$ describes the angular smearing due to finite $\langle p_\perp\rangle$ and is the same quantity found for $q\bar{q}$ jets,

$$\bar{\alpha}(p)=\frac{1-3I_2(p)/2I_0(p)}{1+I_2(p)/2I_1(p)} \tag{26}$$

whereas $\varrho(z)$ depends only on the scaling variable z and reads $z/x=z'$,

$$\varrho(z)=\frac{\int_z^1 dx\sigma_1(x)\frac{z}{x}D\left(\frac{z}{x}\right)}{\int_z^1 dx\sigma_0(x)\frac{z}{x}D\left(\frac{z}{x}\right)}. \tag{27}$$

For numerical purposes we set $b=5$ and chose $z'D(z')=(1-z')^n$ with $n=1,2,3$. The result for $\varrho(z)$ and $\alpha(z)$ is shown in Fig. 5. $\alpha(z)$ vanishes at low z because the finite $\langle p_\perp\rangle$ in a gluon jet gives an isotropic distribution of particles with low $p\ll\langle p_\perp\rangle$. Note that $\alpha(z)\to 1$ as $z\to 1$. For spinless gluons $\alpha(z)\to -1$ as $z\to 1$. We already pointed this out for $\alpha(T)$.

We want to emphasize the weak model dependence of $\alpha(z)$. For $z>1.5\,\mathrm{GeV}/(M_{Q\bar{Q}}/2)\approx[0.3$ for $Y(9.46)]$, $\alpha(z)$ depends hardly at all on either $\langle p_\perp^2\rangle$ for a gluon jet or the gluon fragmentation function. $\alpha(z)$ for $z<1.5\,\mathrm{GeV}/(M_{Q\bar{Q}}/2)$ measures the mean $\langle p_\perp^2\rangle$ in a jet, but is not otherwise model dependent.

Our calculation of $\alpha(z)$ can be easily generalized to a calculation of α_E *including* the effect of finite jet $p_\perp$. We do this by weighting the measured inclusive hadron with its energy (assuming that all particles have small

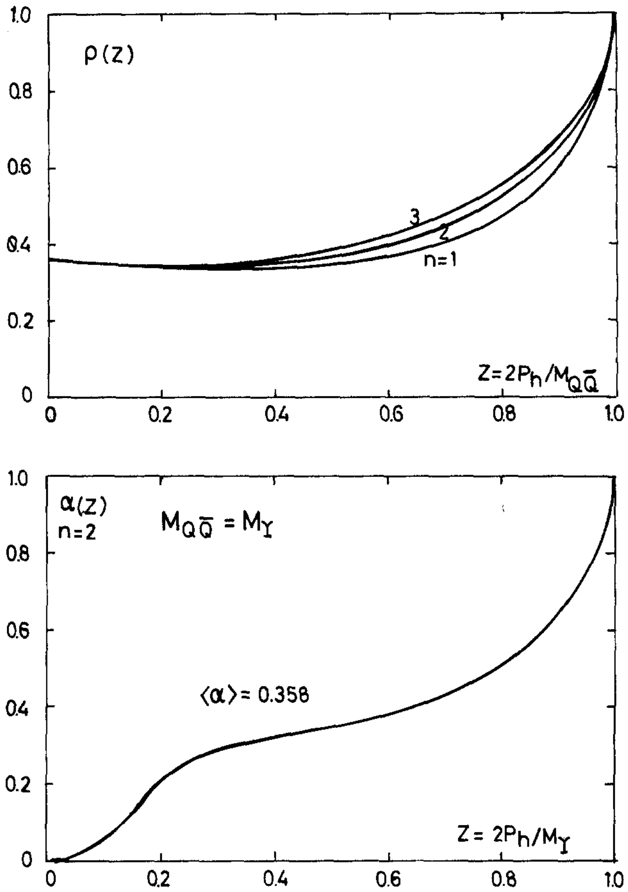

Fig. 5. The quantity $\varrho(z)$ defined in the text, shown for three different choices of the gluon-to-hadron fragmentation function $z'D(z') = (1-z')^n$. Also shown is $\alpha(z)$, defined by Eq. (21) of the text. This describes the polar angle dependence of a single inclusive hadron with $z=2p/M_\Upsilon$

mass). We find for $\langle p_\perp^2\rangle = 0.2\,\mathrm{GeV}^2$

$$\langle\alpha_E\rangle_{p_\perp \text{ smeared}} = \frac{\int_0^1 dz\, z\cdot f_1(z)\,[I_0(p)-(3/2)I_2(p)]}{\int_0^1 dz[zf_0(z)I_0(p)+(1/2)zf_1(z)I_2(p)]} = 0.23$$

$$f_{1,0}(z) = \int_z^1 dx\, \sigma_{1,0}(x)\frac{z}{x} D\left(\frac{z}{x}\right). \tag{28}$$

The reader may wonder whether the decay $Q\bar{Q}\to 1\gamma\to q\bar{q}\to 2$ jets does not interfere with the $Q\bar{Q}\to 3g$ test we have presented. For distributions it does not. This is because once $BR(Q\bar{Q}\to\mu^+\mu^-)$ is known, so is the absolute size of $Q\bar{Q}\to 1\gamma\to q\bar{q}$ (it is R times $Q\bar{Q}\to\mu^+\mu^-$). Then using off-resonance data it is easy to subtract $Q\bar{Q}\to 1\gamma\to q\bar{q}$ in distributions. An event-by-event subtraction is, of course, not possible. But $Q\bar{Q}\to 1\gamma\to q\bar{q}$ does not disturb a 3 jet search in small thrust events because it is negligible there even event-by-event. (Such $QQ\to 2$ jet events have large T.)

We have been discussing single particle distributions so far. Two particle distributions,

$$e^+e^-\to Q\bar{Q}\to 3g\to h(\mathbf{p}_1)+h(\mathbf{p}_2)+\ldots \tag{29}$$

where $h(\mathbf{p}_1)$, $h(\mathbf{p}_2)$ are detected hadrons of momentum $\mathbf{p}_1, \mathbf{p}_2$, contain more physics than (20). But they are also more model dependent. We have already drawn attention to (20) as a way of looking indirectly for 3 jet structure. Now we want to point out how two particle distributions can be used to study $3g$ decay dynamics.

The differential rate for (29) is given by Eq. (14) with some simple changes. θ, χ are the polar and azimuthal angles of $\mathbf{p}_1$ relative to the beam (we order $|\mathbf{p}_1|>|\mathbf{p}_2|$) and the $\mathbf{p}_1$, $\mathbf{p}_2$ plane relative to the plane of $\mathbf{p}_1$ and the beam. The variables x_1 and x_2 are replaced by

$$z_1 = 2|\mathbf{p}_1|/M_{Q\bar{Q}}, \quad z_2 = 2|\mathbf{p}_2|/M_{Q\bar{Q}}$$

and the missing mass against z_1 and z_2 is no longer zero.

The structure functions in (14) are now

$$\sigma_U(z_1,z_2),\ \sigma_L(z_1,z_2),\ \sigma_T(z_1,z_2),\ \sigma_I(z_1,z_2)$$

interpreted as two particle structure functions for the process (29). Choosing $|\mathbf{p}_1|>|\mathbf{p}_2|>\mathbf{p}_{\min}$ as before one can look for azimuthal asymmetries proportional to $\cos\chi$, $\cos 2\chi$ in (14). These are more model dependent than the azimuthal asymmetries we discussed earlier, and we won't embark on a detailed discussion. However we want to point out that the average value of $\cos 2\chi$

$$\langle\cos 2\chi\rangle > 0 \tag{30}$$

due to the positivity of σ_T.

Two particle distributions are also very useful in checking the QCD prediction that gluons have no flavor. We do this as follows. Consider high thrust events which have a clear jet axis. These are collinear $Q\bar{Q}\to 3g\to 2$ jet events. Since these are gluon jets, they carry no net flavor. In particular, knowing the flavor of a hadron in one jet does not determine the flavor of a hadron in the opposite jet. There are no correlations. By contrast, if one goes off the quarkonium resonance the 2 jet process is $e^+e^-\to 1\gamma\to q\bar{q}$. The quanta have flavor, and if one sees a hadron of definite flavor (a π^+ say) in one jet this increases the probability to see a hadron of the opposite flavor in the other jet [16]. Thus we see how to test the flavor of gluon jets: look for a correlation off resonance from $q\bar{q}$ jet and check that there is *no* correlation on resonance. The comparison on and off resonance provides a standard for judging whether the absence of correlation on resonance is significant or not. In carrying out this check it is important to take events with the same range of thrust on and off resonance. This way we look at events which are globally similar.

To be quantitative, take the cross section for a particle of flavor F_1 with fractional momentum z_1 in one jet and F_2, z_2 in the opposite jet. We integrate this two particle distribution over the tips of the two jets, $1 \geqq z_1$, $z_2 \geqq z_0$,

$$C(F_1,F_2) = \int_{z_0}^{1} dz_1 \int_{z_0}^{1} dz_2 \frac{1}{\sigma} \frac{d\sigma(F_1,F_2)}{dz_1\, dz_2}. \tag{31}$$

On resonance $C(F_1,F_2)$ is just a product of single particle distribution. As one specific example out of many,

$$C_{\Upsilon}(\pi^+\pi^+) = C_{\Upsilon}(\pi^+\pi^-). \tag{32}$$

We can calculate $C_{e^+e^-}(\pi^+\pi^+)$ and $C_{e^+e^-}(\pi^+\pi^-)$ from the parton model off resonance [16]. This indicates what sort of correlations we except to see. For z_0 sufficiently large we expect pions to come from $e^+e^- \to u\bar{u}$, $e^+e^- \to d\bar{d}$. (Final states from $s\bar{s}$, $c\bar{c}$ should not have high momentum pions.) We find

$$C(\pi^+\pi^\pm) = \int_{z_0}^{1} dz_1 \int_{z_0}^{1} dz_2 \sum_{q=u,d,\bar{u},\bar{d}} e_q^2 \cdot D_q^{\pi^+}(z_1) D_q^{\pi^\pm}(z_2). \tag{33}$$

After some algebra,

$$\frac{C_{e^+e^-}(\pi^+\pi^-)}{C_{e^+e^-}(\pi^+\pi^+)} = \frac{(p^+/p^-)^2+1}{2(p^+/p^-)} \tag{34}$$

where $p^\pm = \int_{z_0}^{1} dz D_u^{\pi^\pm}(z)$. We use the $D_u^{\pi^\pm}(z)$ from Sehgal [17] and find the resulting ratio (34) shown in Fig. 6 as a function of z_0. It is large for large z_0 because it is very improbable to find a fast π^+ from a u quark in one jet an a π^+ from $\bar{u}$ in the other (π^- is favored). (The same exercise can be carried through for other flavors, kaons for example.) $C(\pi^+\pi^-)/C(\pi^+\pi^+)$ should show a striking change when going on the Υ resonance. It will drop from a large value to nearly unity. (The contribution from $Q\bar{Q} \to \gamma \to q\bar{q}$ is small, and can be subtracted as we have already discussed.*)

In this section we have discussed many experimental QCD tests. Most of them appear to us viable even at a resonance like $\Upsilon(9.46)$ where 3 jet structure is not expected to be dramatic.

IV. Conclusions

This paper is intended as an aid to experimentalists looking for gluon jets and desiring to test QCD in $Q\bar{Q} \to 3g$. We have concentrated on three jet kinematics and on the probabilities of different jet configurations. We have also shown how to check experimentally that gluons are really massless vector particles with no flavor.

All the calculations in this paper are carried out in Born approximation. We think that this is the essential first step. Eventually we expect gluon jets to be found and their distributions measured. Then it will become interesting to look for deviations from our lowest order QCD predictions. We have already mentioned

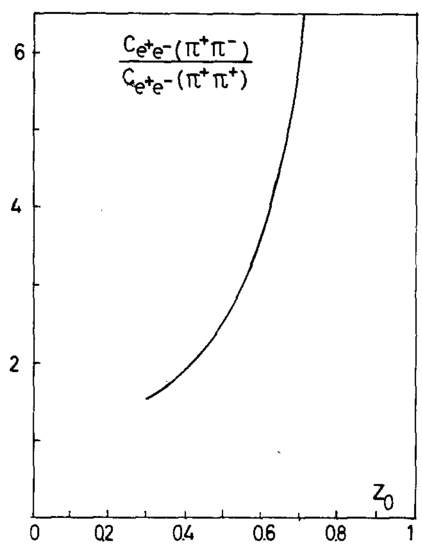

Fig. 6. The ratio of the probabilities to find π^+ and π^- or π^+ and π^+ in opposite $e^+e^- \to q\bar{q}$ jets, as a function of the minimum fractional momentum of a pion, $z_0 = 2p_{\min}/s$

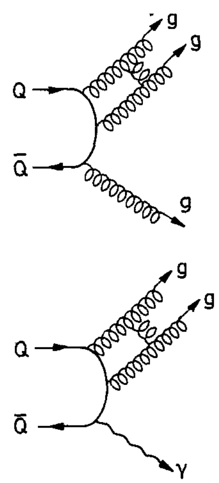

Fig. 7. Two higher order QCD diagrams containing gluon self-interactions

* We caution against choosing z_0 so large that the multiplicity in the final state is small. Then $C(\pi^+\pi^+)$ is artifially suppressed

this in the introduction. A detailed theoretical study of higher order QCD effects is certain to be quite complicated. However, the issues can be appreciated from the following naive observation. We already pointed out that appreciable deviations from the Born approximation are likely to appear only for events near the boundary of the $Q\bar{Q}\to 3g$ jet Dalitz plot. This is where two of the three jets are nearly parallel. The $2g$ invariant mass is small compared to $2M_Q$. If we consider not just the $3g$ process $Q\bar{Q}\to 3g$ but also the radiative process $Q\bar{Q}\to\gamma 2g$ for modest $2g$ mass, then the two gluons nearby in phase space might interact according to Fig. 7. (Of course, there are many diagrams of the same order as those in Fig. 7; we focus attention on these here.) But we see that the color combinations of gg in $Q\bar{Q}\to g+gg$ and $\gamma+gg$ are different. In the radiative process the gg are in a color singlet state. They "attract" one another. In the $3g$ decay process the gg must be in a net color octet state; they repel. If it is possible to study distributions for this kinematic configuration, we will stand to learn about the non-abelian self coupling of gluons. Then $Q\bar{Q}\to 3g$ and $Q\bar{Q}\to\gamma gg$ [3] become a laboratory for the study of QCD.

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References

1. Fritzsch, H. et al.: Phys. Lett. B**47**, 365 (1973); Gross, D.J., Wilczek, F.: Phys. Rev. D**8**, 3497 (1973); Weinberg, S.: Phys. Rev. Lett. **31**, 31 (1973); Appelquist, T., Politzer, H.: Phys. Rev. Lett. **34**, 43 (1975), Phys. Rev. D**12**, 1404 (1975)
2. Koller, K., Walsh, T.F.: Phys. Lett. **72**B, 227 (1977), E**73**B, 504 (1978); DeGrand, T.A. et al.: Phys. Rev. D**16**, 3251 (1977); Bodsky, S. et al.: Phys. Lett. **73**B, 203 (1978); Fritzsch, H., Streng, K.-H.: Phys. Lett. **74**B, 90 (1978); Hagiwara, K.: Nucl. Phys. B**137**, 164 (1978)
3. Koller, K., Walsh, T.F.: DESY preprint 78/16 (to be published in Nucl. Phys. B)
4. Herb, S.W. et al.: Phys. Rev. Lett. **39**, 252 (1977)
5. Berger, Ch. et al.: DESY preprint 78/21; Darden, C.W. et al.: DESY preprint 78/22
6. Meyer, H.: Symposium on Jets in High Energy Collisions, Copenhagen, July 1978; Alexander, G.: XIX Int. Conf. on High Energy Physics, Tokyo 23–30 Aug., 1978; Berger, Ch. et al.: DESY preprint 78/39
7. Born, M.: Z. Physik **37**, 863 (1926), **38**, 803 (1926)
8. Sterman, G., Weinberg, S.: Phys. Rev. Lett. **39**, 1436 (1977)
9. Krammer, M., Krasemann, H.: Phys. Lett. **73**B, 58 (1978)
10. Farhi, E.: Phys. Rev. Lett. **39**, 1587 (1977)
11. De Rujula, A. et al.: Nucl. Phys. B**138**, 387 (1978)
12. Brandt, S., Dahmen, H.: Siegen preprint (1978)
13. Basham, C.L. et al.: Phys. Rev. D**17**, 2298 (1978)
14. Similar results have been obtained independently by F. Steiner, whom we wish to thank for drawing our attention to Ref. [13]
15. Gronau, M. et al.: Nucl. Phys. B**104**, 307 (1976)
16. Walsh, T.F., Zerwas, P.: Nucl. Phys. B**77**, 494 (1974)
17. Sehgal, L.: 1977 Lepton-Photon Symposium (ed. F. Gutbrod, Hamburg 1977)

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