DUALITY PREDICTIONS FOR THE PRODUCTION OF HEAVY QUARK SYSTEMS IN QCD

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Using partonic semi-local duality ideas combined with QCD we derive absolute, parameter-free predictions for the cross sections of heavy quarkonia (J/ψ, ψ', Υ, Υ', ...) production in purely hadronic collisions as well as in photoproduction processes. We also discuss "open" charm ($D\bar{D}$) production and show how the CERN SPS beam dump measurements at $\sqrt{s} = 27.4$ GeV can be naturally reconciled with the predictions of QCD; similarly the recent ISR data are in good agreement with QCD.

Recently, duality ideas combined with QCD have been used [1–4] to relate fundamental parton cross sections to measured J/ψ and Υ production cross sections in hadronic collisions, and in photoproduction processes [3,4]. For hadronic collisions, for example, one assumes the fundamental subprocesses to be $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$ where q denotes "light" quarks (u, d, s), Q the "heavy" quarks (c, b, t), and g the gluon. Denoting the c.m. energy squared of the fundamental subprocess by $\hat{s}$, one integrates over $\hat{s}$ between $\hat{s}_{th} = (2m_Q)^2$ and $\hat{s}$ – the threshold for open flavor Q production ($D\bar{D}$, etc.). The total cross section for bound $Q\bar{Q}$ systems obtained this way exceeded the measured J/ψ cross section by a factor of about 8 [2], and the measured Υ cross section by a factor of 20 [5]. It was originally pointed out by Fritzsch [1] that this, in fact, is expected for J/ψ production since there are many charmonia states in the aforementioned interval of invariant mass-squared $\hat{s}$ of the $c\bar{c}$ system. It is our purpose here to turn this qualitative observation not only into a quantitative one, but also to suggest that this semi-local duality idea can and should be applied to the production of any bound heavy $Q\bar{Q}$ system. In this way we obtain absolute, i.e., parameter-free predictions for the production cross sections of

any quarkonia state which (surprisingly) agree remarkably well with the measured rates in purely hadronic collisions as well as photoproduction processes.

For illustration let us first consider charmonium J/ψ production. Here, $\hat{s}_{th} = (2m_c)^2$ and $\hat{s}_0 = (2m_D)^2 = (3.726)^2$ GeV^2. Then, in the $\hat{s}$ integration region one finds 7 to 8 charmonium levels (depending on whether $\chi(2.83)$ is taken or not), so we suggest to divide the calculated partonic cross sections by this number to obtain the J/ψ cross section. For ψ' production we suggest the $\hat{s}$ integration to be taken between $(m_{\psi'} - \frac{1}{2}\Gamma_{tot}^{\psi'})^2$ and $(2m_D)^2$, where the lower limit has been chosen such that the entire resonance lies within the duality integration region. (For extremely narrow resonances like the J/ψ, the width Γ_{tot} can be safely neglected.) The resulting cross section would now directly predict the ψ' production cross section; no further division should be performed since only ψ' lies in the aforementioned energy interval. The extension of these ideas to the Υ and $\zeta \equiv t\bar{t}$ families is now obvious: One always divides the calculated, i.e., $\hat{s}$-integrated partonic cross sections by the number of levels in the corresponding energy integration interval. This should be valid for *any* beam and target particle since the formation of the final bound $Q\bar{Q}$ state is independent of the initial state.

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To be more quantitative let us first recall the relevant expressions for the various production cross sections. Within the semi-local duality approach [1,2] the cross section for producing a definite $Q\bar{Q}$ bound state in purely hadronic collisions (e.g., pp) is given by

$$\frac{d\sigma}{dy}\bigg|_{y=0} = \frac{1}{N}\frac{1}{s} \int_{\hat{s}_{th}}^{\hat{s}_0} d\hat{s}$$

$$\times \left[2 \sum_{q=u,d,s} \sigma^{q\bar{q}\rightarrow Q\bar{Q}}(\hat{s}) q(\sqrt{\tau},\hat{s})\bar{q}(\sqrt{\tau},\hat{s}) \right.$$

$$\left. + \sigma^{gg\rightarrow Q\bar{Q}}(\hat{s}) G(\sqrt{\tau},\hat{s}) G(\sqrt{\tau},\hat{s}) \right], \quad (1)$$

with $\tau = \hat{s}/s$ and N is the number of bound $Q\bar{Q}$ levels in the invariant $Q\bar{Q}$ mass interval $\hat{s}$ considered. The $Q^2 (\equiv \hat{s})$ dependent quark and gluon distributions $q(x, Q^2)$ and $G(x, Q^2)$, respectively, in hadrons will be discussed below. The cross section for the fundamental $q\bar{q}$ annihilation subprocess is given by the well known expression

$$\sigma^{q\bar{q}\rightarrow Q\bar{Q}} = \frac{2}{9} (4\pi\alpha_s^2/3\hat{s})(1+\tfrac{1}{2}\gamma)\sqrt{1-\gamma}, \quad (2)$$

where $\gamma = 4m_Q^2/\hat{s}$ and $\alpha_s = 12\pi/25 \ln(\hat{s}/\Lambda^2)$ with $\Lambda \approx 0.5$ GeV. For the gluon–gluon fusion process $gg \rightarrow Q\bar{Q}$ one obtains [2]

$$\sigma^{gg\rightarrow Q\bar{Q}} = \frac{\pi\alpha_s^2}{3\hat{s}} \left[\left(1+\gamma+\tfrac{1}{16}\gamma^2\right) \ln \frac{1+\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}} \right.$$

$$\left. - \left(\tfrac{7}{4}+\tfrac{31}{16}\gamma\right)\sqrt{1-\gamma} \right]. \quad (3)$$

Since most experiments on hadronic quarkonia production done so far observe the heavy $Q\bar{Q}$ resonances through their $\mu^+\mu^-$ or e^+e^- decay channel, we have to multiply eq. (1) by the measured [7] leptonic branching ratio $B = \Gamma_{\mu\mu}/\Gamma_{tot}$ for the resonance under consideration. For the Υ and ζ families no measured B's exist and one has to rely on some (potential) model calculations [8–10].

The total cross section for producing a definite $Q\bar{Q}$ bound state in photoproduction processes (e.g., $\gamma p \rightarrow (Q\bar{Q}) + X$) is given by [4,6]

$$\sigma^{\gamma p} = \frac{1}{N}\frac{1}{s} \int_{\hat{s}_{th}}^{\hat{s}_0} d\hat{s}\, \sigma^{\gamma g\rightarrow Q\bar{Q}}(\hat{s})\, G(\tau,\hat{s}), \quad (4)$$

where the fundamental (Bethe–Heitler) pair creation process reads

$$\sigma^{\gamma g\rightarrow Q\bar{Q}} = \tfrac{1}{2} e_Q^2 \frac{4\pi\alpha\alpha_s}{\hat{s}}$$

$$\times \left[\left(1+\gamma-\tfrac{1}{2}\gamma^2\right) \ln \frac{1+\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}} - (1+\gamma)\sqrt{1-\gamma} \right]. \quad (5)$$

For illustration we will consider five different classes of $Q\bar{Q}$ resonance production with the following choice for the parameters and integration limits in eqs. (1) and (4):

(i) J/ψ-production: $\hat{s}_{th} = (2m_c)^2$, $\hat{s}_0 = (2m_D)^2$ with $m_c = 1.5$ GeV and $m_D = 1.863$ GeV, $N = 8$ and $B = 0.07$;

(ii) ψ'-production: $\hat{s}_{th} = (m_{\psi'} - \tfrac{1}{2}\Gamma_{tot}^{\psi'})^2 \approx (3.684 - 0.114)^2$ GeV^2, $\hat{s}_0 = (2m_D)^2$, $N = 1$ and $B = 0.009$;

(iii) Υ-production: $\hat{s}_{th} = (2m_b)^2$, $\hat{s}_0 \approx (10.7 \text{ GeV})^2$ with $m_b \approx \tfrac{1}{2}m_\Upsilon = 4.7$ GeV. The remaining parameters have to be taken from some potential model calculation [8] which suggests the number of $b\bar{b}$ states to be 1s, 1p, 2s, 1d, 2p, 3s, i.e., $N = 18$ and the branching ratio is estimated [8,11] to be $B \approx 0.04$;

(iv) Υ'-production: $\hat{s}_{th} = m_\Upsilon^2 \approx (10.0 \text{ GeV})^2$, $\hat{s}_0 \approx (10.7 \text{ GeV})^2$, and again model calculations suggest [8,11] $N = 11$ and $B \approx 0.02$;

(v) ζ-production: $\hat{s}_{th} = (2m_t)^2$. Here no experimental information whatsoever is available, so we will give predictions for two drastically different potential models. In the case of a purely logarithmic confining potential [9] we used $m_t = 8.5$ GeV and correspondingly $\hat{s}_0 \approx (18 \text{ GeV})^2$ and with the number of $t\bar{t}$ levels to be $N = 22$. In the other extreme case where the confining potential consists of a linear as well as exponential part [10] we took $m_t = 14$ GeV and $\hat{s}_0 \approx (29.8 \text{ GeV})^2$ with $N = 98$. In both cases we used a theoretically estimated [11] leptonic branching ratio $B = 0.08$.

Before discussing the predictions of eqs. (1) and (4) the various parton distribution functions must be specified. We shall employ two different (extreme) sets of parton distributions in nucleons [5], in order to show the dependence of our results on different parametrizations of these distributions. One set is based on 'counting rule-like' input distributions at $Q^2 = Q_0^2 = 1.8$ GeV^2 where the SU(3) symmetric sea and gluon distributions are taken to be $x\xi(x, Q_0^2) = 0.147(1-x)^7$ and $xG(x, Q_0^2) = 2.412(1-x)^5$, respectively. The valence distributions are well known [5] and uncontroversial. The distributions at higher Q^2 values were obtained by the usual method of Mellin inverting the
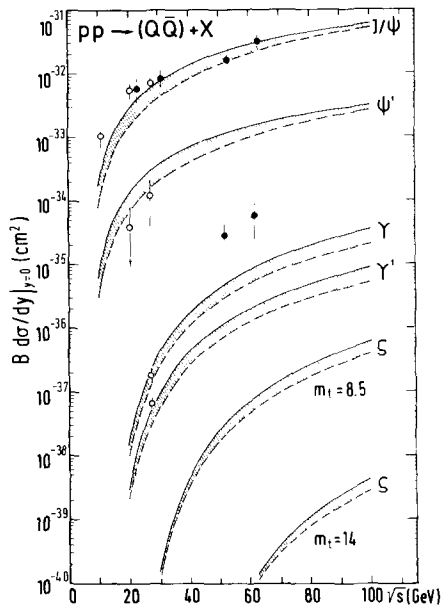
Fig. 1. Predictions for the production of heavy $Q\bar{Q}$ bound states according to eq. (1). The solid curves correspond to the counting rule-like QCD parton distributions, and the dashed curves to the dynamical QCD distributions as described in the text. The pN data (○) are taken from refs. [13,15] whereas the pp data (●) are from refs. [14,16].
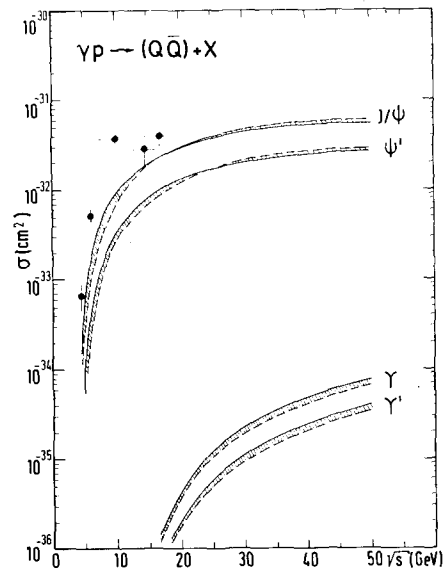

Fig. 2. Predictions for the photoproduction of $Q\bar{Q}$ bound states according to eq. (4). The notation is as in fig. 1, and the data for J/ψ production are from ref. [17].

QCD predicted moments. The other set of parton distributions used correspond to the dynamically calculated [12] QCD predictions. These "dynamical" distributions were obtained, with the help of well known renormalization group techniques, by assuming that at low resolution energies, $Q^2 = \mu^2$, hadrons consist of valence quarks only, i.e., $x\,\xi(x,\mu^2) = xG(x,\mu^2)$ $= 0$. These two different extreme sets of counting rule-like and dynamical parton distributions represent roughly the upper and lower bounds of distributions, respectively, compatible with present experiments on deep inelastic lepton-nucleon scattering. In order to facilitate the numerical calculations, we have used in both cases the simple parametrisations of ref. [5] for the exact x- and Q^2-dependence of parton distributions as predicted by QCD.

In fig. 1 we show the predictions of eq. (1) for the production of heavy $Q\bar{Q}$ bound states. The upper (solid curves) and lower (dashed curves) limits of the shaded areas refer to counting rule-like and dynamical parton distributions, respectively. The agreement with

existing data [13–16] is surprisingly good. It is remarkable that the experimentally observed increase of the production of the 2s bound state relative to the 1s state for increasing quarkonia masses, is naturally explained by our model: $\sigma(\psi')/\sigma(\psi) \approx \frac{1}{10}$ whereas $\sigma(\Upsilon')/\sigma(\Upsilon) \approx \frac{1}{3}$. Our parameter-free predictions stand on firm grounds for $c\bar{c}$ (ψ-family) production since here the number N of bound state levels as well as the leptonic branching ratios B are experimentally reliably well known. This is in contrast to the production of heavier $Q\bar{Q}$ bound states where, in order to compare with existing data on Υ and Υ' production, we have to use some potential-model estimates for B and N. Similarly the predictions for $\zeta = t\bar{t}$ production are heavily dependent on model calculations for N and B, and on the assumed quark mass m_t. It should be noted that the dominant contribution to the production of members of the ψ-family comes from the gluon fusion process $gg \rightarrow c\bar{c}$ which, at $\sqrt{s} = 20$ GeV, is about twice as large as the contribution from $q\bar{q} \rightarrow c\bar{c}$, and increases with energy: At $\sqrt{s} = 100$ GeV the $gg \rightarrow c\bar{c}$ process dominates $q\bar{q} \rightarrow c\bar{c}$ by more than an order of magnitude. At a given energy the dominance of the gluon-fusion subprocess will be reduced for the production of heavier $Q\bar{Q}$ bound states like $b\bar{b}$ and $t\bar{t}$, since the parton distributions are probed at larger values of $x = \sqrt{\tau}$. For $b\bar{b}$ (Υ-family)

Table 1
Semi-local duality predictions for quarkonia production in e^+e^- annihilation processes, according to eq. (6).

| | $\int d\hat{s}\, \sigma^{e^+e^-\to h}(\hat{s})$ | | |
h	J/ψ	ψ'	Υ
theory	3.8×10^{-5}	0.7×10^{-5}	0.3×10^{-5}
exp.	$(18.3\pm2.3) \times 10^{-5}$	$(6.7\pm1.0) \times 10^{-5}$	$(1.6\pm0.5) \times 10^{-5}$

production, for example, the $q\bar{q}$ annihilation subprocess dominates for $\sqrt{s} \lesssim 30$ GeV, whereas both subprocesses become about equal at $\sqrt{s} \approx 50$ GeV; at $\sqrt{s} \approx 100$ GeV gg → $b\bar{b}$ starts to become the dominant production mechanism where it is about twice as large as $q\bar{q} \to b\bar{b}$. Similarly, $q\bar{q} \to t\bar{t}$ dominates ζ-production almost over the whole energy range shown in fig. 1, except for $\sqrt{s} \gtrsim 90$ GeV where $q\bar{q} \to t\bar{t}$ and gg → $t\bar{t}$ are of comparable size.

Since the absolute normalization of the cross-section is now uniquely fixed by our semi-local duality approach, depending only on the $Q\bar{Q}$ *final* state under consideration, the predictions of quarkonia production off nucleons using different beams ($\bar{p}$, $\pi^{\pm}$, $K^{\pm}$) are unambiguous. The latter depend only on the different parton distributions of the initial state. These predicted beam ratios [2,5] turn out to be in excellent agreement with experiment, where the gluon–gluon fusion subprocess gg → $Q\bar{Q}$ plays a dominant role [2] in explaining the data.

The partonic duality predictions for photoproduction of quarkonia according to eq. (4) are shown in fig. 2. Our predictions are comparable with existing scarce data [17] for J/ψ production, but might lie about a factor of 2 below experiment. The same trend holds also for partonic duality predictions of J/ψ, ψ' and Υ production in e^+e^- collisions which are obtained from

$$\int d\hat{s}\, \sigma^{e^+e^-\to h}(\hat{s}) = \frac{1}{N} 4\pi\alpha^2 e_Q^2 \int_{\hat{s}_{th}}^{\hat{s}_0} d\hat{s}\, \frac{1}{\hat{s}}\, (1+\tfrac{1}{2}\gamma)\sqrt{1-\gamma},$$
(6)

where now $\hat{s} \equiv s$ and h denotes a definite $Q\bar{Q}$ bound state. Note that now only the $J^{PC} = 1^{--}$ states in the considered energy interval should be counted: For J/ψ, ψ' and Υ production they are given by $N = 2, 1$ and 4, respectively. The predictions of eq. (6) are compared in table 1 with the experimental values for the integrated total e^+e^- annihilation cross section for

quarkonia production, i.e. with

$$\int d\hat{s}\, \sigma^{e^+e^-\to h}(\hat{s}) = \frac{12\pi^2}{m_h}\, \frac{\Gamma_{ee}\Gamma_h}{\Gamma_{tot}} \approx \frac{12\pi^2}{m_h}\, \Gamma_{ee}\,,$$
(7)

where the measured leptonic widths are taken from [7,18]. The discrepancy between our duality predictions and experiment is now worse than in the case of photoproduction, and lie typically a factor of about 5 below the data. Why semi-local duality should become worse the less hadronic the process is, remains unclear to us. At present we just mention this observation.

So far we have been solely concerned with the bound $Q\bar{Q}$-excitation spectrum, i.e. with the region below the $Q\bar{q} + \bar{Q}q$ continuum. We finally briefly turn to the so called "open" flavor production in the continuum region. The total cross section for open flavor production in hadron–hadron collisions, e.g., pp → $(Q\bar{q} + \bar{Q}q)$ + X is a simple generalization of eq. (1) and is given by

$$\sigma^{pp} = \frac{1}{s} \int_{\hat{s}_0}^{s} d\hat{s} \int_{\tau}^{1} \frac{dx}{x}$$

$$\times \left\{ \sum_{q=u,d,s} \sigma^{q\bar{q}\to Q\bar{Q}}(\hat{s}) [q(x,\hat{s})\bar{q}(\tau/x,\hat{s}) + q \leftrightarrow \bar{q}] \right.$$

$$\left. + \sigma^{gg\to Q\bar{Q}}(\hat{s})\, G(x,\hat{s})\, G(\tau/x,\hat{s}) \right\}.$$
(8)

Similarly the total photoproduction cross section of open heavy flavors is given by eq. (4) with $\hat{s}_{th} \to \hat{s}_0$ and $\hat{s}_0 \to s$, and without the factor N^{-1}. In fig. 3 we show the relevant predictions for open charm production, i.e. using $\hat{s}_0 = (2m_D)^2$. In order to demonstrate the effect of the $Q^2 \equiv \hat{s}$ dependence of parton distributions, we also display the results referring to the "naive" parton model (dotted curves) where the counting rule-like distributions have been calculated at Q_0^2 with the Q^2-dependence switched off (as opposed to the solid curves). Note that for $\sqrt{s} \lesssim 15$ GeV the
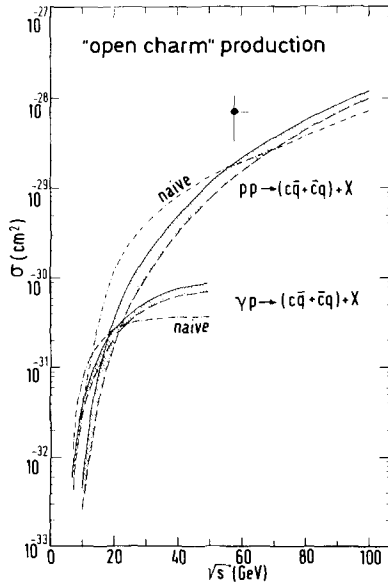
Fig. 3. Predictions for open charm production in proton-proton collisions, eq. (8), and in photoproduction off protons. Solid and dashed curves refer to the counting rule-like and dynamical QCD parton distributions, and the dotted curves correspond to the counting rule-like distributions evaluated at Q_0^2, i.e. with no QCD Q^2-dependence ("naive" parton model). The ISR data point has been taken from ref. [19].

photoproduction of open charm is expected to be almost a factor of 5 larger than charm production in purely hadronic collisions [6]. Our QCD predictions in fig. 3 for open charm production in proton–*proton* collisions are in agreement with recent ISR measurements [19] between $\sqrt{s}$ = 53 and 63 GeV. Furthermore we notice, together with others [3,20,21], that the predictions for hadronic production of open charm are much too low when compared with the CERN beam dump experiments [22]: Scattering protons off heavy nuclei (Cu) the CERN SPS beam dump experiments [22] find the total cross section for open charm production to lie in the range of 20–100 μb at $\sqrt{s}$ = 27.4 GeV, whereas the QCD predictions for proton–*proton* collisions in fig. 3 lie typically around 1–2 μb. This discrepancy remains, regardless of whether one starts the $\hat{s}$ integration at [6,20] $(2m_c)^2$ or at [3,21] $(2m_D)^2$. This latter choice is the more consistent procedure in our opinion. Furthermore it should be emphasized [20,3] that the predictions for open quarkonia production, as well as for bound $Q\bar{Q}$ production, are sensitive to the value of the heavy quark

mass m_Q: Taking m_c = 0.6 GeV instead of 1.5 GeV would easily increase the predictions in fig. 3 by a factor of 5 to 10. However, the success of our bound quarkonia predictions for hadronic reactions in fig. 1 forbids a drastic reduction of m_c and poses also severe limits on any additional "mechanism" which might be suggested in the future; these would have to cope with the saturation of the quarkonia cross sections by the present mechanism. Especially the measured structure of $\sigma(e^+e^- \rightarrow$ hadrons), i.e. the onset of $c\bar{c}$ production, dictates the charmed quark mass to be at least $m_c \approx 1.5$ GeV.

The only sensible mechanism we can think of, which influences quarkonia ($J/\psi, \psi', \Upsilon, ...$) production (taken mainly on light nuclei) differently than open charm production (taken from heavy nuclei, such as Cu) are nuclear enhancement effects. In the extreme case [23] collective nuclear effects cumulatively enhance the c.m. energy-squared $s = 2mp_{lab}$ of a hadron–nucleon collision to $s_{eff} = ns$ for a hadron–nucleus collision, where $n \approx A^{1/3}$ is the number of nucleons which collectively interact with the incoming hadron. Thus nuclear enhancement effects affect only marginally our successful predictions for quarkonia production in figs. 1 and 2, since the Fermilab pN data have been taken mainly from light nuclei such as Be, C and D_2 [13,17]. The beam dump experiments [22] for open charm production, however, will be strongly affected since Cu targets have been used, i.e. the effective energy of this experiment corresponds to about $\sqrt{s_{eff}} \approx 55$ GeV instead of 27.4 GeV. Thus, according to the strongly rising predictions in fig. 3, we expect the total cross section for open charm ($D\bar{D}$) production as measured at CERN SPS to lie in the range of 10 to 20 μb in agreement with experiment [22].

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