Jet-Like Distributions from the Weak Decay of Heavy Quarks*

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Abstract. We consider the production and weak decay of a pair of heavy quarks (mesons) in e^+e^- experiments and study their effect on the various jet distributions. The relative magnitudes of the two-quark-jet and three-quark-jet final states, in the decay of a heavy quark are estimated in the framework of an $SU(2)_L \times U(1)$ model. We find that the three quark configuration dominates over the two quark configuration. For the quark jets resulting from the weak decay of the heavy quarks, we calculate the jet distributions in Sphericity, Spherocity and Thrust for the process $e^+e^- \to Q\bar{Q} \to 6$ quarks. These distributions are compared with the corresponding quantities from the non-perturbative process $e^+e^- \to q\bar{q}$ and the QCD process $e^+e^- \to q\bar{q}g$. We find that the weak decay of heavy quarks is the dominant mechanism for jet broadening in e^+e^- experiments, in the intermediate energy region relevant for PETRA and PEP.

1. Introduction

The Υ (9.46) particle first discovered in proton interactions by Herb et al. [1] and recently observed in electron-positron annihilation by two groups at DORIS [2] points to the existence of a new heavy quark with charge $Q = -1/3$. The mass of Υ (9.46) suggests that there is a rich spectroscopy of new hadrons in the mass range 5–7 GeV and they will be produced copiously in the next generation of e^+e^- colliding beam experiments at PETRA, PEP, and CESR. The lowest lying members of this group of particles will decay only through weak interactions shedding more light on the nature of the quark as well on the new weak interaction.

Since there is now evidence for a new heavy lepton (the τ) [3] and the new heavy quark b it is clear that the standard Weinberg-Salam (WS) model [4] must be extended. The simplest extension allowing for these discoveries is to add a further left-handed doublet to the old ones, so that we now have

$$\begin{pmatrix} u \\ \tilde{d} \end{pmatrix}_L, \begin{pmatrix} c \\ \tilde{s} \end{pmatrix}_L, \begin{pmatrix} t \\ \tilde{b} \end{pmatrix}_L, \begin{pmatrix} v_e \\ e \end{pmatrix}_L, \begin{pmatrix} v_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} v_\tau \\ \tau \end{pmatrix}_L. \quad (1.1)$$

This six quark model has been discussed first by Kobayashi and Maskawa (KM) [5]. Here t and b are new heavy quarks with charge $Q = 2/3$ and $Q = -1/3$ respectively. The $\tilde{d}$, $\tilde{s}$, and $\tilde{b}$ are eigenstates of the weak interaction which are related to the strong interaction eigenstates d, s, and b by a unitary transformation to be specified later.

Several authors have already studied the anticipated properties of the weak interactions associated with the b and t decays in the context of the KM model [6]. In this paper we want to extend these investigations in several respects. First, the recent measurement of leptonic decay width of the Υ (9.46) favours a $b\bar{b}$ interpretation of this resonance and this way fixes the "mass" of the bottom quark. This still leaves the possibility that the mass of the top quark is not much higher but a very naive extrapolation of the s, c, b sequence would suggest that the t quark mass is in the vicinity of 15 GeV. The new hadrons associated with this quark would be in the mass range 15–20 GeV. The decay characteristics of mesons with such mass values are expected to be much different compared to the 5–7 GeV B mesons, associated with the b quark, or the charmed D and F mesons with masses around 2 GeV. The D and F meson presumably decay to a considerable fraction into two-body and quasi-two-body channels [7]. For the B mesons we expect the two- and quasi-two-body channels to contribute just a few percent to the total rate. Then the identification of these mesons with the help of specific two-body chan-

nels will be very difficult. We expect the B mesons as well as the T mesons (with the t quark as constituent) to decay into many particles. With large decay energies available it is then more reasonable to resort to quark-parton model ideas and to interpret the basic weak interaction quark diagrams accordingly.

With the mass of the decaying particle high enough, as in the T region, the quark lines in the final state can be interpreted as jets. In this case the model allows rather specific predictions about the jet distribution in terms of variables which have been introduced in connection with quark jets produced in e^+e^--annihilation, as for example thrust T, spherocity S, sphericity $\tilde{S}$ and others [8]. Experimentally it will be difficult to measure the decay characteristics of isolated B or T mesons which must be identified through their many body decay channels. On the other hand since the parent quark mass and energy are large, leading particle (jet like) effects are anticipated in the decay of heavy quarks. At this early stage it is more useful to calculate the influence of weak decays of charm, bottom or top quark on the global properties of jets produced in e^+e^- annihilation. Because of the weak decay cascade $t\to b\to c\to s$ the number of jets produced by weak interaction increases with the quark threshold. Therefore passing through a new quark threshold will lead to a broadening of the jets. Studying this effect quantitatively might help to locate new quark thresholds.

A related question is that of jet-broadening in e^+e^- experiments. It has been argued in the literature [8] that the QCD phenomenon $e^+e^-\to q\bar{q}g$ may become "visible" in e^+e^- experiments due to jet broadening effects as the centre of mass energy increases. We show that such jet broadening effects are masked due to pair production and subsequent weak decays of heavy quarks. We calculate inclusive quantities like $\langle S\rangle$, $\langle\tilde{S}\rangle$, and $\langle 1-T\rangle$ for the process $e^+e^-\to Q\bar{Q}\to 6$ quarks and compare them with the corresponding quantities for the QCD process $e^+e^-\to q\bar{q}g$. We show that the jet broadening effects due to weak interactions dominate over jet broadening due to the QCD bremsstrahlung process. This is true for energies at least as high as the maximum PETRA, PEP energies.

We start in Sect. 2 with a short description of the KM model and exhibit the basic quark diagrams on which our analysis is based. Also contained in this section are the formulas for heavy quark (meson) decay into two and three quark jets as well as for the decay into one specific particle and two quark jets. We estimate the relative magnitude of these decays and find that the decay of a heavy quark into three quark jets is the dominant process.

In Sect. 3, we present the distributions in thrust and spherocity for the process $Q\to q\bar{q}q$ in the rest frame of the heavy quark. Since the heavy quarks (mesons) will be produced in general in flight, we calculate the effect of Lorentz boost and fragmentation on the distributions in $\tilde{S}$ and $1-T$ for the process $e^+e^-\to c\bar{c},b\bar{b},t\bar{t}\to 6$ quarks. The ensuing distributions are then compared with the ones from the process $e^+e^-\to q\bar{q}g$. Section 4 contains a summary of our results.

2. The Weak Interaction Model

The quark states $\tilde{d}$, $\tilde{s}$, and $\tilde{b}$ in the weak doublets (1.1) are related to the strong interaction eigenstates by a unitary matrix

$$\begin{pmatrix}\tilde{d}\\\tilde{s}\\\tilde{b}\end{pmatrix}=U\begin{pmatrix}d\\s\\b\end{pmatrix}\tag{2.1}$$

with four parameters: three Euler angles θ_1, θ_2, and θ_3 generalizing the Cabibbo angle and a CP violating phase δ [5]. This matrix U, in its most general form, is:

$$U=\begin{pmatrix}c_1 & -s_1c_3 & -s_1s_3\\s_1c_2 & c_1c_2c_3-e^{i\delta}s_2s_3 & c_1c_2s_3+e^{i\delta}s_2c_3\\s_1s_2 & c_1s_2c_3+e^{i\delta}c_2s_3 & c_1s_2s_3-e^{i\delta}c_2c_3\end{pmatrix}\tag{2.2}$$

where $c_i(s_i)\equiv\cos\theta_i(\sin\theta_i)$, $i=1,2,3$.

The corresponding current has the form

$$J_\mu=(\bar{u},\bar{c},\bar{t})\gamma_\mu(1-\gamma_5)U\begin{pmatrix}d\\s\\b\end{pmatrix}.\tag{2.3}$$

In the limit $\theta_1\to\theta_C$, $\theta_2,\theta_3\to 0$ the KM current reduces to the GIM current. For $\delta\to 0$, there is no CP violation induced by (2.3). Furthermore $c_3\simeq 1$ in order to maintain the observed Cabibbo universality. Restrictions on s_2 and δ are discussed in [6]. $|s_2|$ could be larger than $s_1\simeq\sin\theta_C$. For example with $m_t=15\,\text{GeV}$, a value which we shall use in the following, the K_L-K_S mass difference leads to the constraint $s_2^2<0.08$ with $m_c=2\,\text{GeV}$.

With the notation $(\bar{q}_1q_2)=\bar{q}_1\gamma_\mu(1-\gamma_5)q_2$ the current (2.3) leads to the following Hamiltonian for the nonleptonic decay of the b quark:

$$H_{nl}^{|\Delta B|=1}=\frac{G}{\sqrt{2}}\{(c_1c_2s_3+e^{i\delta}s_2c_3)(\bar{c}b)$$

$$\cdot[c_1(\bar{d}u)+(c_1c_2c_3-e^{-i\delta}s_2s_3)(\bar{s}c)]+\text{h.c.}\}\tag{2.4}$$

and similarly for the decay of the quark t:

$$H_{nl}^{|\Delta T|=1}=\frac{G}{\sqrt{2}}\{(c_1s_2s_3-e^{-i\delta}c_2c_3)(\bar{b}t)$$

$$\cdot[c_1(\bar{u}d)+(c_1c_2c_3-e^{i\delta}s_2s_3)(\bar{c}s)]+\text{h.c.}\}.\tag{2.5}$$

In (2.4) and (2.5) we have taken into account only the dominant transitions and neglected all transitions which are of higher order in s_1, s_2 or s_3.

If we want to include strong interaction effects through short-distance enhancement factors we must separate the operators (2.4) and (2.5) into symmetric and anti-symmetric pieces. Then the effective non-leptonic Hamiltonian has the form:

$$H_{nl,eff}^{|\Delta B|=1} = \frac{f_+}{2} [H_{nl}^{|\Delta B|=1} + \text{Fierz transf.}]$$

$$+ \frac{f_-}{2} [H_{nl}^{|\Delta B|=1} - \text{Fierz transf.}] \qquad (2.6)$$

and similarly for $H_{nl,eff}^{|\Delta T|=1}$. Here by "Fierz transf." we mean the transformation of $(\bar{q}_1 q_2)(\bar{q}_3 q_4)$ into $(\bar{q}_1 q_4)(\bar{q}_3 q_2)$. The $f_\pm$ are the "enhancement" factors. They can be calculated with the short-distance expansion [9]. We fix the subtraction points in accordance with Ellis et al. [6]. Then $f_+ = 0.85$, $f_- = 1.4$ for $\Delta B = \pm 1$ transitions and $f_+ = 0.92$, $f_- = 1.18$ in the t region. We see that the enhancement factors are not very important for bottom and top quarks. Therefore we shall neglect them in the following discussion.

The Hamiltonians (2.4) and (2.5) induce the following basic quark transitions:

i) $|\Delta B| = |\Delta C| = 1$, $\Delta S = 0 : b \to c + d + \bar{u}$

ii) $|\Delta B| = |\Delta S| = 1$, $\Delta C = 0 : b \to c + s + \bar{c}$

iii) $|\Delta T| = |\Delta B| = 1$, $\Delta C = \Delta S = 0 : t \to b + u + \bar{d}$ $\qquad (2.7)$

iv) $|\Delta T| = |\Delta B| = |\Delta C| = |\Delta S| = 1 : t \to b + s + \bar{c}$.

The corresponding diagrams for these transitions are shown in Fig. 1. We see that the dominant terms in the weak interaction of b and t with the other quarks obey rather stringent selection rules.

In the following we consider the decay of mesons B composed of a b quark and various antiquarks $\bar{u}$, $\bar{d}$, $\bar{s}$, and $\bar{c}$ and of mesons T composed of a t quark and the antiquarks $\bar{u}$, $\bar{d}$, $\bar{s}$, $\bar{c}$, and $\bar{b}$. We assume that the mesons B and T with the lowest masses have spin-parity 0^-. Of course this depends on details of the spin interactions of the various quarks. Looking at the charm mesons this assumption seems justified. For these pseudoscalar mesons we adopt the following notation

$$B_u^- = b\bar{u}, \quad B_d^0 = b\bar{d}, \quad B_s^0 = b\bar{s}, \quad B_c^- = b\bar{c},$$

$$T_u^0 = t\bar{u}, \quad T_d^+ = t\bar{d}, \quad T_s^+ = t\bar{s}, \quad T_c^0 = t\bar{c}, \quad T_b^+ = t\bar{b}. \qquad (2.8)$$

On the basis of the most simple quark diagrams these mesons can decay by the following two mechanisms:

(α) Annihilation of the heavy quark and the light anti-quark producing new $q\bar{q}$ pairs [diagram (a) and (b) in Fig. 2].
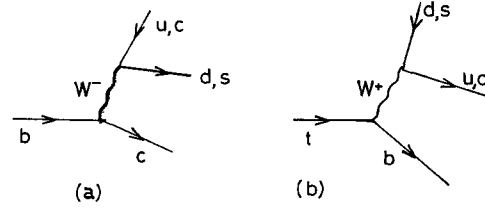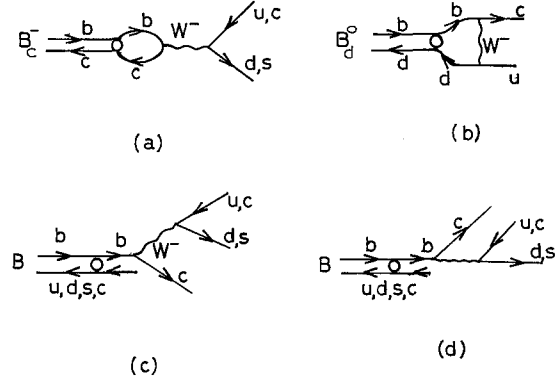

Fig. 1. Basic quark diagrams for b and t decay

Fig. 2. Two and three jet final states in B-decay

(β) Decay of a free b or t quark. Here the anti-quark acts as a spectator while the heavy quark acts as if free [diagram (c) and (d) in Fig. 2].

In this section we shall calculate total decay rates and other decay characteristics for all the mesons (2.8). The quarks in the final states will be interpreted as jets. Then according to mechanism (α) the pseudoscalar meson decays into two jets and with mechanism (β) into three jets and additional slow particles represented by the spectator quark which we shall assume can be disregarded in the calculation of jet properties. We shall start with the two jet process. The formulas are similar to $\pi \to \mu \nu$. Only the pseudoscalar decay constant enters

$$\langle 0 | J_\mu^+ | B_c^- \rangle = -i f_{B_c} p_\mu. \qquad (2.9)$$

The diagrams in Fig. 2b can be reduced to diagrams of Fig. 2a by a Fierz transformation. In this way the matrix element gets an extra factor $1/3$. As an example for a decay according to diagram 2a we consider $B_c^- \to s + \bar{c}$. The formula for the width is

$$\Gamma(B_c^- \to s + \bar{c})$$

$$= \frac{3 G^2}{4\pi} |g_c|^2 f_{B_c}^2 p[(m_c^2 + m_s^2) - (m_c^2 - m_s^2)^2 / m_{B_c}^2]$$

$$\simeq \frac{3 G^2}{8\pi} |g_c|^2 f_{B_c}^2 m_{B_c} m_c^2 (1 - m_c^2 / m_{B_c}^2)^2. \qquad (2.10)$$
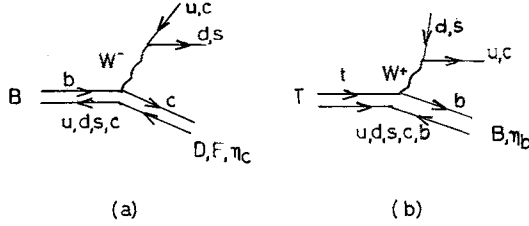
Fig. 3. B and T meson decay into one meson and two quark jets

Here g_c stands for the following combination of KM angles

$$g_c = (c_1 c_2 s_3 + e^{i\delta} s_2 c_3)(c_1 c_2 c_3 - e^{-i\delta} s_2 s_3) \simeq s_3 + s_2$$

$$(2.11)$$

and p is the decay momentum. To proceed further we need to know the particle and quark masses. For the quark masses we take the constituent quark masses

$$m_u = m_d = 0.33 \, \text{GeV}, \qquad m_b = 4.7 \, \text{GeV},$$

$$m_s = 0.51 \, \text{GeV}, \qquad m_t = 15 \, \text{GeV}, \qquad (2.12)$$

$$m_c = 1.55 \, \text{GeV}.$$

The meson masses are assumed to be given simply by the appropriate sums of the constituent quark masses. For the decay constants f_B and f_T we take the estimate [10, 11]

$$f_P = m_P / \sqrt{32\pi}$$

$$(2.13)$$

where we put $m_P = 5 \, \text{GeV}$ for B mesons and $m_P = 15 \, \text{GeV}$ for T mesons.

Under these assumptions the decay $B_c^- \rightarrow d + \bar{u}$ is negligible compared to $B_c^- \rightarrow s + \bar{c}$. The only remaining two jet decays for B mesons are $B_c^- \rightarrow s + \bar{c}$, $B_d^0 \rightarrow c + \bar{u}$, $B_s^0 \rightarrow c + \bar{c}$, whereas B_u^- has no two-jet decay in this approximation. The relative rates of these decays are:

$$B_d^0 : B_s^0 : B_c^- = \frac{1}{9} : \frac{2}{9} : 1$$

$$(2.14)$$

if we neglect m_u and m_s compared to m_c. Similarly for T mesons only the following mesons have two-jet decays: $T_u^0 \rightarrow b + \bar{d}$, $T_c^0 \rightarrow b + \bar{s}$, and $T_b^+ \rightarrow c + \bar{s}$. Their relative rates are

$$T_u^0 : T_c^0 : T_b^+ = \frac{1}{9} : \frac{1}{9} : \frac{m_c^2}{m_b^2}.$$

$$(2.15)$$

Notice that T_d^+ and T_s^+ have no two-jet decays in this approximation. The absolute rates for these decays will be discussed later. We remark that only a subgroup of the four B mesons and the five T mesons has these two-jet events. Furthermore the jets have particular flavour content. For example, in the B decays open or hidden charm should be present and in T_u^0 and T_c^0 open or

hidden b flavour should occur. The interpretation of the two-quark final state as two jets is presumably justified for the T mesons and much less for the B mesons. If we compare with $e^+ e^-$ annihilation then we expect visible jets for rest masses above 7 GeV.

We proceed now to the three-jet decay. Under the assumption that the initial meson $B(T)$ transfers its total energy to the heavy quark b (or t) with unit probability the decay of the meson is identical to free quark decay into three other quarks which is similar to ordinary μ decay. As an example, we consider $b \rightarrow c + d + \bar{u}$ (see Fig. 1). The quarks in the final state have momenta p_1, p_2, p_3 and masses m_1, m_2, m_3. The decay matrix element M is

$$M = \frac{G}{\sqrt{2}} \langle \bar{u} d | J_\mu^+ | 0 \rangle \langle c | J^\mu | b \rangle$$

$$= \frac{G}{\sqrt{2}} \sqrt{3} g_c \bar{u}_d(p_2) \gamma_\mu (1 - \gamma_5) v_u(p_3)$$

$$\cdot \bar{u}_c(p_1) \gamma^\mu (1 - \gamma_5) u_b(p_1).$$

$$(2.16)$$

For the total decay width one obtains, using $m_2 = m_3 = 0$

$$\Gamma = \frac{3 G^2 |g_c|^2}{192 \pi^3} m_b^5 f(x)$$

$$(2.17)$$

where $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x$ and $x = m_1 / m_b$ [11].

Further details which characterize the final three jets are contained in the double energy distribution (Dalitz distribution). For example, for the energies E_1 and E_2, it is:

$$\frac{d^2 \Gamma}{dE_1 dE_2} = \frac{3 G^2 |g_c|^2}{2 \pi^3} (m_b - E_1 - E_2)$$

$$\cdot [2 m_b(E_1 + E_2) - m_b^2 - m_1^2 - m_2^2 + m_3^2]. \quad (2.18)$$

The single-jet energy distribution derived from (2.18) looks rather complicated. It simplifies again for $m_2 = m_3 = 0$, with the result:

$$\frac{d\Gamma}{dE_1} = \frac{3 G^2 |g_c|^2}{12 \pi^3}$$

$$\cdot p_1 [3 E_1(m_b^2 + m_1^2) - 2 m_b m_1^2 - 4 m_b E_1^2]. \quad (2.19)$$

It is obvious that these formulas are directly applicable also to $t \rightarrow b + u + \bar{d}$ with the appropriate changes in notation. These formulas will be utilised in the next section to derive distributions in terms of the new jet variables spherocity and thrust.

A subclass of processes generated by diagrams (c) and (d) in Fig. 2, are states where the spectator quark contained in the initial meson combines with the c quark in b decay (or b quark in t decay) to form a single meson in the final state. The relevant diagrams are shown in Fig. 3. The final state consists of one specific

Table 1. Relarive contribution of the various final states in the decay of pseudoscalar bottom mesons. Last column contains lifetime estimates in units of $|g_c|^2 = (s_2^2 + s_3^2 + 2s_2 s_3 \cos\delta)$

Meson \ Modes	Leptonic			Semi-Leptonic			Non-Leptonic			Lifetime (sec) $\times 10^{-14}$
	$e\nu_e$	$\mu\nu_\mu$	$\tau\nu_\tau$	$c\,(e\nu_e)$	$c\,(\mu\nu_\mu)$	$c\,(\tau\nu_\tau)$	2 quarks	$c\,(\bar{u}d)$	$c\,(\bar{c}s)$	
B_d^0	0	0	0	1	1	0.3	0.2	3	0.6	4.2
B_u^-	~ 0	~ 0	~ 0	1	1	0.3	~ 0	3	0.6	4.3
B_s^0	0	0	0	1	1	0.3	0.3	3	0.6	4.1
B_c^-	~ 0	~ 0	~ 0.6	1	1	0.3	2.4	3	0.6	2.8

particle and two quark jets. In particular we generate the following decays with this mechanism:

(I)

$$B_u^- \to D^0 + d + \bar{u}, \qquad D^0 + s + \bar{c}$$
$$B_d^0 \to D^+ + d + \bar{u}, \qquad D^0 + s + \bar{c} \tag{2.20}$$
$$B_s^0 \to F^+ + d + \bar{u}, \qquad F^+ + s + \bar{c}.$$

(II)

$$T_u^0 \to B_u^- + u + \bar{d}, \qquad B_u^- + c + \bar{s}$$
$$T_d^+ \to B_d^0 + u + \bar{d}, \qquad B_d^0 + c + \bar{s}$$
$$T_s^+ \to B_s^0 + u + \bar{d}, \qquad B_s^0 + c + \bar{s} \tag{2.21}$$
$$T_c^0 \to B_c^- + u + \bar{d}, \qquad B_c^- + c + \bar{s}$$
$$T_b^+ \to \eta_b + u + \bar{d}, \qquad \eta_b + c + \bar{s}.$$

In (2.21) the particle η_b, the analogue of η_c, is the pseudoscalar $\bar{b}b$ state. Instead of pseudoscalar mesons D, F, η_c, B, and η_b we can also have vector mesons, D^*, F^*, Ψ's, B^*, and Υ's or axial vector mesons, tensor mesons etc. in the final state. In the following we shall write down the formulas for energy distributions and total rates in the case that the final meson is an 0^- or 1^- state. The matrix element for a typical process of (2.20) is written as:

$$M = \frac{G}{\sqrt{2}} g_c \langle \bar{u}d | J_\mu^+ | 0 \rangle \langle D^0 | J^\mu | B_u^- \rangle \tag{2.22}$$

and similarly for $B_u^- \to D^{*0} + d + \bar{u}$. The current matrix elements in (2.22) are:

$$\langle \bar{u}d | J_\mu^+ | 0 \rangle = \sqrt{3}\,\bar{u}_d(p_2)\gamma_\mu(1 - \gamma_5)v_u(p_3) \tag{2.23}$$

$$\langle D^0 | J_\mu | B_u^- \rangle = f_+(p + p_1)_\mu \tag{2.24}$$

$$\langle D^{*0} | J_\mu | B_u^- \rangle = f_A \varepsilon_\mu(D^*). \tag{2.25}$$

The momenta of $D^0(D^{*0})$, u, and $\bar{d}$ are p_1, p_2, p_3 respectively and p is the momentum of the B^-. The q^2 dependent form factor of the vector transition in (2.24) is approximated by a constant $f_+ = 1$, since the q^2 involved is small compared to $m_{B_c^*}^2$. For the axial vector form factor we shall take $f_A = 2m_{D^*}$ as follows from charge symmetry at infinite momentum [7]. The dou-

ble differential distribution with respect to the energies E_2 and E_3 of the two jets u and $\bar{d}$ come out to be (for $B_u^- \to D^0 + u + \bar{d}$):

$$\frac{d^2\Gamma}{dE_2 dE_3} = \frac{3G^2 |g_c|^2 f_+^2}{8\pi^3}$$
$$\cdot m_B(4E_2 E_3 + (m_B^2 - m_D^2) - 2m_B(E_2 + E_3)) \tag{2.26}$$

and for $B_u^- \to D^{*0} + u + \bar{d}$:

$$\frac{d^2\Gamma}{dE_2 dE_3} = \frac{3G^2 |g_c|^2 f_A^2}{32\pi^3 m_{D^*}^2 m_B}$$
$$\cdot \left[m_{D^*}^2(m_{D^*}^2 - m_B^2 + 2m_B(E_2 + E_3)) \right.$$
$$\left. + (m_B^2 - m_{D^*}^2 - 2m_B E_2)(m_B^2 - m_{D^*}^2 - 2m_B E_3) \right]. \tag{2.27}$$

For the calculation of the total rates we use the approximation $m_u = m_d = 0$. For $B_u^- \to D^0 + u + \bar{d}$ one has:

$$\Gamma = \frac{3G^2 |g_c|^2 f_+^2}{768\pi^3} m_B^5 f(x) \tag{2.28}$$

with the same function $f(x)$ as in (2.17) and $x = \dfrac{m_D}{m_B}$. For the vector meson decay we obtain,

$$\Gamma = \frac{3G^2 |g_c|^2}{768\pi^3} \frac{f_A^2}{4m_{D^*}^2} m_B^5 f_1(x) \tag{2.29}$$

where $f_1(x) = 1 + 72x^4 - 64x^6 - 9x^8 + (72x^4 + 96x^6)\ln x$ and $x = \dfrac{m_{D^*}}{m_B}$ in this case. It is obvious that these formulas can be applied also to the corresponding T meson decays.

The formulas for the total rates give us estimates of the contributions of the various jet-channels. First from (2.17) we get for the rate $b \to c + \bar{u} + d \to 3$ jets putting $g_c = 1$.

$$\Gamma(b \to 3 \text{ jets}) = 1.2 \cdot 10^{14}\,\text{s}^{-1}. \tag{2.30}$$

Since $|g_c| \simeq s_2 + s_3$ and $s_2^2 < 0.08$, we have $\Gamma(b \to 3\,\text{jets}) < 10^{13}\,\text{s}^{-1}$. The same estimate for $t \to 3$ jets gives

$$\Gamma(t \to 3\,\text{jets}) = 4 \cdot 10^{16}\,\text{s}^{-1}. \tag{2.31}$$

Since for $t \rightarrow b + u + \bar{d}$ (or $b + c + \bar{s}$) the angle factor is $g_c \simeq 1$, all T mesons are expected to have lifetimes at least three orders of magnitudes shorter than the B mesons.

Comparing (2.28) with (2.17) it is clear that for B and T decay the branching ratio for 0^- meson $+2$ jets to 3 jets is the same:

$$\frac{\Gamma(B \rightarrow D + 2\,\text{jets})}{\Gamma(B \rightarrow 3\,\text{jets})} = \frac{\Gamma(T \rightarrow B + 2\,\text{jets})}{\Gamma(T \rightarrow 3\,\text{jets})} \simeq \frac{1}{4}. \tag{2.32}$$

The branching ratio of $B \rightarrow D^* + 2$ jets to $B \rightarrow 3$ jets (and similarly for T's) depends crucially on the choice of the axial vector form factor. With our assumption $f_A = 2m_{D^*}$ (or $f_A = 2m_{B^*}$ for T decay) [12] we obtain:

$$\frac{\Gamma(B \rightarrow D^* + 2\,\text{jets})}{\Gamma(B \rightarrow 3\,\text{jets})} \simeq 0.33 \tag{2.33}$$

$$\frac{\Gamma(T \rightarrow B^* + 2\,\text{jets})}{\Gamma(T \rightarrow 3\,\text{jets})} \simeq 0.38. \tag{2.34}$$

For (2.24) we took $m_{B^*} = 5\,\text{GeV}$. If one uses the nonrelativistic quark model value $f_A = (m_B + m_{D^*})$ or $(m_T + m_{B^*})$ respectively the ratio in (2.33) is multiplied by 3.3 and in (2.34) by a factor 4. We conclude from this that the SU(6) value for f_A [12] is unreasonable for very large quark masses. Presumably even the assumption $f_A = 2m_{D^*}$ might lead to an overestimate for the rate into $D^* + 2$ jets.

The branching ratio for the two-jet final state is estimated with (2.10). With $f_{B_c} = 0.5\,\text{GeV}$ according to (2.13) we obtain (using $b \rightarrow c + \bar{u} + d \rightarrow 3$ jets)

$$\frac{\Gamma(B_c \rightarrow 2\,\text{jets})}{\Gamma(B_c \rightarrow 3\,\text{jets})} = 0.6 \tag{2.35}$$

and

$$\frac{\Gamma(T_b \rightarrow 2\,\text{jets})}{\Gamma(T_b \rightarrow 3\,\text{jets})} = 0.45. \tag{2.36}$$

In Table 1, we present the relative rates for all the important decay modes of the top and bottom pseudoscalar mesons, as well as their lifetime estimates. The table serves to show that the dominant contribution to the non-leptonic decays of the top and bottom mesons comes from the three quark final states. In the next section, we interpret these quarks as jets and calculate the jet distributions ensuing from the weak decay of a heavy quark, $Q \rightarrow q\bar{q}q$.

3. Jet-Distributions from Heavy Quark Decays

In this section we shall discuss the distributions which are due to the production and subsequent weak decay of a pair of heavy quarks in e^+e^- annihilation experi-

ments. Weak decays of heavy quarks produce large effects near the threshold of the quarks. Hence, the 2-jet like distribution before the onset of a threshold will get distorted. Of course, much above such thresholds, an effective two-quark jet structure will again be established. Our aim is to study the various jet distributions quantitatively and compare them with the ones due to the QCD process $e^+e^- \rightarrow q\bar{q}g$ and from the non-perturbative process $e^+e^- \rightarrow q\bar{q}$ $(q=u,d,s)$. We shall study two kinematic regions,

a) threshold region, where the heavy quark (meson) decays in its rest frame

b) deep inelastic region, where the pair of heavy quarks first fragments into heavy mesons, which subsequently decay into six quarks.

a) Rest Frame Characteristics

We start with a study of heavy quark decay in its rest system. The matrix elements for the decay into two and three quarks have been given in the last section together with the resulting decay distributions. It has been shown there that the decay into three quarks, which in our interpretation means decay into three jets, is the dominant channel. For two quark decay channel the relevant variables for a jet analysis — spherocity S and thrust T — have trivial values, i.e. $T = 1$ and $S = 0$ except for non-perturbative effects. Therefore we discuss in the following only the decay into three quark jets.

The definitions for spherocity S and thrust T have been given in [8]. These definitions of S and T have been written down in connection with a "3-particle" final state in e^+e^- annihilation but can be taken over to the case of the decay of a heavy quark into "three particles". The center-of-mass energy in e^+e^- annihilation is now replaced by the mass of the decaying quark. Thrust is then defined as

$$T = \max(x_1, x_2, x_3), \qquad x_i = \frac{2E_i}{m_b} \tag{3.1a}$$

and the expression for spherocity is

$$S = \frac{64}{\pi^2 T^2} (1 - x_1)(1 - x_2)(1 - x_3). \tag{3.1b}$$

Table 2. Jet characteristics from the decay of b and t quarks in their rest frame

	$b \rightarrow c + \bar{u} + d$, $m_b = 5\,\text{GeV}$	$t \rightarrow b + u + \bar{d}$, $m_t = 15\,\text{GeV}$
$\langle 1 - T \rangle$	0.13	0.125
$\langle S \rangle$	0.18	0.18
$\langle E_{\text{jet 1}} \rangle$	2.23 GeV	6.98 GeV
$\langle E_{\text{jet 2}} \rangle$	1.3 GeV	4.23 GeV
$\langle E_{\text{jet 3}} \rangle$	1.46 GeV	3.77 GeV

We also give the expression for sphericity $\tilde{S}$ which is used frequently in the analysis of e^+e^- experiments. For a "three particle" final state $\tilde{S}$ takes the form:

$$\tilde{S} = \frac{12(1-x_1)(1-x_2)(1-x_3)}{x_1^2(x_1^2+x_2^2+x_3^2)}. \tag{3.2}$$

The double differential distribution for the heavy quark decay (in S and T) is then given by:

$$\frac{1}{\Gamma}\frac{d^2\Gamma}{dSdT} = \frac{3\pi^2 T}{8(1-T)[1-\pi^2S/16(1-T)]^{1/2}}$$
$$\cdot\{(2-T-x_{2+})(T+x_{2+}-1)$$
$$+(2-T-x_{2-})(T+x_{2-}-1)+T(T-1)\}. \tag{3.3}$$

We have divided the distribution by the total decay rate Γ for the decay into three quarks which has been given in (2.17). $x_{2\pm}$ are functions of S and T which are covering different regions of phase space and are given by:

$$x_{2\pm} = 1 - \frac{T}{2}(1\pm(1-\pi^2S/16(1-T))^{1/2}). \tag{3.4}$$

For a three particle event S and T are limited to $2/3 \leq T \leq 1$

$$\frac{64(1-T)^2(2T-1)}{\pi^2 T^2} \leq S \leq \frac{16}{\pi^2}(1-T). \tag{3.5}$$

Integrating over S we obtain the single differential decay distribution

$$\frac{1}{\Gamma}\frac{d\Gamma}{dT} = 12(\tfrac{4}{3} - 10T + 18T^2 - 9T^3) \tag{3.6}$$

which fullfills $\left.\dfrac{1}{\Gamma}\dfrac{d\Gamma}{dT}\right|_{T=2/3} = 0$ and the normalization condition:

$$\int_{2/3}^{1} dT \frac{1}{\Gamma}\frac{d\Gamma}{dT} = 1.$$

The form of the analogous formulas for the case of massive quarks in the final state is somewhat clumsy; the major difficulty being the square root in the energy momentum relation, $|\mathbf{p}_i| = (E_i^2 - m_i^2)^{1/2}$. Instead, we consider the case with two masses equal to zero ($m_1 \neq 0$, $m_2 = m_3 = 0$). This is relevant for the decays $b \to c + \bar{u} + d$ as well as for $t \to b + u + \bar{d}$ and $t \to s + u + \bar{d}$. Using the definition

$$x_i = \frac{2|\mathbf{p}_i|}{\sum_j |\mathbf{p}_j|}, \quad \sum_j x_i = 2 \tag{3.7}$$

which agrees in the massless case with $x_i = 2E_i/m_b$; the relations (3.1) still hold, as well as the transformation

between x_1, x_2 and S, T:

$$dx_1 dx_2 = \frac{\pi^2 T}{64(1-T)}(1-\pi^2S/16(1-T))^{-1/2}dSdT. \tag{3.8}$$

The relations between x_i and E_i are, however, complicated in this case.

$$dE_1 dE_2 = \frac{dx_1 dx_2 x_1}{4(1-x_1)^2(2-x_1)(2-x_2)}$$
$$\cdot\{m_b(2-x_1)-[m_b^2-x_1^2+4m_1(1-x_1)]^{1/2}\}^3$$
$$\cdot\{x_1^2[2m_b^2(2+x_1^2-2x_1)+4m_1^2(1-x_1)$$
$$-2m_b(2-x_1)(m_b^2x_1^2+4m_1^2(1-x_1))^{1/2}]$$
$$+16m_1^2(1-x_1)^2\}^{-1/2} \tag{3.9}$$

and

$$E_2 = \frac{x_2}{4(1-x_1)}\{m_b(2-x_2)-(m_b^2x_1^2+4m_1^2(1-x_1))^{1/2}\} \tag{3.10}$$

$$E_1 = \frac{1}{4(1-x_1)}\{x_1^2[2m_b^2(2+x_1^2-2x_1)+4m_1^2(1-x_1)$$
$$-2m_b(2-x_1)(m_b^2x_1^2+4m_1^2(1-x_1))^{1/2}]$$
$$+16m_1^2(1-x_1)^2\}^{1/2}.$$

Starting from the double differential distribution (2.18) with $m_2 = m_3 = 0$

$$\frac{d^2\Gamma}{dE_1 dE_2} = \frac{3G^2|g_c|^2}{2\pi^3}(m_b - E_1 - E_2)$$
$$\cdot(2m_b(E_1+E_2)-m_b^2-m_1^2) \tag{3.11}$$

we can use (3.9) and (3.10) to replace E_1 and E_2 by x_1 and x_2 and then use (3.1), (3.8), and (3.4) to express x_1 and x_2 through S and T. The resulting formula for $d^2\Gamma/dSdT$ for the heavy quark decay in the rest frame is cumbersome and is not presented here. We have calculated this distribution and the average values of S and T numerically.

The main results are presented in Table 2 for the two cases of interest, namely $b \to c + \bar{u} + d$ and $t \to b + u + \bar{d}$, where we have assumed $m_b = 5.0\,\text{GeV}$ and $m_t = 15.0\,\text{GeV}$. The numbers for $\langle 1-T\rangle$ and $\langle S\rangle$ for the QCD process $q\bar{q}g$ are 0.08 for $E_{c.m.} = 10\,\text{GeV}$ and 0.06 for $E_{c.m.} = 30\,\text{GeV}$. Thus the distribution in T and S from the weak decay of a heavy quark are broader as compared to the QCD bremsstrahlung process.

There are several comments in order at this point. First, note that the available energy per quark is rather small for b-quark (B-meson) decay so that the dressing of the final quarks (non-perturbative confinement effects) will considerably broaden the distributions. This will lead to even bigger values for $\langle 1-T\rangle$ and $\langle S\rangle$ for the quark decay case. Viewed in this way, our quark-parton approach may amount to underestimating
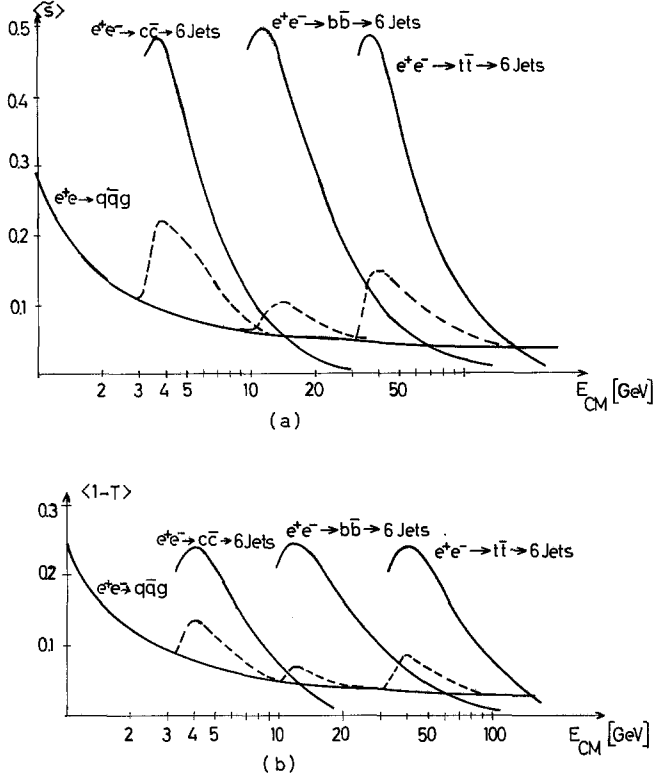
Fig. 4. a Average sphericity for $e^+e^- \to q\bar{q}g$ and $e^+e^- \to c\bar{c} \to 6$ jets, $e^+e^- \to b\bar{b} \to 6$ jets and $e^+e^- \to t\bar{t} \to 6$ jets; dotted line: total average sphericity $\langle \tilde{S} \rangle$ from $e^+e^- \to c\bar{c}, b\bar{b}, t\bar{t} \to 6$ quarks and $e^+e^- \to q\bar{q}g$ with the normalization as explained in Sect. 3. b $\langle 1 - T \rangle$ distributions

$\langle 1 - T \rangle$ and $\langle S \rangle$ for the b-quark. The quark-parton model, however, is expected to reproduce the actual situation much more faithfully for the decay of the t-quark. It is conceivable that a six jet final state may actually be seen near the threshold of a $T\bar{T}$ meson in e^+e^- experiments. Second, note that $\langle S \rangle$ and $\langle 1 - T \rangle$ are approximately the same for the decays $b \to c + \bar{u} + d$ and $t \to b + u + \bar{d}$. This stems from the fact that both $\langle S \rangle$ and $\langle 1 - T \rangle$ are sensitive to the ratio m_b/m_c and m_t/m_b only, which with our input are roughly the same. Finally, $\langle 1 - T \rangle$, $\langle S \rangle$ and $\langle E_{\text{jet}} \rangle$ are not sensitive to the chirality of the bcW^- or the tbW^+ vertex $((V - A)$ vs. $(V + A))$, with the exception that $\langle E_{\text{jet}(\frac{2}{3})} \rangle_{(V - A)}$ $= \langle E_{\text{jet}(\frac{1}{3})} \rangle_{(V + A)}$.

b) e^+e^- Annihilation into Quark Jets

We now turn to the problem of a heavy quark pair production in e^+e^- experiments much above their threshold. The framework we use here to calculate the various jet distributions is the quark-parton model. The basic process is described by the 1-photon production of a pair of heavy quarks

$$e^+e^- \to Q\bar{Q} \tag{3.12}$$

followed by independent fragmentation of both the heavy quarks. Each heavy quark fragments into a heavy hadron, having the same flavour, plus a number of soft quarks (light mesons). The momentum distribution is given by a fragmentation function. $D_Q^h(z)$ with $z = |\mathbf{p}_h|/|\mathbf{p}_Q| = |\mathbf{p}_h|/(q^2/4 - m_Q^2)^{1/2}$. The non-leptonic decay of these heavy hadrons is then described by the three quark final state, as discussed in the preceding section.

The Lorentz-invariant density matrix for the process (3.12) is given by

$$|M|^2 = \frac{\alpha^2}{q^4} \{ (l_+ p_1)(l_- p_2) + (l_+ p_2)(l_- p_1) + m_Q^2 q^2 / 2 \}. \tag{3.13}$$

Here $l_-(l_+)$ is electron (positron) momentum and $p_1(p_2)$ is the momentum of $Q(\bar{Q})$. $q = p_1 + p_2 = l_- + l_+$ and the quark mass is denoted by m_Q.

The fragmentation function, $D_Q^h(z)$ satisfies the normalization condition

$$\int_0^1 dz \, z D_Q^h(z) = 1. \tag{3.14}$$

In conformity with the usual practice, only the parallel component of p_Q is assumed to fragment. With this definition

$$\int_0^1 dz D_Q^h(z) = n \tag{3.15}$$

where n is the (particle) multiplicity from one quark. For the fragmentation of a heavy quark it is generally anticipated that in the process

$$Q \to (Q\bar{q}) + q$$

the meson h containing the heavy quark Q carries the bulk of the parent quark momentum. Such a picture is also supported by the inclusive lepton momentum measurements in v-dimuon [13] and e^+e^- colliding beam experiments [14], where the data favours

$$D_c^D(z) = \text{constant}. \tag{3.16}$$

On the other hand Bjorken [15] has argued that for super heavy quarks, one anticipates

$$\langle z_h \rangle = 1 - (1 \text{ GeV})/m_Q.$$

With this picture in mind we choose

$$D_Q^h(z) = z \tag{3.17}$$

for the top and bottom quarks [15].
Finally, the dynamics of the weak decay process

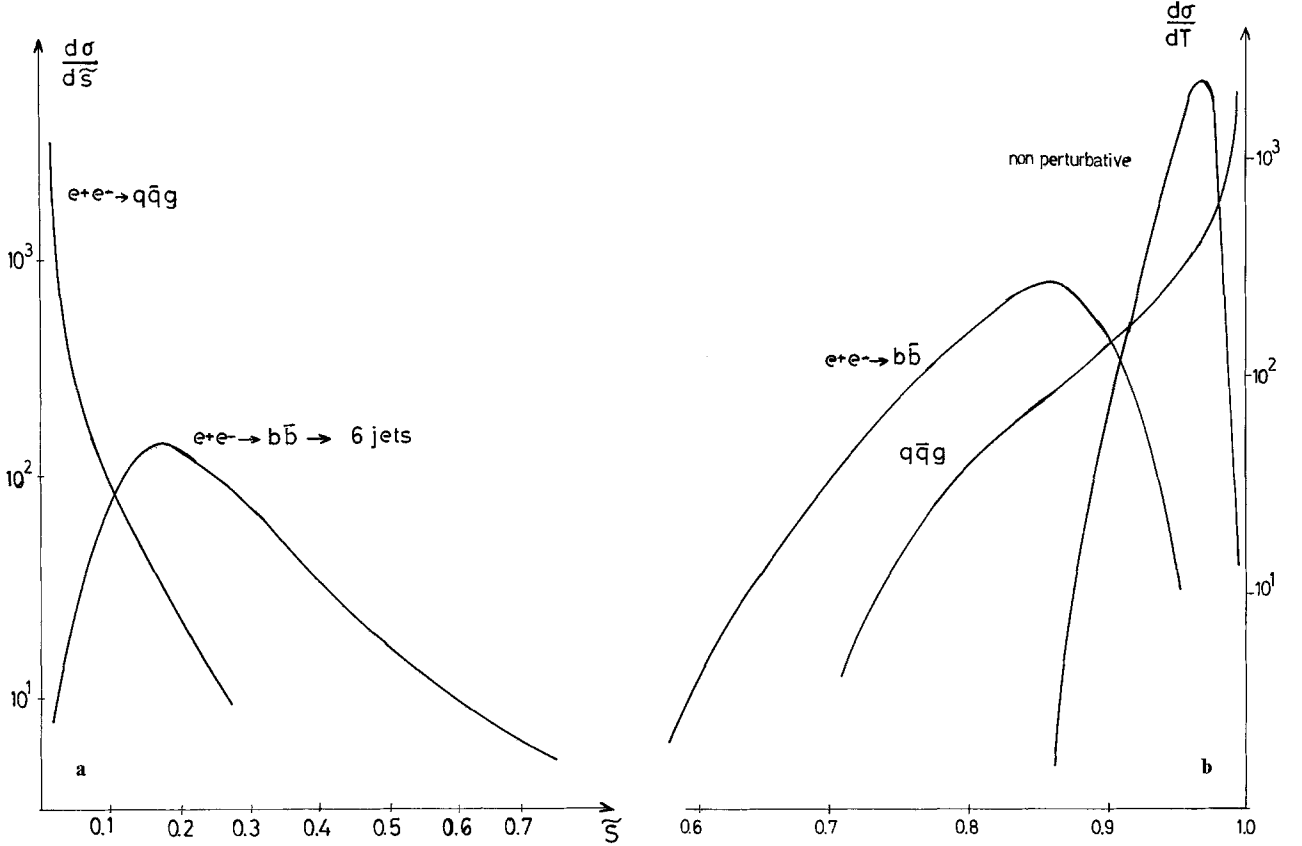$$Q(p) \to q_1(q_1) + \bar{q}_2(q_2) + q_3(q_3) \tag{3.18}$$

Fig. 5. a Sphericity distribution at $E_{c.m.} = 20\,\text{GeV}$ from $e^+e^- \to q\bar{q}g$ (for $m_q = 0$) and from $e^+e^- \to b\bar{b} \to 6$ jets. **b** Thrust distribution at $E_{c.m.} = 20\,\text{GeV}$ from $e^+e^- \to q\bar{q}g$, $e^+e^- \to b\bar{b} \to 6$ jets and non-perturbative effects. (The non-perturbative curve has been taken from De Rujula et al. in [8]). Relative normalization of $b\bar{b}$ and $q\bar{q}g$ distributions given by multiplying $q\bar{q}g$ distribution by a factor 10 according to Eq. (3.21)

is given by the following (Lorentz invariant) density matrix:

$$|M|^2 = 32 G^2 \{ (q_1 q_2)(pq_3)(1+\varepsilon)$$
$$+ (q_1 q_3)(pq_2)(1-\varepsilon) \} \tag{3.19}$$

where $\varepsilon = \pm 1$ for $V \pm A$, and we have suppressed an overall normalization factor due to weak angles.

We have adopted the following procedure for the numerical calculations. We use a Monte-Carlo phase space program, FOWL, to generate the process (3.12) at a given center of mass energy. Next, we implement the fragmentation procedure using the fragmentation function given by (3.16) for each quark independently. Then events are generated for the decay process (3.18) in the rest frame of the heavy hadron, h, with the invariant density matrix given by (3.19). Finally, the events are Lorentz-boosted to the frame of their respective heavy hadrons. We then calculate the distributions in sphericity, $\tilde{S}$, and thrust, using an algorithm proposed in [16].

We have refrained from calculating the distribution in spherocity, S, despite its being an infra-red finite quantity, due to the unavailability of a reliable algor-

ithm to determine the spherocity axis. A detailed discussion of the properties of the jet variables, S, $\tilde{S}$, and T is contained in [16].

Our results are contained in Figs. 4–6. In Fig. 4, we have plotted the average value of sphericity, $\langle \tilde{S} \rangle$, and thrust, $\langle 1 - T \rangle$ for the decay of c, b, and t quarks and compared them with the corresponding quantities from the QCD process $e^+e^- \to q\bar{q}g$. It is clear that for the region above charm threshold extending up to PETRA, PEP energies the "jet broadening" effects from the weak decays of heavy quarks are sizeable, and most probably will swamp the QCD process, $q\bar{q}g$. Also plotted in Fig. 4 are the normalized $\langle \tilde{S} \rangle$ and $\langle 1 - T \rangle$ distributions. For $c\bar{c}$ production we have added the contribution of $e^+e^- \to c\bar{c} \to 6$ jets and $e^+e^- \to q\bar{q}g$ according to the formula (for $\langle \tilde{S} \rangle$ for example)

$$\langle \tilde{S} \rangle_{\text{tot}} = \frac{\frac{6}{3}\sigma(e^+e^- \to q\bar{q})\langle \tilde{S} \rangle_{q\bar{q}g} + \frac{4}{3}\sigma(e^+e^- \to c\bar{c})\langle \tilde{S} \rangle_{6\,\text{jets}}}{\frac{6}{3}\sigma(e^+e^- \to q\bar{q}) + \frac{4}{3}\sigma(e^+e^- \to c\bar{c})} \tag{3.20}$$

as follows from the quarks charges for u, d, s, and c quarks. Here $\sigma(e^+e^- \to q\bar{q})$ is the pointlike cross section

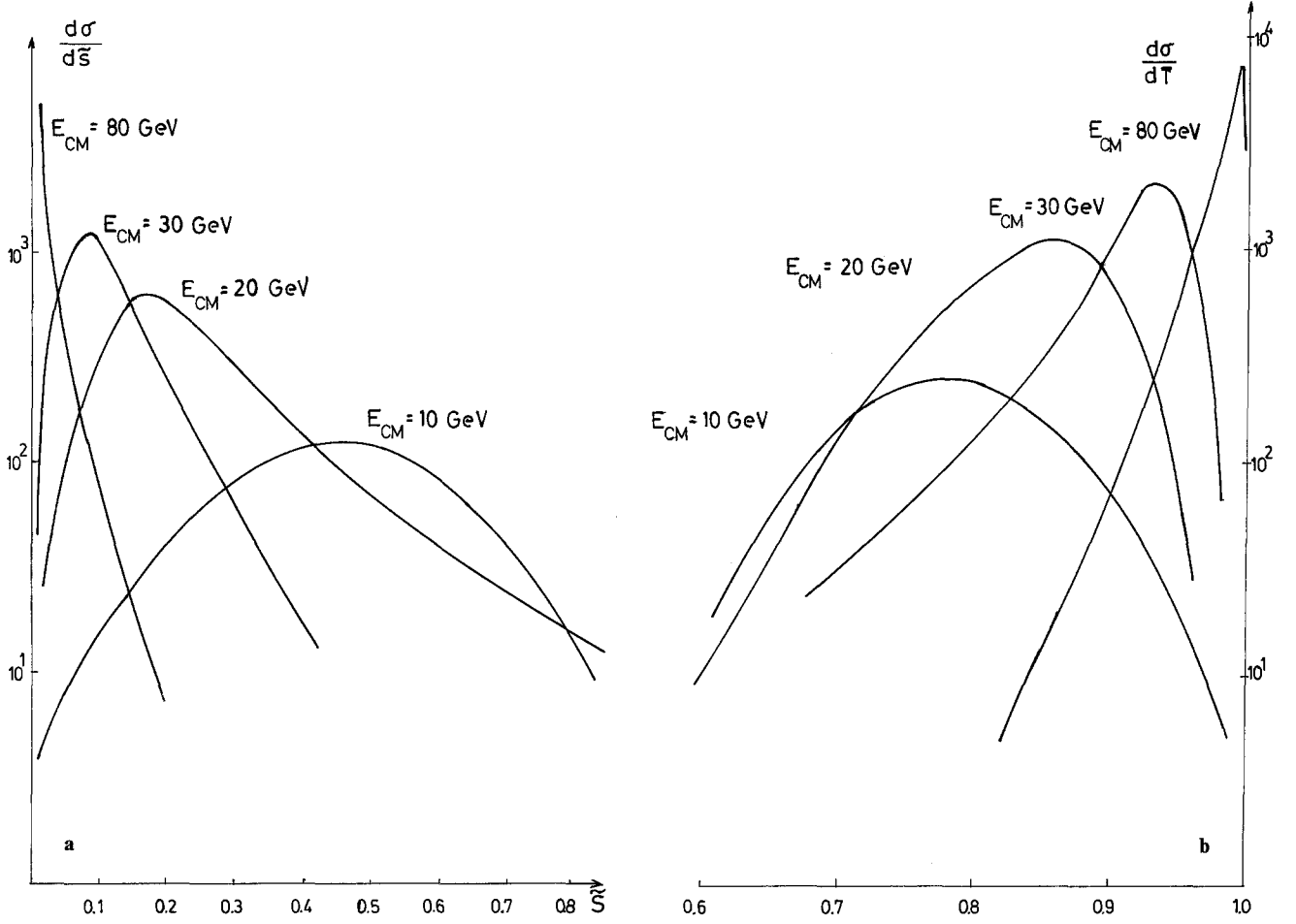**Fig. 6. a** Sphericity distributions from the process $e^+e^- \to b\bar{b} \to 6$ jets for different energies in the e^+e^- colliding beam experiments. $E_{c.m.}$ $= 10\,\text{GeV}$ gives the decay in the rest frame of the bottom meson (quark). (The distributions are not normalized.) **b** The same for the thrust distribution

for the production of a pair of quarks with mass m_q and unit charge.

Close to threshold, the $c\bar{c}$ point cross section in (3.20) only averages the experimental cross section which shows direct channel resonance effects. However, for higher energies the $c\bar{c}$ point cross section should be a good approximation to the experimental charm production cross section. Note that (3.20) results in a shoulder above the $c\bar{c}$ threshold which has actually been observed [17], though in this region the non-perturbative effects are rather large. Since we have not included the non-perturbative effects and our $c\bar{c}$ point cross section does not include direct channel resonance effects, we don't expect to reproduce the correct value of $\langle \tilde{S} \rangle = 0.35$ measured at $E_{c.m.} = 4\,\text{GeV}$ [17].

At the $b\bar{b}$ threshold, and more so at the $t\bar{t}$ threshold, the non-perturbative effects are smaller. Thus, a measurement of average sphericity or thrust may be a rather good measure of detecting these thresholds. For the $b\bar{b}$ case, however, the weak interaction shoulder

may be suppressed compared to $c\bar{c}$, assuming $Q(b) = -1/3$, though direct channel $b\bar{b}$ resonance effects may enhance it.

The normalized distribution in $\langle \tilde{S} \rangle$ corresponding to the $b\bar{b}$ and $t\bar{t}$ production are also plotted in Fig. 4. Using the expressions

$$\langle \tilde{S} \rangle_{tot}$$
$$= \frac{\frac{10}{3}\sigma(e^+e^- \to q\bar{q})\,\langle \tilde{S} \rangle_{q\bar{q}g} + \frac{1}{3}\sigma(e^+e^- \to b\bar{b})\,\langle \tilde{S} \rangle_{6\,jets}}{\frac{10}{3}\sigma(e^+e^- \to q\bar{q}) + \frac{1}{3}\sigma(e^+e^- \to b\bar{b})} \quad (3.21)$$

and

$$\langle \tilde{S} \rangle_{tot}$$
$$= \frac{\frac{11}{3}\sigma(e^+e^- \to q\bar{q})\,\langle \tilde{S} \rangle_{q\bar{q}g} + \frac{4}{3}\sigma(e^+e^- \to t\bar{t})\,\langle \tilde{S} \rangle_{6\,jets}}{\frac{11}{3}\sigma(e^+e^- \to q\bar{q}) + \frac{4}{3}\sigma(e^+e^- \to t\bar{t})} .$$

In Fig. 5 we have plotted $d\sigma/d\tilde{S}$ and $d\sigma/dT$ for $E_{c.m.} = 20\,\text{GeV}$, obtained from the weak decays of the b-quark. Note that the distributions from the b quark

decay are very different from the other competing processes $e^+e^- \to q\bar{q}g$ and non-perturbative effects (the curve for $d\sigma/dT$ (non-perturbative) has been taken from de Rujula et al. [8]). These distributions might be useful to experimentalists for choosing cuts on sphericity and thrust so as to enhance the effects due to new quark production.

In Fig. 6 we have plotted the distribution in $\tilde{S}$ and T for various e^+e^- colliding beam energies, which serve to show the effect of Lorentz boost quantitatively, taking into account the quark masses explicitly.

Let us remark that a realistic treatment of the jet distributions requires inclusion of non-perturbative effects as well as effects of the heavy quark cascades. The cascade decay of the b-quark

$$b \to c + \bar{u} + d$$
$$\quad \hookrightarrow s + u + \bar{d}$$

as well as of the t-quark

$$t \to b + u + \bar{d}$$
$$\quad \hookrightarrow c + \bar{u} + d$$
$$\qquad \hookrightarrow s + u + \bar{d}$$

tend to increase $\langle S \rangle$, $\langle \tilde{S} \rangle$, and $\langle 1 - T \rangle$, as well as broaden the distributions in S, $\tilde{S}$, and T. We hope to return to these questions in a later publication. However, our simplified calculations do serve the purpose of underlining our main point that the weak decays of heavy quarks may be the major source of jet-broadening in e^+e^- experiments at PETRA, PEP energies.

4. Summary

In the preceding sections we have discussed the consequences of electromagnetic production of a pair of heavy quarks in e^+e^- annihilation and their subsequent weak decays on the various jet distributions. Weak decays generate large $p_\perp$ effects and consequently change the 2-jet like distributions that one anticipates from the pair production of ordinary u, d and s quarks. Since the QCD process $e^+e^- \to q\bar{q}g$ is expected to become "visible" at higher energies where new quark thresholds are also opening, a trustworthy test of the QCD process $e^+e^- \to q\bar{q}g$ calls for a careful estimate of the background generated by weakly decaying heavy quarks. We have investigated the phenomenon of jet-broadening from weakly decaying heavy quarks, assuming a quark parton model. In particular, we have calculated the distributions in sphericity, spherocity and thrust and compared them with the corresponding distributions from the QCD process $q\bar{q}g$. We have found that inclusive measurements of

these quantities are not very good measures of unambiguously verifying the effect of the $q\bar{q}g$ vertex. Our calculations show that for energies as high as the endpoint PETRA energy, the jet-broadening phenomenon due to the production and weak decay of bottom and top quarks is at least as important as the QCD process $e^+e^- \to q\bar{q}g$. What happens much beyond the PETRA-PEP energies obviously depends on the number of quark flavours present in nature.

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