

Dynamics of Charm Molecules

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Abstract. We calculate in a dynamical model based on coupled channels with charm-anticharm mesons $D\bar{D}$, $D\bar{D}^*$ and $D^*\bar{D}^*$ and exchanges of light mesons π , η , ρ and ω the spectrum of spin-parity states up to $J=2$ for $C=I=0$ and $C=2, I=0, 1$. They are compared with experimentally known hidden charm states.

1. Introduction

Experimental results at SPEAR and DORIS indicate that some of the structures observed in e^+e^- annihilation in the energy range 3.7–4.4 GeV are resonances that decay mainly into a pair of charmed mesons $D\bar{D}$, $D\bar{D}^* + D^*\bar{D}$ or $D^*\bar{D}^*$ [1]. A number of authors suggested that these peaks are genuine charmed meson-antimeson resonances [2]. In particular, in Ref. 3 the peak at 4.028 GeV was interpreted as a resonance between D^* and \bar{D}^* lying just above the $D^*\bar{D}^*$ threshold. Such states, which in quark language are four quark composites $(c\bar{q})(\bar{c}q)$ (with q being the old u, d and s quarks) are usually referred to as “charmonium molecules” in distinction to the lower lying $c\bar{c}$ bound states of ψ 's and χ 's, also called “charmonium atoms”. Of course, whether such $(c\bar{q})(\bar{c}q)$ mesons really exist is a question of dynamics and cannot be answered easily. For example, one could start from a basic theory, like Quantum-chromodynamics, in which the low lying $q\bar{q}$, $c\bar{q}$ and $c\bar{c}$ states are generated by the basic color confining mechanism. The remaining “Van der Waals” type forces would then produce the molecular boundstates $(c\bar{q})(\bar{c}q)$ (or any other pairing of the four quarks) at threshold. Without an understanding of the confining

forces it is not clear how to apply QCD directly to four quark systems. Therefore one is bound to use more phenomenological means, for example the bag model [4]. Unfortunately the bag model is suitable only for S -wave molecules. It has been applied to charmonium molecules recently [5]. In connection with e^+e^- annihilation we are primarily interested in P -wave charmonium molecules. For them we can expect a more “diatomic” structure than for S -waves, where there is no centrifugal barrier so that all four quarks overlap. For P and higher waves loosely bound systems of colorless atoms $c\bar{q}$ and $\bar{c}q$ possibly exist also from the quark dynamics point of view on the basis of qualitative arguments. It has been suggested by several authors to describe the dynamics of such loosely bound $D\bar{D}$, $D\bar{D}^*$ and $D^*\bar{D}^*$ (and conceivably $F\bar{F}$, $F\bar{F}^*$ and $F^*\bar{F}^*$) states by the exchange of a few light mesons [6]. The strong decay $D^* \rightarrow D\pi$ has been observed experimentally. So there is no doubt that pion exchange forces exist in $D\bar{D}^* \rightarrow D^*\bar{D}$ and presumably also for $D^*\bar{D}^* \rightarrow D^*\bar{D}^*$. In the $D\bar{D}$ system we can exchange ρ and ω instead. Then we have a similar situation as in the nucleon-nucleon (or nucleon-antinucleon) system which is successfully described by light meson exchange forces.

So far the molecular states are considered completely distinct from the quark model $c\bar{c}$ states, which describe $J/\psi, \psi', \psi''$ etc., and it is assumed that the $c\bar{q}\bar{c}q$ states can be calculated without taking into account the mixing with the $c\bar{c}$ states. On the other hand some authors found it necessary to introduce mixing of $D\bar{D}$ components with the $c\bar{c}$ wave function to account for radiative transitions in the charmonium “atomic” spectrum [7]. In such a theory the problem of the transition potential between $c\bar{c}$ and $D\bar{D}$ states is completely open. Due to our lack of understanding the confinement mechanism to calculate these mixing forces in QCD seems to be as difficult as the calculation of the four quark states. Furthermore in Eichten

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et al. [7] the direct forces between the open charm states $D\bar{D}$ were neglected completely.

At this point one might ask whether it is possible to abandon the QCD framework completely and describe all J/ψ and χ states above and below the charm threshold as bound states or resonances of charm-anticharm mesons. Whether such a framework leads to a reasonable theory of the ψ and χ particles cannot be decided beforehand. It depends very much which channels of charm-anticharm mesons are included and whether the approximation of the binding forces by a small number of meson exchanges is sufficient. For spin 1/2 constituents with weakly spin-dependent forces the level ordering clearly is $\{0^-, 1^-\}, \{0^+, 1^+, 2^+\}, \dots$, which we take as the empirical ordering although the crucial 0^- states are far from being settled. It is a natural guess that the level ordering with boson constituents is completely different. In a boson-antiboson bound state model the observed level ordering would not arise naturally and could come about only through complicated dynamics of many coupled channels. In this sense a boson constituent model, even if it worked, would not look very natural. On the other hand states near charm production threshold, like the 3.77 and 4.03 resonances certainly decay into $D\bar{D}, D\bar{D}^*$ or $D^*\bar{D}^*$ particles with strong coupling if phase space is available for them. Then the assumption that these states contain very large components of charm-anticharm mesons is natural.

One of the main objectives of this paper is to investigate whether the resonances 3.77 and 4.03 can be understood as resonances with $D\bar{D}, D^*\bar{D}$ and $D^*\bar{D}^*$ constituents. For this we consider a meson exchange model with π, η, ρ and ω exchange forces between the mesonic constituents, mentioned above, and we ask the question, whether the existence of resonances in the 4 GeV region, details of the spectrum and decay properties of the resonances can be understood with reasonable choices for the coupling constants between the various mesons. The J/ψ and the ψ' are still interpreted as atomic $c\bar{c}$ states. We call this approach a weak coupling model. Of course, here we make the assumption that there is negligible mixing between "molecular" and "atomic" states. This assumption of no mixing between $c\bar{c}$ and meson-antimeson channels is not examined further in our framework. To make the division between "atomic" and "molecular" states just near 3.7 GeV might look artificial. On the other hand the 3.77 and 4.03 are just the states which are nearest to the $D\bar{D}, D^*\bar{D}$ and $D^*\bar{D}^*$ thresholds. Certainly we must expect that the molecular channels will influence also the properties of the ψ' (and perhaps even the J/ψ). It is clearly desirable that mixing should be taken into account in an improved version of this work. In this sense our calculations are just a first step towards understanding the dynamics of molecular states.

We study also a strong interaction model. Here we

ask the question whether all known J/ψ and χ states, usually interpreted as $c\bar{c}$ bound states, can be obtained as bound states of charm-anticharm mesons. The purpose of this investigation is to see whether the usual prejudice that only with spin 1/2 constituents one can understand the "charmonium" spectrum is really true.

The basic assumptions for both models are as follows. We consider only the three channels $D\bar{D}, D\bar{D}^* + D^*\bar{D}$ and $D^*\bar{D}^*$, where D and D^* are the 0^- and 1^- mesons with $C = 1$. As forces we take all one particle exchanges with spins 0^- and 1^- lighter than 1 GeV. Bound states and resonances are calculated with the Blankenbecler-Sugar approximation to the Bethe-Salpeter equation.

After the forces have been determined from the $C = 0, I = 0$ states we calculate the states with $C = 0, I = 1$ and $C = 2, I = 0, 1$. The forces for such states are directly related to the forces in the $C = I = 0$ channel using the known isotopic spins and G parities of the exchanged bosons.

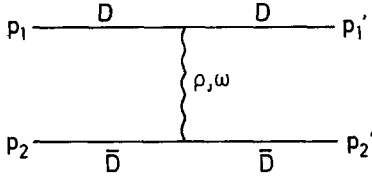
The concept of the two models is very similar to the familiar nucleon-antinucleon boson exchange models which are often used to interpret recently found baryonium resonances [8]. Of course, in this case, one is able to use the information on couplings of exchanged bosons from the nucleon-nucleon analysis. We go more the opposite way and try to use information about the $C = 0$ system to predict the $C = 2$ system. Naturally the formal aspects of our model are very similar to the well-known NN and $N\bar{N}$ treatment [9].

The layout of our paper is as follows. In Sect. 2 we give the dynamical equations and describe the structure of the exchanges used as input. The results are presented and discussed in Sect. 3.

2. One Boson Exchange Potentials (OBEP) in Momentum Space

The definition of the one boson exchange potential is very similar to equivalent descriptions of the low energy nucleon-nucleon system. The channels $D\bar{D}, D\bar{D}^*, D^*\bar{D}$ and $D^*\bar{D}^*$ are considered on equal footing. We include the exchange of all mesons lighter than 1 GeV with spins 0^- and 1^- . These are the non-strange member of the SU(3) nonets of pseudoscalar (π, η, η') and vector mesons (ρ, ω, ϕ). We leave out the scalar mesons (δ, ϵ, S^*), since their experimental verification as resonances is rather unclear. The exchange of mesons with higher spins ($J \geq 2$) and other parity (1^+), such as f, A_2, f', g, A_1 and B are expected to have little influence because of the short range of the corresponding forces.

The potential of the $D\bar{D}$ etc. interaction is determined by identifying it with the off-energy-shell Born term of the Feynman amplitude in the center-of-mass system. The corresponding diagram, as a representative the graph for $D\bar{D} \rightarrow D\bar{D}$, is shown in Fig. 1.

Fig. 1. Diagram for $D\bar{D}$ scattering with ρ and ω exchange

Here \vec{p} and \vec{p}' are initial and final state three momenta and p_0 and p'_0 the corresponding energies in the c.m. system. The notation for the other channels $D\bar{D}^*$ and $D^*\bar{D}$ is analogous. To calculate the complete list of Feynman graphs for $D\bar{D} \rightarrow D\bar{D}$, $D\bar{D} \rightarrow D^*\bar{D}$, $D\bar{D}^* \rightarrow D\bar{D}^*$, $D^*\bar{D} \rightarrow D\bar{D}^*$ and $D^*\bar{D}^* \rightarrow D^*\bar{D}^*$ we need the couplings of the D and D^* with the pseudo-scalar and vector meson fields π, η, ρ and ω . These couplings are:

$$H_{VPP} = ig_{VPP} \{ \mathcal{D}^\dagger \tau \bar{\partial}^\mu \mathcal{D} \rho_\mu + \mathcal{D}^\dagger \bar{\partial}_\mu \mathcal{D} \omega^\mu + (\pi \bar{\partial}^\mu \mathcal{D}^\dagger \tau \mathcal{D}_\mu^* + h.c.) + (\eta \bar{\partial}_\mu \mathcal{D}^\dagger \tau \mathcal{D}_\mu^* + h.c.) \} \quad (2.1)$$

$$H_{VVP} = g_{VVP} \epsilon^{\kappa\lambda\mu\nu} \{ \pi \partial_\kappa \mathcal{D}_\lambda^{\dagger\dagger} \tau \partial_\mu \mathcal{D}_\nu^* + \eta \partial_\kappa \mathcal{D}_\lambda^{\dagger\dagger} \partial_\mu \mathcal{D}_\nu^* + (\mathcal{D}^\dagger \tau \partial_\kappa \mathcal{D}_\lambda^* \partial_\mu \rho_\nu + h.c.) + (\mathcal{D}^\dagger \partial_\kappa \mathcal{D}_\lambda^* \partial_\mu \omega_\nu + h.c.) \} \quad (2.2)$$

$$H_{VVV} = -i \left[f_1 \{ g^{ab} \mathcal{D}_a^{\dagger\dagger} \tau \bar{\partial}^\mu \mathcal{D}_b^* \rho_\mu + g^{ab} \mathcal{D}_a^{\dagger\dagger} \bar{\partial}_\mu \mathcal{D}_b^* \omega_\mu \} + f_2 \{ \mathcal{D}_\alpha^{\dagger\dagger} \tau \mathcal{D}_\beta^* (g_\mu^\alpha \partial^\mu - g_\mu^\beta \partial^\beta) \rho_\mu + \mathcal{D}_\alpha^{\dagger\dagger} \mathcal{D}_\beta^* (g_\mu^\alpha \partial^\mu - g_\mu^\beta \partial^\beta) \omega_\mu \} + \frac{f_3}{m^2} \{ \mathcal{D}_\alpha^{\dagger\dagger} \tau \bar{\partial}^\mu \mathcal{D}_\beta^* \partial^\alpha \partial^\beta \rho_\mu + \mathcal{D}_\alpha^{\dagger\dagger} \bar{\partial}^\mu \mathcal{D}_\beta^* \partial^\alpha \partial^\beta \omega_\mu \} \right] \quad (2.3)$$

Writing down these couplings we assumed SU(4) invariance and ideal mixing in the couplings and for ω, ϕ, η and η' . Then the ϕ does not couple to the D and D^* mesons and the η in (2.1) to (2.3) stands for the $1/\sqrt{2}(u\bar{u} + d\bar{d}) \equiv \eta_\sigma$ component of the true η and η' . To use these couplings for the latter particles one has to multiply with the amount of η_σ component in the η or η' respectively.

For the coupling H_{VVV} of three vector mesons we use the decomposition introduced in Ref. 10. It contains three independent coupling constants f_1, f_2 and f_3 . In the analogous electromagnetic case the three constants f_1, f_2 and f_3 are related to the charge f_c , the magnetic moment f_m and the electric quadrupole form factor f_q respectively [10]. The connections are:

$$\begin{aligned} f_1 &= f_c + 2/3 \eta f_q \\ f_2 &= f_m \\ f_3 &= (2(1 - \eta))^{-1} [(1 - 2/3 \eta) f_q - f_c + f_m] \end{aligned} \quad (2.4)$$

where $\eta = m_\rho^2/4m_{D^*}^2$

The evaluation of the OBEP matrix elements is conveniently carried out in momentum space. This way we have the option to take into account relativistic and meson retardation effects on a later stage.

The spin properties of the potentials are worked out in the helicity basis. The expressions for the five transition potentials are rather lengthy and will be given in a separate publication [11]. The next step is the angular momentum decomposition of the OBE potentials. We choose the incident direction \vec{p} along the z -axis and the outgoing momentum \vec{p}' in the xz -plane. Then the expansion is:

$$\begin{aligned} &\langle \lambda'_1, \lambda'_2, \vec{p}' | V | \lambda_1, \lambda_2, \vec{p} \rangle \\ &= \sum_J (2J+1) d_{\lambda\lambda'}^J(\Theta) \langle \lambda'_1, \lambda'_2, p' | V^J | \lambda_1, \lambda_2, p \rangle \end{aligned} \quad (2.5)$$

with $\lambda = \lambda_1 - \lambda_2$ and $\lambda' = \lambda'_1 - \lambda'_2$. The potentials for fixed J are obtained by inverting (2.5):

$$\begin{aligned} &\langle \lambda'_1, \lambda'_2, p' | V^J | \lambda_1, \lambda_2, p \rangle \\ &= \frac{1}{2} \int_0^\pi d\Theta \sin \Theta d_{\lambda\lambda'}^J(\Theta) \langle \lambda'_1, \lambda'_2, \vec{p}' | V | \lambda_1, \lambda_2, \vec{p} \rangle \end{aligned} \quad (2.6)$$

The same expansion (2.5) is used for the whole amplitude $\langle \lambda'_1, \lambda'_2, \vec{p}' | T | \lambda_1, \lambda_2, \vec{p} \rangle$. The basic integral which occurs in (2.6) is

$$\frac{1}{2} \int_{-1}^{+1} d(\cos \Theta) P_J(\cos \Theta) / (m_\alpha^2 - t) = \frac{1}{2pp'} Q_J(z_\alpha) \quad (2.7)$$

with

$$z_\alpha = \frac{1}{2pp'} (2p_0 p'_0 - (p_0^2 - p'^2) - (p_0^2 - p^2) + m_\alpha^2) \quad (2.8)$$

and (see Fig. 1)

$$t = (p_1 - p'_1)^2 \quad (2.9)$$

The completely off-shell partial wave projections of the various potentials have been calculated. The expressions are too lengthy to be reproduced here. They will be given elsewhere [11].

As is familiar from low energy nucleon-nucleon dynamics based on OBE potentials most exchange amplitudes are too singular for p' or $p \rightarrow \infty$ to guarantee a solution of the dynamical equations. Therefore we modify the OBEP by form factors which represent the combined effect of vertex and propagator corrections. We multiply every potential term by an universal form factor of monopole type

$$F(t) = \frac{\Lambda^2}{m_\alpha^2 + \Lambda^2 - t} \quad (2.10)$$

with a form factor mass Λ to be fixed later. The same form factor is used to modify the two propagators in the Bethe-Salpeter equation. The bound and resonance states are calculated on the basis of the Blankenbecler-Sugar approximation [12] to the Bethe-Salpeter equation.

In the case of equal masses m it has the following form:

$$\begin{aligned} T_{ij}^J(p, \vec{p}) &= V_{ij}^J(p, \vec{p}) \\ &+ \frac{1}{2\pi} \sum_j \int \frac{d p' p'^2}{(m^2 + p'^2)^{1/2} (p'^2 - k^2)} V_{ij}^J(p, p') T_{j'j}^J(p', \vec{p}) \end{aligned} \quad (2.11)$$

Here $k^2 = s/4 - m^2$ and $T_{ij}^J(p, \bar{p})$ is the partial wave projected off-shell amplitude taken as relative energy = 0 limit of the Bjorken-Drell [13] invariant amplitude (divided by 4π). In the calculation the momentum \bar{p} is also chosen off-shell in order to avoid left-hand cut singularities in $V^J(p, \bar{p})$. The indices i, j refer to both different particle combinations as well as different helicity states. The equation was solved only for a fixed j (e.g. the $D\bar{D}$ state for $\tau_p = \tau_c = (-1)^j$). For the unequal mass state $D\bar{D}^*$ we replaced m by twice the reduced mass. In (2.11) the additional form factors coming with the D -propagators are left out for convenience.

Before solving the T -matrix equations we did some further approximations. Since we neglected the retardation in the propagators in the Bethe-Salpeter equation we consequently do the same with the retardation in the meson propagators (including the form factor (2.10)). Furthermore we replace all energies on- and off-shell by their masses. $p_0 = p'_0 = m_D = 1.87 \text{ GeV}$, as one would do in a nonrelativistic approximation. Then z_α in Eq. (2.8) reduces to

$$z_\alpha = \frac{1}{2pp'}(p'^2 + p^2 + m_\alpha^2) \quad (2.12)$$

In channels with D and D^* present we use the average of m_D and m_{D^*} , $\bar{m} = 1.95 \text{ GeV}$ and use this value for $p_0 = p'_0 = \bar{m}$. In matrix elements with longitudinal polarization of the vector mesons ($\lambda_1, \lambda_2, \lambda'_1$ or $\lambda'_2 = 0$) we must renormalize the polarization vectors accordingly. In the matrix elements with only D^* mesons in the initial and final state we also use the average mass \bar{m} for simplicity.

Of course the equations used reduce to the familiar Lippman-Schwinger equation in the non-relativistic approximation. Equations of the form (2.11) have also been used in low energy NN scattering theory [9]. To obtain the energies of the bound states and the resonance positions we proceed as follows. Bound states and resonances manifest themselves as poles in the T -matrix. The T -matrix equation (2.11) is solved by iteration with subsequent application of Padé approximants. Then bound states or resonances appear as poles in these Padé approximants.

3. Results

As advertised in Sect. 1 we first consider the weak coupling model in which the 3.77 state is the lowest 1^{--} state. We have calculated the levels up to $J = 2$ and for the three cases $C = I = 0$; $I = 0, C = 2$ and $I = 1, C = 2$. For $C = 0, I = 1$ no bound states or resonances are possible, as will be seen later. Obviously the results depend on the coupling constants of the charmed mesons on the various particles exchanged in the crossed channel. We have chosen our coupling constants in such a way that they are roughly compatible with relations following from SU(4) symmetry and ideal mixing. Whereas masses are

badly broken in SU(4) it is usually assumed that coupling constants are impaired much less by SU(4) breaking [14]. For the coupling constants g_{VPP} these symmetry relations are [15]:

$$\begin{aligned} g(\omega D^+ D^-) &= -g(\rho^0 D^+ D^-) = -g(D^{*+} \eta_\sigma D^-) \\ &= +g(D^{*+} \pi^0 D^-) = -\frac{1}{2}g(\rho^0 \pi^+ \pi^-) \end{aligned} \quad (3.1)$$

and for g_{VVP} :

$$\begin{aligned} g(D^{*+} D^{*-} \eta_\sigma) &= -g(D^{*+} D^{*-} \pi^0) = g(D^{*-} \omega D^+) \\ &= -g(D^{*-} \rho^0 D^+) = \frac{1}{2}g(\rho^- \omega \pi^+) \end{aligned} \quad (3.2)$$

The coupling constants for VVV fulfill the same symmetry relations as the VPP coupling constants in (3.1) with appropriate change of notation. For $f_1(\rho^0 \rho^+ \rho^-)$ we expect that it is roughly equal to $g(\rho^0 \pi^+ \pi^-)$ if the electromagnetic form factor of the ρ^+ is dominated by ρ^0 exchange as the π^+ form factor is. The particle $\eta_\sigma = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ is the ideal mixed η .

The relation of the physical η and η' to η_σ are well known. $f_2(VVV)$ is fixed in such a way, so that approximately $f_2(VVV) = 2f_1(VVV)$. This relation is obtained in the quark model or in current algebra for the ρ meson [16]. $f_3(VVV)$ will be neglected throughout. For simplicity we take $m_{\eta'} = m_\eta = m_\pi$ and of course $m_\rho = m_\omega$.

A further input is the form factor mass Λ in (2.10). We have chosen $\Lambda^2 = 2 \text{ GeV}^2$ as is suggested by other studies of hadron properties with the Bethe-Salpeter equation [17]. The same value for Λ^2 was used in the form factor for the propagators in the Bethe-Salpeter equation. All three factors are necessary to correct the singular high energy behaviour of the potentials due to the vector exchange. The value for the form factor mass and the magnitude of the coupling constants are not independent. We have checked that the spectrum is changed very little when Λ^2 is varied in the vicinity of 2 GeV^2 by $\pm 1 \text{ GeV}^2$. The overall strength of the potential had to be lowered if Λ^2 was increased. We now shall look into the isospin and charm content of the potentials.

For $C = 0, I = 1$ the potential vanishes because we have $m_\pi = m_\eta = m_{\eta'}$. Even if we relax this constraint the potential is so weak so that no bound states or resonances are formed. One would need a very strong breaking of the relations of π and η and of ρ and ω couplings respectively as given in (3.1) and (3.2) to produce bound states for $C = 0, I = 1$. From this we conclude that charm molecules with $C = 0$ and $I = 1$ are very unlikely. In the four channels $C = 0, 2, I = 0, 1$ the contributions of ω, ρ, π and η, η' (which is the η_σ) appear in $V(C, I)$ with the following factors:

$$\begin{aligned} V(0, 0) &= (\eta_\sigma + 3\pi) + (\omega + 3\rho) \\ V(0, 1) &= (\eta_\sigma - \pi) + (\omega - \rho) \\ V(2, 0) &= (\eta_\sigma - 3\pi) + (-\omega + 3\rho) \\ V(2, 1) &= (\eta_\sigma + \pi) + (-\omega - \rho) \end{aligned} \quad (3.3)$$

Clearly the $C = I = 0$ potential is the most attractive. For $C = 2$ the relative magnitude of pseudoscalar versus vector exchange determines whether the potentials are attractive or not. Of course these remarks, based on (3.3) do not hold without exceptions since the sign of the potential depends also on the spin-parity state and the influence of nondiagonal versus diagonal terms in the potential. From (3.4) we expect the strongest binding for $C = I = 0$ and less binding for $C = 2, I = 0, 1$.

Since $m_D = 1.87$ GeV all states above 3.74 GeV are resonances. It was not difficult with the symmetry relations (3.1) and (3.2) to reduce $g(\rho^0 D^+ D^-)$ and $g(D^{*+} D^{*-} \pi^0)$ (as compared to the known values of $g(\rho^0 \pi^+ \pi^-) = 6$ and $g(\rho^- \omega \pi^+) = 17.5$ GeV $^{-1}$) in such a way that the lowest 1^{--} state appeared at 3.77 GeV. With $f_2(VVV) \simeq 3f_1(VVV)$ a second 1^{--} state is generated at 4.04 GeV which we identify with the experimentally found state at 4.03 GeV. The mass values for all other J^{PC} states for $C = I = 0$ and $C = 2, I = 0, 1$ are presented in Table 1. The values of the coupling constants which are used as input are given in Table 2.

The width of the 1^{--} state at 3.77 GeV is $\Gamma = 23$ MeV which has to be compared with $\Gamma = (24 \pm 5)$ MeV, measured by DELCO [18] and $\Gamma = (28 \pm 5)$ MeV, measured by SLAC-LBL [19]. For the width of the 4.04 resonance we obtain $\Gamma \approx 60$ MeV. This number could not be determined better because of computational problems which appear above the $D^* \bar{D}^*$

Table 1. Calculated mass values for various spin-parity states for the weak coupling model. In the second row we have given the experimental mass values of the charmonium spectrum [21] and the measured excited states in the 1^{--} state

J^{PC}	$C = I = 0$		$C = 2$	$C = 2$
	Experiment	$C = I = 0$	$I = 0$	$I = 1$
2^{+-}		4.15	4.37	none
1^{-+}		none	none	3.87
2^{--}		> 4.30	none	none
2^{++}	3.55	3.81 4.10	4.18	none
1^{++}	3.505	3.69	none	4.15
0^{++}	3.413	3.40 3.80	3.44 3.95	4.05
1^{--}	3.77 4.03	3.77 4.04	4.00	none
2^{-+}		none	none	none
1^{+-}		3.52 4.30	none	4.10
0^{-+}	3.455	3.71	none	3.99 4.10

Table 2. Coupling constants for the results in Table 1

VPP	$g(\rho^0 D^+ D^-) = 3.5$	$g(D^{*+} \pi^0 D^-) = 3.5$
VVP	$g(D^{*+} D^- \rho^0) = 7.3$ GeV $^{-1}$	$g(F^{*+} D^{*+} \pi^0) = 7.3$ GeV $^{-1}$
VVV	$f_1(\rho^0 D^{*+} D^{*-}) = 3.5$	$f_2(\rho^0 D^{*+} D^{*-}) = 11.5$

threshold. Its value agrees well with the $\Gamma_{\text{exp}}(4.04) = (52 \pm 10)$ MeV as measured by the DASP Group at DORIS [20]. It is clear that the 3.77 resonance can decay only into $D \bar{D}$ states as experimentally observed [21]. The 4.03 resonance can decay in all three channels $D \bar{D}, D \bar{D}^* + D^* \bar{D}$ and $D^* \bar{D}^*$. The measured fractions are $0.09 \pm 0.05 : 0.58 \pm 0.06 : 0.33 \pm 0.08$ [22]. We obtain: 0.15 : 0.48 : 0.37, which agrees quite well with the experimentally determined ratios. In calculating these numbers we inserted the experimentally determined mass values of D and D^* in the threshold factors.

This situation can be compared with the charmonium scheme of Eichten et al. [7], in which a coupling of $c\bar{c}$ with open charm states is included, but no direct forces between charm-anticharm mesons are present. They also obtain 1^{--} states at 3.77 and ~ 4.05 –4.10 GeV, being essentially 3D_1 and 3S_1 in the $c\bar{c}$ mode, and can also account, at least qualitatively, for the branching fractions $D \bar{D}/D^* \bar{D}/D^* \bar{D}^*$ of the 4.03 level. The mechanism, which suppresses the $D \bar{D}$ production at 4.03 GeV in their model, originates from a radial node of the 3S_1 wave function near 4 GeV [23]. In our model the suppression of the $D \bar{D}$ -decay of the 4.03-resonance is a consequence of the diagonalization of the various states: The forces in the $D \bar{D}$ -state are the strongest ones, therefore this state resonates first. The wave functions of the higher resonances have to be orthogonal to the lower ones, which qualitatively explains their small admixture of the $D \bar{D}$ -state.

A distinction between the two suppression mechanisms is very difficult to achieve. The node in the quark wave function will depend on the range of the quark potential, especially on the strength of its short range singular part which is not yet fixed definitely. Whether it works also for the higher resonances remains to be seen.

Whether after inclusion of $c\bar{c}$ -states we would get too many states in the 4 GeV-region, we must leave as an open question. We only remark that experimentally the number of resonances is not clear (perhaps there are four) and that on the theoretical side states do not simply add: If the coupling between a $c\bar{c}$ and $D \bar{D}$ state is appreciable, the higher “bare” state will be shifted to higher masses and may become very broad.

Certainly there are many more details which could be calculated in this model. For example, the leptonic widths of the 3.77 and the 4.03 state are known experimentally and could be compared. Unfortunately

leptonic widths are very dependent on details of the wave function at small distances which, we think, are not reliably described in our model.

The other spin-parity states are located where one has states also in the charmonium model, in particular for 0^{-+} the η'_c in charmonium, 1^{+-} , 0^{++} and 1^{++} . This means that for these states the $c\bar{c}$ component and the $D\bar{D}$ component will mix strongly. For the 2^{++} the mixing seems to be less important. Certainly a complete treatment of these states requires a model incorporating these two components with realistic forces in all channels.

We notice that we predict a “wrong” C -parity state 2^{+-} resonance above threshold. Such states cannot be generated by $c\bar{c}$ states at all. The resonance energies of the $C=2$ exotic states are rather high except a 0^{++} state with $I=0$ at 3.44 GeV.

Now we come to the strong coupling model. Here we varied the coupling constants in the vicinity of the symmetry values (3.1) and (3.2) with $g(\rho\pi\pi)=6$ and $g(\omega\rho\pi)=17.5 \text{ GeV}^{-1}$ until we obtained 1^{--} states at 3.1 and 3.6 GeV and a 0^- state at 2.82 GeV. The energies of all the other levels for all three cases $C=I=0$; $C=2, I=0, 1$ are shown in Table 3. The corresponding coupling constants are presented in Table 4.

The symmetry values for the VPP coupling, first line in Table 4, is 3.0, for the VVP coupling, the second

Table 3. Calculated mass values for various spin-parity states for the strong coupling model. In the second row are the experimental masses [21]

J^{PC}	$C=I=0$		$C=2$	$C=2$
	Experiment	$C=I=0$	$I=0$	$I=1$
2^{+-}		3.55 4.15	none	none
1^{+-}		$\geq 4.$	none	3.60
2^{--}		4.01	none	none
2^{++}	3.550	3.05 3.80	3.92 4.05	none
1^{++}	3.505	3.28	3.67	3.89 4.11
0^{++}	3.413	very low 2.75 3.60	2.10 3.10 3.85	2.90 4.30
1^{--}	3.096 3.685 3.772	3.10 3.64 3.88	3.61 4.00	4.20
2^{-+}		none	none	none
1^{+-}		2.84 3.71	3.70	4.01 4.30
0^{-+}	2.83 3.455	2.82 3.71	none	3.25 3.95

Table 4. Coupling constants for the results in Table 3

VPP	$g(\rho^0 D^+ D^-) = 4.0$	$g(D^{*+} \pi^0 D^-) = 6.0$
VVP	$g(D^{*+} D^- \rho^0) = 8.7 \text{ GeV}^{-1}$	$g(D^{*+} D^{*+} \pi^0) = 12.4 \text{ GeV}^{-1}$
VVV	$f_1(\rho^0 D^{*+} D^{*-}) = 6.0$	$f_2(\rho^0 D^{*+} D^{*-}) = 10.7$

line in Table 4 is 8.75 GeV^{-1} and for the VVV coupling $f_1 = f_2/2 = 3.0$. We see that the values in Table 4 do not differ very much from these values and that the 50% increase of the $D^* \pi D$ coupling can be justified with the work of Thews [24].

From Table 3 we observe that for $I=C=0$ almost all states up to $J=2$ have bound states or resonances. Since $m_D = 1.87 \text{ GeV}$ all states above 3.74 GeV are resonances. It was not difficult to find coupling constants reproducing almost perfectly the spectrum of 1^{--} states (of course $J/\psi = 3.1 \text{ GeV}$ served as input). Also the 0^{-+} states are reasonable. But the states 0^{++} , 1^{++} and 2^{++} lie lower than the measured states (see first column of Table 3) on the basis of the usual assignments [21]. In particular the lowest 0^{++} state has a very small mass. This feature, too strong binding in the states 0^{++} , 1^{+-} and 2^{++} , presumably is characteristic for this model based on boson constituents. For 0^{++} states in $D\bar{D}$ and $D^* \bar{D}^*$ channels contribute in S -waves and similarly for 1^{+-} do the $D\bar{D}^* + D^* \bar{D}$ and $D^* \bar{D}^*$ and also 2^{++} has S -wave contributions in $D^* \bar{D}^*$. The fact that the 2^{++} state has a much larger mass than the 0^{++} state is caused by the tensor forces. We conclude from the results in Table 3 that for an understanding of the low mass part of the charmonium spectrum spin 1/2 constituents are essential. Nevertheless we have computed also the exotic $C=2$ states which are produced by the couplings in Table 4. We see that the $I=0$ states are in average more strongly bound than the $I=1$ states. In $I=0$ we have stable molecules with charm equal two in 1^{+-} at 3.7 GeV, 1^{--} at 3.61 GeV, in 1^{++} at 3.67 GeV. The prediction for 0^{++} is unreliable, since here in $C=0$ the forces are already much too strong. We also mention that states with “wrong” C parity, which cannot be generated by $c\bar{c}$ states are possible in this model. In 2^{+-} we have a bound state at 3.55 GeV. The first 1^{+-} and 2^{+-} occur in the channels $D\bar{D}^*$ and $D^* \bar{D}^*$ as P - and D -waves respectively.

From our analysis we can draw two conclusions. The use of spin 1/2-constituents to explain the lower part of the charmonium spectrum is essential. On the other hand we showed that OBE-potentials in the $D\bar{D}$, $D\bar{D}^* + \bar{D}D^*$ and $D^* \bar{D}^*$ channels are capable of forming bound states and narrow resonances. Our approach provides a quite natural way to understand the widths and branching ratios of $\psi(3.77)$ and $\psi(4.03)$. Higher 1^{--} resonances are possible and might be due to the coupling of D - to F -meson channels via K -exchanges.

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