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# The Transverse Momentum Profile of a Quark Jet

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**Abstract.** The first and second moments of the transverse momentum distribution of pions in $e^+e^-$-annihilation at fixed longitudinal momentum with respect to the jet axis are discussed, treating the hadronic final state evolving from the 2-quark system in the framework of a fireball-model. Good agreement with the published data is found. A comparison with the $p_T$-distribution of electroproduction is made, showing qualitative agreement. This casts serious doubts on previous conclusions favoring a large parton $k_T$ inside the proton.

The study of jet structures in multiparticle final states is now attracting a lot of interest both on the experimental and the theoretical side [1]. The jets observed in $e^+e^-$-annihilation, deep inelastic lepton scattering in the virtual photon fragmentation region and in hadron-hadron scattering at large transverse momentum are considered as fragments of quarks which have suffered a large momentum transfer in the process. In parton models, the emphasis is put on a description of the longitudinal distribution in the scaling region, whereas the transverse distribution, if considered at all, is introduced in a more or less ad hoc fashion. In the framework of QCD statements on $p_T$ are obscured by confinement and nonperturbative effects presently not calculable from QCD. Here usually quantities like spherocity or thrust are considered as a measure for jet-like behavior. However, a much more direct insight in the structure of a jet is in our opinion provided by studying the transverse momentum distribution at fixed longitudinal momentum, or alternatively the moments $\langle p_T^n(p_L)\rangle$, where $p_T$ and $p_L$ are defined with respect to the jet axis.

In this note we present results for the moments $\langle p_T(z,W)\rangle$ and $\langle p_T^2(z,W)\rangle$, where $z=2p_L/W$ and $W=$ total $e^+e^-$-CMS- energy, obtained from the model of [2] for $e^+e^-$-annihilation. This model describes the final state evolving from the intermediate 2-quark state produced by the timelike virtual photon as a system of hadron-balls. The jets then appear as the decay products of these hadron-balls, which have to be thought of as localized objects consisting of excited hadronic matter and decaying according to a thermodynamic law in their rest system. The basic assumption was that two such objects corresponding to the two intermediate quarks are produced with equal mass and opposite momenta, and one further object with fixed mass $M_0$ in the low momentum region. A possible explanation of the third object in the context of source theory has been given in [3]. Scaling for the inclusive spectrum is obtained by choosing for the fast hadron-balls a mass distribution like

$$\frac{d\rho}{dM}\sim\frac{1}{M^2} \tag{1}$$

The relative normalisation is fixed by the energy conservation sum rule.

With this picture it was possible to fit the shape and energy dependence of the inclusive pion spectrum between total CMS-energies $W=3.5$ and $7.4\,\mathrm{GeV}$ with only 3 free parameters. Furthermore, as a jet axis is inherently given by the momentum direction of the moving hadron-balls, the integrated longitudinal and transverse momentum distributions could be evaluated without further freedom. For both, good agreement with the data could be obtained. It is a remarkable property of the model that longitudinal and transverse momentum behavior of the jets are closely interrelated.

The $n^{\text{th}}$ moment of the $p_T$-distribution is defined by

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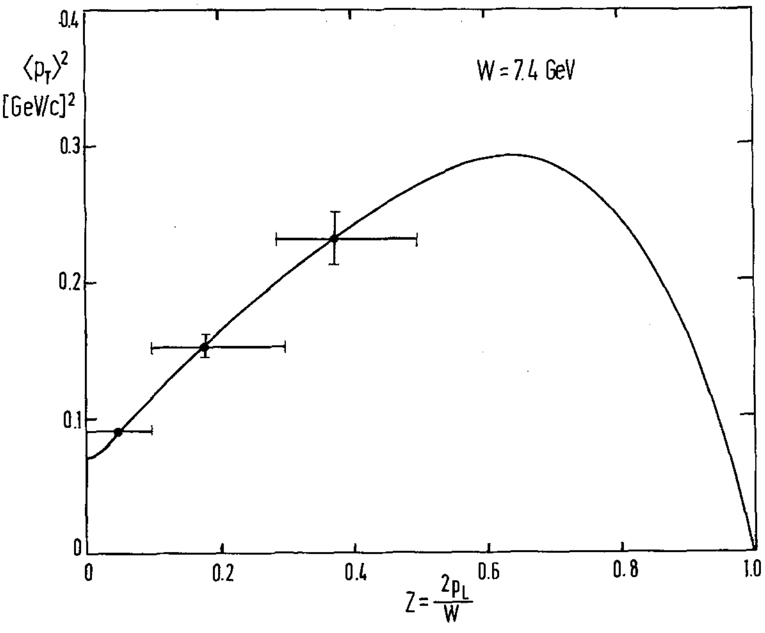


**Fig. 1.** Comparison of our result for $\langle p_T(z)\rangle^2$ with the SPEAR data [4] at $W=7.4$ GeV

$$\langle p_T^n(z,W)\rangle = \frac{\int_0^{\hat{p}_T} p_T^n G(p_T,z,W)\,dp_T}{\int_0^{\hat{p}_T} G(p_T,z,W)\,dp_T} \tag{2}$$

where $\hat{p}_T$ = maximum transverse momentum for given $z$ and $W$ and $G$ is related to the invariant cross section by

$$G=\frac{p_T}{E}\frac{1}{C\sigma_{\mathrm{tot}}}\left(E\frac{d^3\sigma}{d^3p}\right) \tag{3}$$

$C$ being a constant. From formula (2) of [2] follows:

$$G=\frac{p_T}{E}\left[M_0 e^{-\beta E}+\frac{E_J}{E_J/M_1-1}\cdot\left\{\int_1^{E_J/M_1}\frac{d\varepsilon}{\varepsilon}e^{-\beta(\varepsilon E-p_L\sqrt{\varepsilon^2-1})}+\int_1^{E_J/M_1}\frac{d\varepsilon}{\varepsilon}\cdot e^{-\beta(\varepsilon E+p_L\sqrt{\varepsilon^2-1})}\right\}\right] \tag{4}$$

Here, $E_J$ is the energy of a jet, $E_J=(W-M_0)/2$ and $E=\sqrt{p_L^2+p_T^2+\mu^2}$ the pion energy. The parameters from [2] are unchanged:

$M_0=1.46$ GeV, $M_1=1$ GeV, $\beta=1/T=5\ \mathrm{GeV}^{-1}$.

The results for the moments $\langle p_T(z,W)\rangle^2$ and $\langle p_T^2(z,W)\rangle$ are shown in Figs. 2 and 3 for different energies. In Fig. 1 a comparison is made with the data from SPEAR [4] for $\langle p_T(z)\rangle^2$ at $W=7.4$ GeV. We emphasize that our curves are absolute predictions of the model of [2]. The excellent agreement with the SPEAR data strongly supports the basic assumptions of the model. Our results show a "seagull-effect", i.e. an increase of $\langle p_T\rangle$ and $\langle p_T^2\rangle$ with increasing $z$ at small $z$ and a kinematical fall-off towards the kinematical boundary $z=1$. This behavior is trivial if the moments are calculated at fixed values of the radial scaling variable $x=2|\mathbf{p}|/W$, as they have to vanish for $x\to 0$ for kinematical reasons [5]

For fixed $z$, the seagull-effect is nontrivial, as it depends

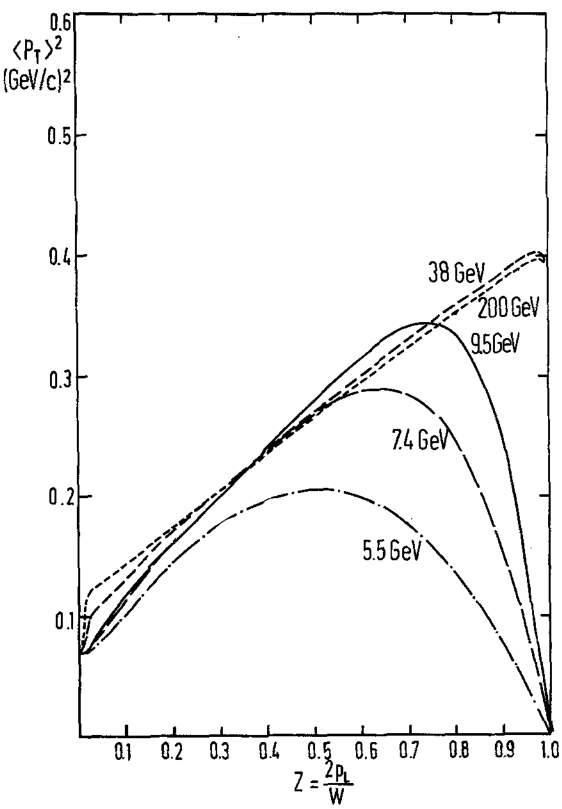


**Fig. 2.** Our results for $\langle p_T(z)\rangle^2$ for $W=5.5, 7.4, 9.5, 38$ and 200 GeV

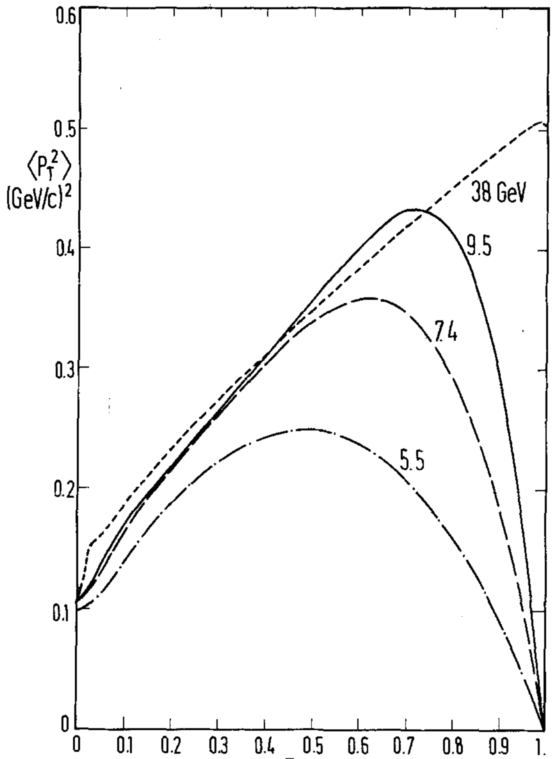


**Fig. 3.** Our results for $\langle p_T^2(z) \rangle$ for $W = 5.5, 7.4, 9.5$ and 38 GeV

on the dynamics. The basic reason for the increase at small $z$ is not the jet structure of (4). This can be seen from a study of the spherical symmetric case

$$G \sim e^{-\beta E} \tag{5}$$

Here the moments can be evaluated analytically and an increase of $\langle p_T(z) \rangle$ and $\langle p_T^2(z) \rangle$ similar to to the one obtained from Eq. (4) is found. In contrast, for a Gaussian distribution

$$G \sim e^{-\lambda(p_L^2 + p_T^2)} \tag{6}$$

as well as for all factorizing distributions $G = G_L(z) G_T(p_T)$ like for instance in the naive parton model, all moments are $z$-independent, apart from the influence of the kinematical boundary near $z = 1$. This means that the Boltzmann-factor $\exp(-\beta E)$ is responsible for the seagull-property.
For the second moment, the asymptotic behavior can be found explicitly,

$$\langle p_T^2(z, W \to \infty) \rangle = 4T^2 \left(1 + \frac{M_1}{2T} z\right) \tag{7}$$

showing a finite limit at fixed $z$ with a linear $z$-dependence, which is already visible at finite $W$ in a limited $z$-range, see Fig. 3. In contrast, QCD predicts scaling, i.e. an increase of $\langle p_T^2(z) \rangle$ roughly proportional to $W^2$, which should already become visible at PETRA-energies [6]. The reason for the different behavior is that our model has an intrinsic mass scale determined by $T$ which governs the high energy behavior.

Finally, as an application of our results we compare the transverse momentum profile of the jets from $e^+e^-$-annihilation with the jets observed in deep inelastic $ep$-or $\mu p$-scattering in the virtual $\gamma$ fragmentation region. The similarity of the longitudinal structure functions has been demonstrated by several experimental groups [7]. In the case of electroproduction the target structure function is of importance in addition to the quark fragmentation function. Thus the transverse momentum $P_T$ of an inclusive hadron produced by an $e$ or $\mu$ receives contributions from two sources:

(i) the transverse momentum $k_T$ of the quark inside the target proton,

(ii) the transverse momentum $p_T$ of the hadron emitted from the struck quark.

The dependence of $\langle P_T^2 \rangle$ on $\langle k_T^2 \rangle$ and $\langle p_T^2 \rangle$ can be found from a geometrical consideration [8].

$$\langle P_T^2 \rangle = \langle p_T^2(z, Q^2) \rangle + z^2 \langle k_T^2(x, Q^2) \rangle \tag{8}$$

Here $x$ is the usual Bjorken variable for electroproduction and $z$ the longitudinal momentum fraction of the observed hadron.

Data for $\langle P_T(z) \rangle$ are shown in Fig. 4. They show a qualitatively similar behavior as $\langle p_T(z) \rangle$ in $e^+e^-$. In a first analysis based on Eq. (8), a $z$-independent $\langle p_T^2 \rangle$ was assumed, explaining the whole variation

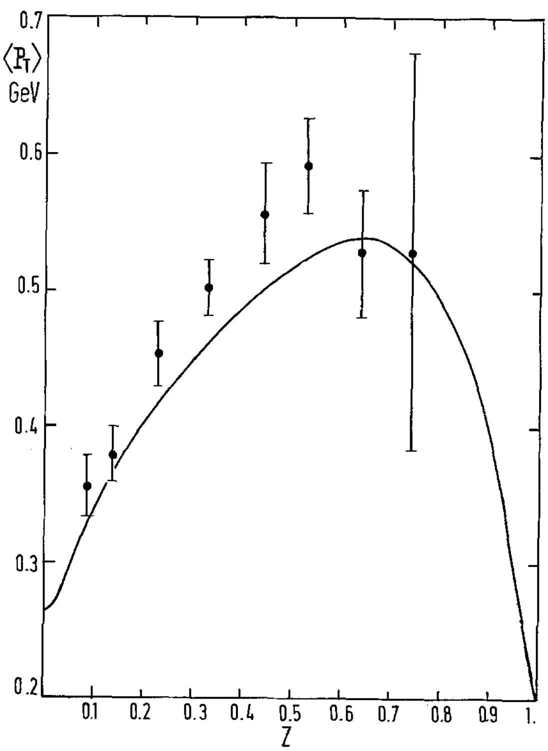


**Fig. 4.** Data for $\langle P_T(z) \rangle$ from $\mu p$-scattering [11]. The curve shows $\langle p_T(z) \rangle$ for $e^+e^-$ at $W = 7.4$ GeV. The similarity shows, according to Eq. (8), that $\langle k_T \rangle$ of the quark inside the proton is small

of the data as being due to the explicit $z$-dependence of (8), [12]. The value of $\langle k_T \rangle$ thereby found was $950 \pm 150$ MeV, in qualitative agreement with the value extracted from lepton pair production [9]. However, this latter result has been critisized [10], as the treatment of nuclear effects in [9] is not reliable, showing that a modified procedure can easily yield much smaller values for $\langle k_T \rangle$. In contrast to [12], we are now able to repeat the analysis with a realistic function $\langle p_T(z) \rangle$, given by our model. A comparison of $\langle P_T(z) \rangle$ and $\langle p_T(z) \rangle$, which corresponds to the omission of the $\langle k_T^2 \rangle$-term in (8), shows qualitative agreement. This means that the seagull-effect observed in electroproduction is intimately related to the one in $e^+e^-$, leaving not much space for the parton $\langle k_T \rangle$ inside the proton. A quantitative discussion of this question will be given elsewhere.

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