THE THREE-GLUON VERTEX OF QCD

K. KOLLER
Sektion Physik, Universität München, Germany

T.F. WALSH
Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

and

P.M. ZERWAS
Institut für Theoretische Physik, Technische Hochschule, Aachen, Germany

Received 16 January 1979

We show how the Q^2 evolution of gluon jets can be used to provide indirect but strong evidence for the three-gluon vertex of QCD. We propose looking for this evolution in the $Q\bar{Q} \to 3G \to$ hadrons decay of successive $1\,^3S_1$ quarkonium states. The results apply to other processes if G-jets can be isolated.

One of the most attractive features of quantum chromodynamics (QCD) [1] is that it makes precise experimental predictions ‡1. If the theory is wrong it can be rejected. If correct, it will be possible to produce objective experimental evidence for the essential elements of the theory: colored quarks, massless colored vector gluons and the self-coupling of the gluons. This last feature is central to QCD; it is a consequence of local color gauge invariance.

Finding direct evidence for QCD's 3G-vertex may be difficult. In this note we discuss a potentially simple indirect way of looking for effects of this self-coupling. The idea is the following. The decay (fig. 1)

$$e^+e^- \to 1\,^3S_1\ Q\bar{Q} \to 3G \to h(\boldsymbol{p}) + \text{anything} \tag{1}$$

provides a pure source of gluon jets at discrete quarkonium masses $Q^2 = M^2 = M^2_{J/\psi}, M^2_{\Upsilon}, \ldots$. (It is necessary to subtract the effects of $1\,^3S_1\ Q\bar{Q} \to 1\gamma \to q\bar{q} \to$ hadrons, but this is now routine [3]). The normalized gluon distribution for $Q\bar{Q} \to G + \ldots$, $(1/\Gamma_{3G})\,d\Gamma_{3G}/dx_G$ ($x_G = 2|\boldsymbol{p}_G|/M$) [4] is independent of M in QCD. However, the jet of hadrons from $G \to h(\boldsymbol{p}) + \ldots$ evolves with Q^2 or M^2 (thus there are violations of scaling in $z = 2|\boldsymbol{p}|/M$). The M^2 evolution of G-jets depends sensitively on the 3G vertex, as we will show in the course of this note.

If we describe the hadron distribution for $G \to h(\boldsymbol{p})$ ... by $D_G(x, M^2)$, $x = |\boldsymbol{p}|/|\boldsymbol{p}|$ ‡2, then the distribution $\Gamma(z, M^2)$ of hadrons in reactions (1) is given by

$$\Gamma(z, M^2) = \int_0^1 dx_G \int_0^1 dx\, \delta(x x_G - z) \times \frac{1}{\Gamma_{3G}} \frac{d\Gamma_{3G}}{dx_G} D_G(x, M^2)\,, \tag{2}$$

and on taking moments, defined by

$$\Gamma(n, M^2) \equiv \int_0^1 dz\, z^{n-1} \Gamma(z, M^2)\,,$$

$$\gamma(n) \equiv \int_0^1 dx_G\, x_G^{n-1} \frac{1}{\Gamma_{3G}} \frac{d\Gamma_{3G}}{dx_G}\,, \tag{3}$$

‡1 More accurately, the perturbative approach [2] to QCD (small effective coupling $\alpha_s(Q^2)$ at distances $\sim 1/\sqrt{Q^2}$) yields detailed expectations for large Q^2 processes. Without this feature, QCD would be divested of most of its predictive power.

‡2 $D_G(x, M^2)$ is summed over hadron species and satisfies $D_G(x, M^2) = \Sigma_{h_i} D_G^{h_i}(x, M^2)$, $\int_0^1 dx\, x\, D_G(x, M^2) = 1$.

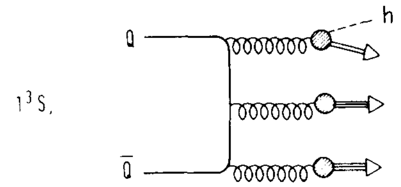

Fig. 1. The decay $Q\bar{Q} \to 3G \to h(p) + \ldots$; the detected hadron can come from any of the three-gluon jets.

$$D_G(n, M^2) \equiv \int_0^1 dx\, x^{n-1} D_G(x, M^2)\,,$$

we find that $\Gamma(n, M^2) = \gamma(n) D_G(n, M^2)$. As a consequence of this factorization we have

$$\Gamma(n, M^2)/\Gamma(n, M_0^2) = D_G(n, M^2)/D_G(n, M_0^2)\,. \tag{4}$$

In other words, *the ratio of moments of* $Q\bar{Q} \to 3G \to$ *hadron distributions at* M^2 *and* M_0^2 *is just the ratio of the moments of the gluon fragmentation functions themselves*. Comparison of QCD and experiment is simplest if one extracts the moments $\Gamma(n, M^2)$ from data. We will exploit eq. (4) in the following, where we discuss $D_G(n, M^2)$ ‡3.

The evolution of gluon jets in M^2 is coupled to the evolution of quark jets. This is depicted in fig. 2. The M^2 change of $D_G(z, M^2)$ comes from $G \to GG$ and $G \to q\bar{q}$, where the observed hadron comes from a final G or q or $\bar{q}$. Similarly, $q \to Gq$ yields an M^2 dependence of the "singlet" quark fragmentation function

$$D_S(x, M^2) \equiv \sum_{h_i} \sum_{q,\bar{q}} D_q^{h_i}(x, M^2)\,,$$
$$D_S(n, M^2) \equiv \int_0^1 dx\, x^{n-1} D_S(x, M^2)\,, \tag{5}$$
$$D_S(2, M^2) = 2N_F\,.$$

The solution to this jet mixing problem is abundantly discussed in the literature [5,6]. The moments $D_G(n, M^2)$ and $D_S(n, M^2)$ obey coupled homogeneous differential equations in the variable $s = \ln[\ln(M^2/\Lambda^2)/\ln(M_0^2/\Lambda^2)]$ ‡4, whose solution is

‡3 Eq. (4) has been noted independently by J. Ellis (private communication).

‡4 Eqs. (6) are reliable to lowest order in α_s if the quanta in fig. 2 give rise to jets. However, the interpretation of Λ may be uncertain. We believe it to be an effective nonperturbative jet $\langle p_\perp \rangle \sim O(1/2\ \mathrm{GeV})$. We set $\Lambda = 0.5$ GeV in the following.

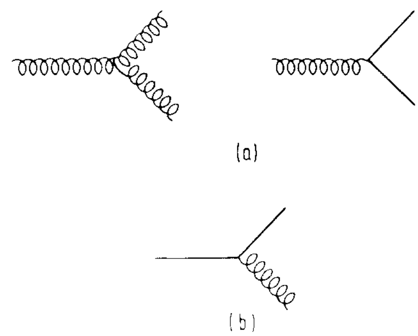

Fig. 2. (a) Perturbation diagrams entering in the evolution of a G-jet. (b) The evolution of a q-jet.

$$\frac{D_G(n, M^2)}{D_G(n, M_0^2)} = \frac{\mu_+ e^{\lambda_+ s} - \mu_- e^{\lambda_- s}}{\mu_+ - \mu_-} - \frac{\mu_+ \mu_- (e^{\lambda_+ s} - e^{\lambda_- s})}{\mu_+ - \mu_-} \frac{D_S(n, M_0^2)}{D_G(n, M_0^2)}\,, \tag{6}$$
$$\frac{D_S(n, M^2)}{D_S(n, M_0^2)} = \frac{\mu_+ e^{\lambda_- s} - \mu_- e^{\lambda_+ s}}{\mu_+ - \mu_-} + \frac{e^{\lambda_+ s} - e^{\lambda_- s}}{\mu_+ - \mu_-} \frac{D_G(n, M_0^2)}{D_S(n, M_0^2)}\,,$$

where

$$2\lambda_\pm = -d_n^{qq} - d_n^{GG} \pm [(d_n^{GG} - d_n^{qq})^2 + 4 d_n^{Gq} d_n^{qG}]^{1/2}\,,$$
$$4N_F d_n^{Gq} \mu_\pm = d_n^{GG} - d_n^{qq} \mp [(d_n^{GG} - d_n^{qq})^2 + 4 d_n^{Gq} d_n^{qG}]^{1/2}\,, \tag{7}$$

and

$$d_n^{qq} = \frac{1}{2\pi b} \frac{2}{3} \left[1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^{n} \frac{1}{j}\right],$$
$$d_n^{GG} = \frac{1}{2\pi b} \frac{3}{2} \left[\frac{1}{3} - \frac{4}{n(n-1)} - \frac{4}{(n+1)(n+2)} + 4 \sum_{j=2}^{n} \frac{1}{j} + \frac{2N_F}{q}\right], \tag{8}$$
$$d_n^{Gq} = -\frac{1}{2\pi b} \frac{4}{3} \left[\frac{n^2 + n + 2}{n(n^2 - 1)}\right],$$
$$d_n^{qG} = -\frac{1}{2\pi b} N_F \left[\frac{n^2 + n + 2}{n(n+1)(n+2)}\right],$$

are anomalous dimensions, where $2\pi b = (33 - 2N_F)/6$, with N_F the number of quark flavors. These anomalous dimensions are moments of lowest order QCD fragmentation functions $q \to qG$, $G \to GG$, $q\bar{q}$ for physical transverse G polarization [‡5].

From eqs. (6) we see that knowing $D_G(n, M_0^2)$, $D_S(n, M_0^2)$ determines $D_G(n, M^2)$, $D_S(n, M^2)$ for any large M^2. The evolution in M^2 from eqs. (6) is shown in fig. 3. Fig. 3a shows $D_G(n, M^2)/D_G(n, M_0^2) = \Gamma(n, M^2)/\Gamma(n, M_0^2)$ and fig. 3b shows $D_S(n, M_0^2)$ for comparison. We set $M^2 = 100\ \mathrm{GeV}^2 \approx M_\Upsilon^2$, $D_G(n, M_0^2) = D_S(n, M_0^2)/2N_F$, and present the ratios from $M^2 = 10\ \mathrm{GeV}^2 \approx M_{J/\psi}^2$ to $M^2 = 5000\ \mathrm{GeV}^2$. (Somewhere in the range above M^2 we hope for at least one more new $Q\bar{Q}$ state.) From fig. 3 it is evident that quark and gluon jets evolve quite differently. The evolution of gluon jets is essentially determined by $G \to GG$ (whose strength relative to $G \to q\bar{q}$ is $33 : 2N_F$; here and in the following we set $N_F = 3$).

A word about the input to fig. 3 is in order. From eqs. (6) we see that both $D_S(n, M_0^2)$ and $D_G(n, M_0^2)$ affect the evolution of $D_G(n, M^2)$. In principle, $D_G(n, M_0^2)$ and $D_S(n, M_0^2)/2N_F$ can be fixed by experiment at M_0^2. We have chosen them equal at $M_\Upsilon^2 \approx M_0^2$. This appears consistent with experiment [3]. (An exception may be large n moments, corresponding to $z \to 1$, where data hardly exist.) In order to test the sensitivity of our results to $D_S(n, M^2)/2N_F D_G(n, M^2)$, we varied this ratio from 1 to 4 for $n > 2$. No significant change in the results in fig. 3 is observed. Thus our results for $\Gamma(n, M^2)/\Gamma(n, M_0^2)$ are insensitive to large variations in $D_S(n, M_0^2)/2N_F D_G(n, M_0^2)$, and are dominated by the parameter-independent part of eqs. (6).

The M^2 evolution of $\Gamma(n, M^2)$ depends in an essential way on the 3G-vertex. In order to illustrate this, we have constructed two alternative models. The anomalous dimensions (8) for these models are found in a straightforward way by calculating the fragmentation functions $q \to qG$ (or $G \to GG$) for physical transverse G polarization. The models are:

(1) QCD with no 3G-vertex. (We have actually varied the strength of the $G \to GG$ vertex using the above prescription; only results for $G \not\to GG$ are shown.) In

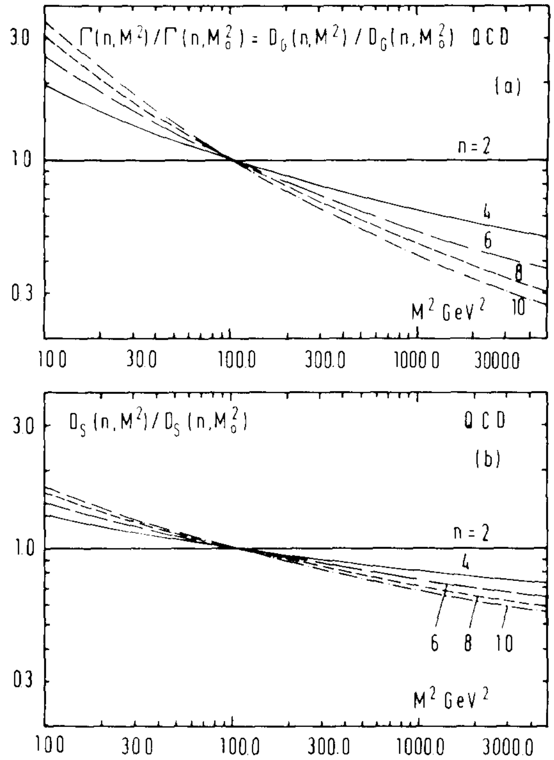

Fig. 3. (a) The QCD ratio $\Gamma(n, M^2)/\Gamma(n, M_0^2)$ for $M_0^2 = 100\ \mathrm{GeV}^2$, three flavors and $n = 2, 4, 6, 8, 10$. As initial condition, $D_S(n, M_0^2)/2N_F D_G(n, M_0^2) = 1$. (b) The QCD ratio $D_S(n, M^2)/D_S(n, M_0^2)$ corresponding to (a).

addition, we have replaced $\alpha_s(M^2) \to \alpha_\infty$ = const. (a fixed-point value) [‡6]. With $\alpha_\infty = \alpha_s(100\ \mathrm{GeV}^2)$ we obtain the results shown in fig. 4a. The scaling violations in gluon jets practically disappear! (This also holds if we let "α" increase or decrease slowly with M^2.)

(2) An abelian model with colored quarks and a massless, colorless vector gluon [7] [‡7]. Because "G" $\to q\bar{q}$ now counts flavors *and* colors, the scaling violations in $D_G(n, M^2)$ will be increased somewhat over case (1). Results for $\alpha_\infty = 0.1$ are shown in fig. 4b. Besides these small scaling violations, we find that $D_S(n, M^2)$ and $D_G(n, M^2)$ are hardly distinguishable for any α_∞, unlike the case for QCD shown in figs. 3a and 3b. (Here, as previously, we set $D_S(n, M_0^2)/2N_F D_G(n, M_0^2) = 1$.)

‡5 $q \to q$ and $G \to G$ fragmentation functions must be regularized at $x \to 1$ [5]. Thus d_n^{GG} depends on the 3G-vertex and on the $Gq\bar{q}$-vertex.

‡6 The relevant variable in eqs. (6) is then $t = \ln M^2/M_0^2$.

‡7 We do not represent this as a realistic alternative to QCD. It possesses an unpleasant color degeneracy of the "hadrons", and does not fit deep-inelastic data [7].

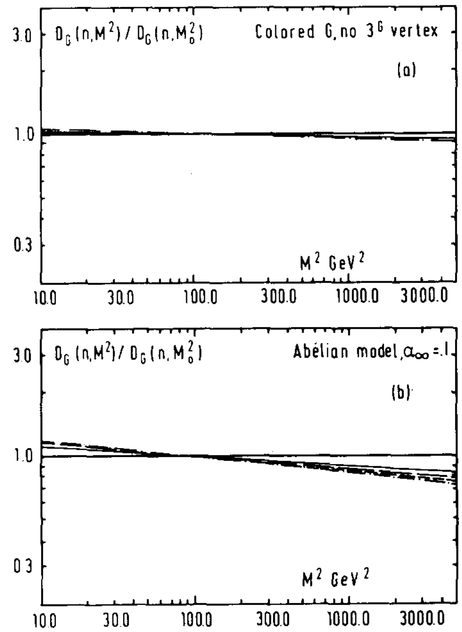

Fig. 4. (a) The ratio $D_G(n, M^2)/D_G(n, M_0^2)$ for a QCD-like model but with no 3G-vertex. Here also, M_0^2 = 100 GeV^2, $D_S(n, M^2)/2N_F D_G(n, M^2)$ = 1, and N_F = 3. We chose α_∞ fixed, $\alpha_\infty = \alpha_s$(100 GeV^2). (b) The same ratio for an abelian model described in the text.

We have found that the M^2 evolution of gluon fragmentation functions is extremely sensitive to the strength of the 3G-vertex of QCD. Observation of these effects will offer quite good indirect evidence for this 3G-coupling. This test is especially appealing, as gluon jet evolution in QCD is dominated by the anomalous dimensions of the theory alone.

Our remarks apply to gluon jets observed in any process. It is, however, essential to isolate gluon jets. This appears easiest in quarkonium decay, where one can compare moments of hadron distributions at J/ψ, ϒ, ...

K.K. and P.M.Z. wish to express their thanks for the hospitality of the theory group during the summer, when this work was begun.

References

[1] H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B (1973) 365;
D.J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3497;
S. Weinberg, Phys. Rev. Lett. 31 (1973) 31.
[2] H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346;
D.J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3497;
T. Appelquist and H.D. Politzer, Phys. Rev. D12 (1975) 1404;
H.D. Politzer, Caltech preprint CALT-68-628.
[3] Ch. Berger et al., PLUTO Collaboration, DESY preprint 78/71 (November 1978).
[4] K. Koller and T.F. Walsh, Nucl. Phys. B140 (1978) 449.
[5] G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.
[6] J.F. Owens, Phys. Lett. 76B (1978) 85;
T. Uematsu, Kyoto preprint;
R.P. Feynman, R.D. Field and G.C. Fox, Caltech preprint CALT-68-651;
T. De Grand, Santa Barbara preprint.
[7] M. Glück and E. Reya, Phys. Lett. 69B (1977) 77.