CAN RECENTLY MEASURED MOMENTS OF $F_3^{\nu N}$ DISCRIMINATE BETWEEN FIELD THEORIES OF STRONG INTERACTIONS?

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It is demonstrated that moments of the flavor *non-singlet* structure function $F_3^{\nu N}$ cannot discriminate, contrary to recent claims, between QCD and other nontrivial fixed point theories of strong interactions, even by taking into account the CDHS precision measurements. Only quantities, such as $F_2(x, Q^2)$, which receive also flavor *singlet* contributions (dominant at small x) can provide us with discriminative tests of strong interaction theories using present experiments; it is not sufficient to study just moments ($n > 2$) of F_2, since these are sensitive mainly to the non-singlet (large-x) contribution.

It has become a widespread belief [1–3] that measurements [4] of moments of the *non-singlet* structure function $F_3^{\nu N}$ constitute already a unique test of QCD and strongly disfavor conventional (fixed point) vector and scalar gluon field theories of strong interactions. It is the purpose of this note to demonstrate that such conclusions are incorrect, even by taking into account the very recent high-statistics precision measurements of the CDHS collaboration [5], as long as only flavor non-singlet quantities are considered. Flavor non-singlet structure functions yield at best a consistency check [4,5] of QCD and are equally well in agreement with all asymptotically non-free fixed point theories known – at least in the presently accessible range of Q^2. Specifically, we show that these $F_3^{\nu N}$-moment data can be accounted for by abelian vector-gluon ($\bar\psi\gamma_\mu\psi A^\mu$), non-abelian scalar-gluon ($\bar\psi\lambda_a\psi\phi_a$) and abelian scalar-gluon ($\bar\psi\psi\phi$) theories as well. Such results rest of course on the (so far unproven) assumption that there exists a fixed point coupling g^* as $Q^2 \to \infty$, i.e. $\beta(g^*) = 0$, such that the effective coupling constant $g^*/16\pi^2 \ll 1$ – a necessary requirement to get approximate scaling in such theories.

In a four-flavor QCD the anomalous dimension [6]

$$a_n = \frac{4}{25}\left[1 - \frac{2}{n(n+1)} + 4\sum_{j=2}^{n} \frac{1}{j}\right] \tag{1}$$

governs the Q^2-dependence of the moments of non-singlet (valence) structure functions such as

$$M_n(Q^2) \equiv \int_0^1 \mathrm{d}x\, x^{n-2} x F_3^{\nu N}(x, Q^2)\,. \tag{2}$$

The renormalization-group improved prediction is that [6]

$$M_n^{-1/a_n} \sim \ln Q^2/\Lambda^2\,, \tag{3}$$

i.e. the $(-1/a_n)$th power of the nth moments are expected to lie along straight lines when plotted against $\ln Q^2$ with a common intercept $\ln Q^2 = \ln \Lambda^2$. These predictions have been found to be in very good agreement [5] with the data for $\Lambda \approx 0.5$ GeV. Similar conclusions have been reached from analyzing [4] the BEBC data but it should be emphasized, however, that these latter results rely heavily on measurements between $Q^2 = 0.6$ and 2 GeV^2 – a region neither appropriate for the parton model nor for the legitimacy of perturbative calculations. Therefore, in the present analysis we shall mainly concentrate on the recent CDHS precision measurements [5] where $Q^2 \geqslant 6.5$ GeV^2, and use only those ($Q^2 > 2$ GeV^2) BEBC data [4] which are not seriously plagued by elastic events as well as by ill-understood binding and target mass effects.

The non-singlet anomalous dimension of an abelian vector-gluon theory is given by [7,8]

$$a_n^{\text{vector}} = (25\alpha^*/16\pi)\, a_n\,, \tag{4}$$

where the fixed point $\alpha^* = g^{*2}/4\pi$ has to be fixed by experiment. In contrast to eq. (3) the Q^2-dependence of M_n is now predicted to be

$$M_n(Q^2)^{-1/a_n} = C_n(Q_0^2)(Q^2/Q_0^2)^{25\alpha^*/16\pi}\,, \tag{5}$$

where the unknown normalization constants $C_n(Q_0^2) \equiv M_n(Q_0^2)^{-1/a_n}$ have to be fitted to the data at an arbitrary value of $Q^2 = Q_0^2$. Similarly, the non-singlet anomalous dimensions of a non-abelian scalar-gluon theory is given by [7,8]

$$a_n^{\text{scalar}} = (\alpha^*/8\pi)\,\tilde{a}_n\,, \tag{6}$$

with

$$\tilde{a}_n = \tfrac{4}{3}[1 - 2/(n(n+1))]\,,$$

which gives

$$M_n(Q^2)^{-1/\tilde{a}_n} = \tilde{C}_n(Q_0^2)(Q^2/Q_0^2)^{\alpha^*/8\pi}\,. \tag{7}$$

The results corresponding to an abelian (Yukawa) scalar-gluon theory are obtained from eqs. (6) and (7) by simply multiplying α^* with a factor of 3/4.

From fig. 1 it can be seen that the predictions of eqs. (5) and (7) of vector- and scalar-gluon field theories, respectively, are in equally good agreement with experiment as the straight line fits [4,5] in ln Q^2 anticipated by QCD in eq. (3). The normalization constants C_n and $\tilde{C}_n$ in eqs. (5) and (7) have been adjusted to the data at $Q_0^2 = 10$ GeV^2. Our conclusions remain essentially unaltered by fitting these constants to, say, $Q_0^2 = 5$ or 20 GeV^2. Considering only non-singlet quantities such as $F_3^{\nu N}$, it is therefore impossible to distinguish between fixed point theories and QCD in the presently measured range of Q^2 ($\lesssim 100$ GeV^2), not even with the recent high precision measurements of the CDHS group [5]. The allowed values of the fixed point coupling suggested by fig. 1 are $0.4 \lesssim \alpha^* \lesssim 0.6$ and $4 \lesssim \alpha^* \lesssim 6$ for abelian vector- and non-abelian scalar-gluon theories, respectively. (A fixed point for abelian vector-gluon theories in eq. (5) as large as [1] $\alpha^* = 16\pi/25 \approx 2$ is of course in striking disagreement with experiment.) It is not too surprising that non-singlet quantities cannot easily discriminate between asymptotically free theories and fixed point theories (unless precision measurements can be extended to $Q^2 = 200$ or 300 GeV^2, say), since their Q^2-dependence is uniquely determined by just *one* anomalous dimension. Therefore, quantities such as M_n^{-1/a_n} are only sensitive to differences in a logarithmic and a power-like behavior in Q^2. This is in contrast to structure functions which receive also contributions from flavor-*singlet* Wilson operators, such as $F_2(x, Q^2)$, the Q^2-dependence of which is determined by *three* different anomalous dimensions [6,8]; thus the different Q^2-behavior of QCD and fixed point theories manifests itself not simply in the difference between a logarithmic and power-like Q^2-dependence but also in the very different n-dependence of singlet (dominant for small n, i.e. small x) and non-singlet anomalous dimensions of the various field theories of strong interactions. In other words one has to consider the *whole* Bjorken-x region and not just large values of x ($\gtrsim 0.4$) where only the non-singlet part plays the dominant role. Therefore, studying only moments ($n > 2$) of any structure function will not suffice to discriminate between strong interaction theories using present experiments ($Q^2 \lesssim 100$ GeV^2), since these moments test mainly the large-x region of structure functions where the non-singlet contribution always dominates. Previous analyses [8,9] of $F_2^{e,\mu N}(x, Q^2)$ have already shown that deep inelastic data strongly favor QCD over other field theories.

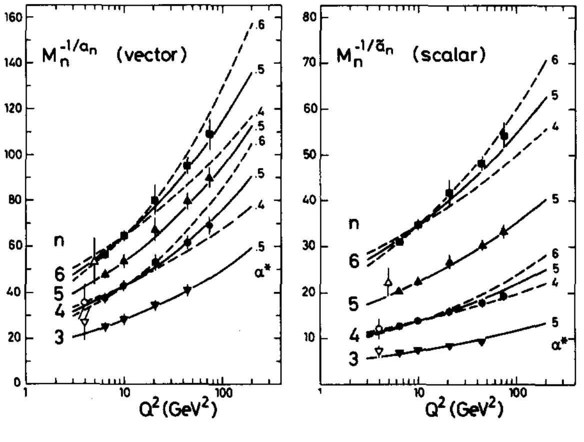

Fig. 1. Comparison of measured [5] $F_3^{\nu N}$-moments with the predictions of abelian vector-gluon theories, eq. (5), and of non-abelian scalar-gluon theories, eq. (7), for various choices of the fixed point α^*. The open circles and triangles are the data of ref. [4]; at higher values of Q^2, these low-statistics data are of course consistent with the high-statistics CDHS data [5] shown.

Another theoretically very attractive test of field theories, which measures ratios of anomalous dimensions directly, is obtained [4] by comparing the logarithms of two moments M_n and $M_{n'}$ which should result in straight lines with slopes $a_n/a_{n'}$ for (non-abelian and abelian) vector-gluon theories or $\tilde{a}_n/\tilde{a}_{n'}$ for (non-abelian and abelian) scalar-gluon theories. These slopes are obviously independent of Λ and α^*, as well as of the number of flavors. Note, that in this way one can discriminate only between vector and scalar gluons, but not between subtleties such as their abelian or non-abelian group-theoretic structure. The slopes of the ordinary Cornwall–Norton moments defined in eq. (2) are found [5] to be in good agreement with the predictions of vector-gluon theories. However, even in the CDHS experiment [5], where $Q^2 \geqslant 6.5$ GeV^2, kinematical target mass effects play a non-negligible role: Assuming that these effects can be in part accounted for by using Nachtmann moments [10,11], their slopes *decrease* by more than 10% [5] as compared to the ones corresponding to the moments in eq. (2). On the other hand, the predictions of scalar theories ($\tilde{a}_n/\tilde{a}_{n'}$) are typically about 20% smaller than those of vector theories ($a_n/a_{n'}$). Thus, at present, scalar-gluon theories cannot be clearly ruled out even by the high-statistics CDHS experiment.

To summarize, we have demonstrated that recent measurements of *non-singlet* moments of $F_3^{\nu N}$ can only be considered as a (necessary) consistency check of QCD, but are not sufficient to discriminate between other field theories of strong interactions. Furthermore as long as *only moments* ($n > 2$) of any structure function, e.g. F_2, are considered, it will be impossible to discriminate between strong interaction theories with present experiments, since these moments test mainly the large-x region of structure functions where the non-singlet contribution always dominates. Apart from having numerous theoretical arguments [12] in favor of QCD, the only deep inelastic structure functions which provide us with discriminative tests of strong interaction theories are those which receive also flavor-singlet contributions, such as $F_2(x, Q^2)$ [8,9]. This is, because for example the lowest ($n = 2$) moment of such quantities exhibit a Q^2-dependence so much different [8] in fixed point theories (increasing) than in QCD (decreasing). The recent precision measurement [13] of $F_2(x, Q^2)$ should prove unique in providing us with clean discriminative tests of QCD.

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