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SEMILEPTONIC DECAYS OF HEAVY QUARKS IN QUANTUM CHROMODYNAMICS

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We investigate the semileptonic decays of heavy quarks in the leading non-trivial order in quantum chromodynamics. Effects of gluon corrections and the initial quark Fermi motion on the semileptonic rates and decay distributions are calculated. The resulting lepton energy spectrum for the charm semileptonic decay is compared with data to extract the mass of the charm quark. This is combined with the semileptonic branching ratio to predict the charm-quark lifetime. We find the lepton energy spectrum very stable with respect to gluon corrections. Expected spectra from the semileptonic decays of bottom and top quarks are presented. We also study the semileptonic decay process $Q \to q\ell\nu_\ell + G$, involving the emission of a single hard non-collinear gluon. This process should be observable with a branching ratio of a few percent in the decays of top (and heavier) quarks.

1. Introduction

Recently, there have been two independent attempts [1,2] to investigate the effects of gluon radiative corrections to the semileptonic decays of heavy quarks (hadrons) in the context of quantum chromodynamics, QCD. The rationale of this approach rests on the QCD inspired hope that for sufficiently heavy quarks, Q, the process

$$Q \to q\ell\nu_\ell \tag{1.1}$$

is expected to closely approximate the semileptonic decays of heavy hadrons containing Q. One is then tempted to assume that the major corrections to the quark-parton process (1.1) could be calculated in the leading order of α_s, the (running) strong interaction coupling constant. That one-gluon corrections to the rate of (1.1) are free of infrared singularities (in the limit $m_q \to 0$) follows from the Kinoshita

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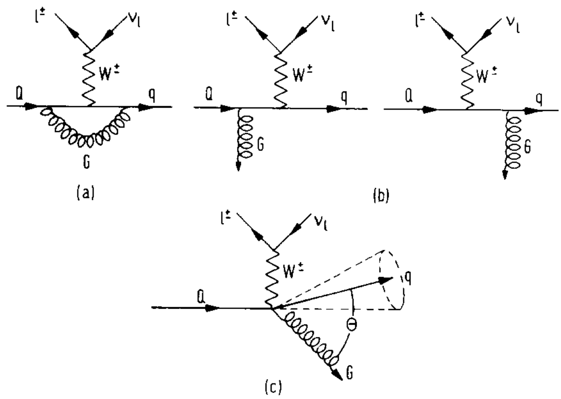

Fig. 1. Feynman graphs for $O(\alpha_s)$ corrections to the semileptonic decay of a quark: (a) virtual gluon correction; (b) gluon bremsstrahlung; (c) hard non-collinear gluon bremsstrahlung in the process $Q \to q\ell\nu_\ell + G$ with $X_G = 2E_G/m_Q > \epsilon$ and $\cos\theta_{qG} < 1 - \delta$.

theorem [3]. The infrared finiteness of these corrections implies that Γ_{SL} is calculable in QCD and it supports the choice of the relevant α_s, as $\alpha_s(m_Q^2)$ [2].

The radiative corrections to Γ_{SL} are also expected to be ultraviolet finite for a V–A four-Fermi weak interaction on the basis of a comparison with μ-decay [4]. In fact there is a one-to-one correspondence between the (μ-decay matrix element and) Feynman diagrams for the radiative QED corrections to μ-decay and those for the charge $+\frac{2}{3}$ heavy quarks in QCD.

This correspondence can be seen through figs. 1a or else from the Lagrangians written in the charge retention form [1,2] if one substitutes *:

$$(\mu^-, e^-, \bar{\nu}_e, \nu_\mu) \leftrightarrow (Q, q, \ell^+, \nu_\ell) \quad .$$

One can then calculate Γ_{SL} and the quark energy distribution $(1/\Gamma_{SL})(d\Gamma/dE_q)$ from the existing literature on μ-decay [4] by simply replacing:

$$\alpha \to \tfrac{1}{3}\alpha_s \,\mathrm{Tr} \sum_{i=1}^{8} \lambda_i\lambda_i = \tfrac{4}{3}\alpha_s, \tag{1.2}$$

$$\alpha_s = \frac{12\pi}{(33 - 2n_f)\ln(M_Q^2/\Lambda^2)} \quad , \tag{1.3}$$

where n_f = number of effective flavours, M_Q = mass of the heavy quark and Λ is the renormalization point $\simeq$500 MeV. The $O(\alpha_s)$ corrections so calculated decrease Γ_{SL} for the charm quark by ~35% [1,2].

* The corresponding substitution for the decay of charge $-\frac{1}{3}$ quarks (bottom) is

$(\mu^-, e^-, \bar{\nu}_e, \nu_\mu) \leftrightarrow (Q, q, \bar{\nu}_e, e^-)$.

Another quantity which is also reliably calculable in QCD is the inclusive lepton energy spectrum $(1/\Gamma)\, d\Gamma/dE_\ell$. Apart from being infrared finite, $(1/\Gamma)\, d\Gamma/dE_\ell$ does not suffer from the fragmentation effects of the final quark, which is the case for the inclusive hadron energy distribution ($(1/\Gamma)\, d\Gamma/dE_h$ can only be got by folding the $(1/\Gamma)\, d\Gamma/dE_q$ distribution with the non-perturbative $q \to h$ fragmentation). Of course, such a picture is expected to work for sufficiently heavy quarks only, since only then is one in the deep inelastic region, with the q fragmenting into a jet of hadrons. The quark-parton process (1.1) (together with the gluonic corrections) should be taken in a duality sense, averaging the effects of multibody final hadronic states much the same way that the QCD corrected value of R gives the average hadronic cross section in e^+e^- annihilation experiments. In this vein, the decay $c \to s\ell\nu_\ell$ is expected to be a marginal case since the decays $D \to K\ell\nu_\ell$ and $D \to K^*\ell\nu_\ell$ apparently saturate Γ_{SL}. Even in this case we get a satisfactory description of the lepton spectrum when non-perturbative effects are taken into account. The method has clearly more merit for the heavier bottom and top quarks, where perturbative QCD would be the only effect of strong interactions on the lepton energy spectrum.

The shape of the lepton energy spectrum depends on the masses m_Q and m_q and one could extract them, in principle, from the observed lepton energy spectrum. Since (unlike the non-leptonic process $Q \to q\bar{q}q$) the semileptonic process (1.1) does not have any short- or long-distance enhancement effects in the operator product expansion sense [5], the information on the (inclusive) lepton energy spectrum and the semileptonic branching ratio can be transcribed, using perturbative QCD, to get both the masses and lifetime of heavy quarks *. To the extent that the non-perturbative (confinement) effects can be neglected, the lifetime of the heavy hadrons $(Q\bar{q})$ should simply be given by τ_Q. A reliable comparison with the experimental data to extract m_Q and m_q requires both the computation of $O(\alpha_s)$ corrections in QCD to $(1/\Gamma)\, d\Gamma/dE_\ell$, as well as the non-perturbative effects due to the heavy quark motion inside the heavy hadron, i.e., the Fermi motion of Q.

Unfortunately, the counterpart of $(1/\Gamma)\, d\Gamma/dE_\ell$ for (1.1) is the distribution $(1/\Gamma)\, d\Gamma/dE_{\bar{\nu}_e}$ for the μ-decay, which has not been calculated for obvious reasons! A first attempt towards calculating $(1/\Gamma)\, d\Gamma/dE_\ell$ to $O(\alpha_s)$ was made in ref. [1]. However, apart from an (unexplained) crude assumption we disagree with the comparison made there with charm data. We present our calculation for the distribution $(1/\Gamma) \times d\Gamma/dE_\ell$ and find it is remarkably stable against $O(\alpha_s)$ gluon corrections. In particular, we do not find the drastic softening effects in the lepton energy distribution due to $O(\alpha_s)$ corrections, as claimed in ref. [1]. The softening of the lepton energy spectrum from charm semileptonic decays should then be attributed mainly to non-perturbative effects. This is exemplified by assuming a gaussian for the Q momentum distribution in the initial heavy hadron. We find that the Fermi motion effect in

* The masses m_Q and m_q and the branching ratio determine lifetimes of heavy quarks (mesons) up to a mixing angle. For charm and top mesons this will not introduce any significant error since $\cos^2\theta_c \simeq 1$ and $\cos^2\theta_{tb} \simeq 1$ in the Kobayashi-Maskawa model.

$(1/\Gamma)\, d\Gamma/dE_\ell$ is noticeable for charm decay but it vanishes rapidly as m_Q increases, in conformity with most other confinement effects at higher energies. The lepton energy spectrum from the semileptonic decays of top (and heavier) quarks is thus an unambiguous prediction of QCD, stable against infrared and confinement effects.

We apply our analysis to the observed electron spectrum measured around ψ'' (3.77) [6]. A reasonable χ^2 fit is obtained with the charm quark mass in the range 1.8 GeV $\geqslant m_c \geqslant$ 1.6 GeV and $m_s = 0.5$ GeV. Assuming a charm semileptonic branching ratio of 10%, this leads to the charm quark (hadron) lifetime estimate 3.78×10^{-13} sec $\leqslant \tau_c \leqslant 8.0 \times 10^{-13}$ sec in the GIM model [7]. The $O(\alpha_s)$ corrected spectra for the bottom and top quark semileptonic decays are also presented.

Next, we study the semileptonic decay process of a heavy quark *involving the emission of a hard non-collinear gluon*:

$$Q \to q\ell\nu_\ell + G. \tag{1.4}$$

To that end we first define the lowest-order semileptonic jet process corresponding to (1.1), interpreting the final quark as a jet. (See fig. 1b.) This is done by including in it the $O(\alpha_s)$ soft and collinear gluons through cuts on the gluon energy E_G, and the gluon-quark angle, θ_{qG}. Since, we have the complete $O(\alpha_s)$ corrected rate for the process (1.1), we can obtain the rate for the process (1.4) by simply subtracting the $O(\alpha_s)$ jet rate corresponding to (1.1). In this way we are led to an estimate of the branching ratio of a few percent for the process (1.4), for reasonable cuts on θ_{qG} and E_G. We present angular distributions $(1/\Gamma)\, d\Gamma/d\cos\theta_{G\ell}$ and $(1/\Gamma)\, d\Gamma/$

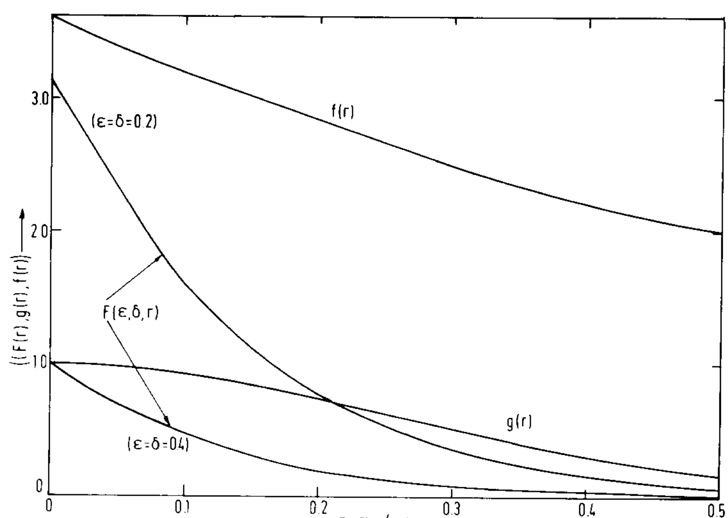

Fig. 2. The functions $f(r)$, $g(r)$ and $F(\epsilon, \delta, r)$ where $r = m_q/m_Q$. For the definitions of these functions see sects. 2, 3.

d cos θ_{qG}, which though somewhat modified by non-perturbative effects, should help in the search for gluons in the semileptonic decays of top (and heavier) quarks.

The paper is organized as follows. In sect. 2, we describe our analysis for the $O(\alpha_s)$ corrections to the process (1.1). The corrections to Γ_{SL} can be calculated through the function $g(r)$ and $f(r)$ which are shown in fig. 2 ($r = m_q/m_Q$). The lepton energy spectrum is compared to charm data in fig. 3 and predictions for the bottom and top quark induced lepton energy spectra are presented in figs. 4, 5, respectively. Fig. 6 contains plots for acceptable ranges of m_c, m_s and ΔP_c (charm quark Gaussian momentum width) with χ^2 of 10/7 d.f. or less. In sect. 3, we describe our analysis for the gluon emission process (1.4). The rate for (1.4) for a given cut on θ_{qG} and E_G is a function of r and shown in fig. 2 for $0 \leqslant r \leqslant 0.5$. Figs. 7 and 8 contain respectively the distributions $(1/\Gamma_{(\epsilon,\delta)})\, d\Gamma/d\cos\theta_{G\ell}$ and $(1/\Gamma_{(\epsilon,\delta)})\, d\Gamma/d\cos\theta_{qG}$ for the process (1.4). Sect. 4 contains a brief summary of our results.

2. Lepton energy spectra and semileptonic rates

We begin by defining the square of the invariant matrix element for the $O(\alpha_s)$ corrected process (1.1). To be definite, we present the results for the semileptonic decay of a heavy quark with electric charge $+\frac{2}{3}$ into a lighter quark with electric charge $-\frac{1}{3}$ and a lepton pair, $\ell^+\nu_\ell$. The $O(\alpha_s)$ calculations are done by giving the gluon a small mass λ. The $O(\alpha_s)$ corrected rate for the process (1.1) in our normalization can be expressed as:

$$d\Gamma = (2\pi)^{-5}(16 m_Q E_p E_\ell E_\nu)^{-1}\, d^3p_q\, d^3p_\ell\, d^3p_\nu$$
$$\times \{\delta(p_Q - p_q - p_\ell - p_\nu)(|M_0|^2 + M_1)$$
$$+ (2\pi)^{-3}(2E_G)^{-1}\, d^3p_G\, \delta(p_Q - p_q - p_\ell - p_\nu - p_G)\,|M_2|^2\} \,. \tag{2.1}$$

$|M_0|^2$, M_1 and $|M_2|^2$ are respectively, the zeroth order, $O(\alpha_s)$ contribution from the virtual gluon correction to the zeroth order, and the single bremsstrahlung diagram contributions. These are listed below:

$$|M_0|^2 = 64 G_F^2 (p_q \cdot p_\nu)(p_Q \cdot p_\ell)\,, \tag{2.2}$$

$$|M_2|^2 = 512 G_F^2 (\tfrac{4}{3}\pi\alpha_s)\left[\frac{B_1}{D_1^2} + \frac{B_2}{D_1 D_2} + \frac{B_3}{D_2^2}\right], \tag{2.3}$$

where

$$B_1 = (p_q \cdot p_\nu)[2(p_\ell \cdot p_G)(p_Q \cdot p_G) + 2m_Q^2 p_\ell \cdot (p_G - p_Q) - \lambda^2 p_Q \cdot p_\ell]\,,$$

$$B_2 = (p_q \cdot p_\nu)[2(p_\ell \cdot p_G)(p_Q \cdot p_q) - 2(p_q \cdot p_\ell)(p_Q \cdot p_G) + \lambda^2 (p_Q \cdot p_\ell) + 2(p_Q \cdot p_\ell)(p_q \cdot p_G - p_Q \cdot p_G - 2p_Q \cdot p_q)] + (p_q \cdot p_Q)[-2(p_\nu \cdot p_G)(p_Q \cdot p_\ell) + \lambda^2 p_\ell \cdot p_\nu] + (p_\nu \cdot p_Q)[2(p_q \cdot p_G)(p_Q \cdot p_\ell) - \lambda^2 p_q \cdot p_\ell] , \quad (2.4)$$

$$B_3 = (p_Q \cdot p_\ell)[2(p_\nu \cdot p_G)(p_q \cdot p_G) - 2m_q^2 (p_\nu \cdot p_G) - 2m_q^2 (p_\nu \cdot p_q) - \lambda^2 (p_q \cdot p_\nu)] ,$$

$$D_1 = \lambda^2 - 2p_Q \cdot p_G , \qquad D_2 = \lambda^2 + 2p_q \cdot p_G .$$

The terms B_1 and B_3 in eq. (2.4) are simply one-gluon bremsstrahlung from the initial and final quark lines respectively. B_2 is the interference term.

2.1. Virtual gluon correction (to $O(\alpha_s)$*):*

$$M_1 = -\tfrac{64}{3} G_F^2 \frac{\alpha_s}{\pi} \{4(p_q \cdot p_\nu)(p_Q \cdot p_\ell)[G_1 + G_5 - (p_q \cdot p_Q) \times (2G_2 - G_3 + G_4)] + G_2 m_q^2 [(p_Q \cdot p_q)(p_\ell \cdot p_\nu) - (p_Q \cdot p_\nu)(p_q \cdot p_\ell) + 4(p_Q \cdot p_\nu)(p_Q \cdot p_\ell) - 3(p_Q \cdot p_\ell)(p_q \cdot p_\nu)] + 2G_3 m_q^2 [m_Q^2 (p_\ell \cdot p_\nu) - (p_\ell \cdot p_\nu)(p_Q \cdot p_q) + (p_Q \cdot p_\nu)(p_q \cdot p_\ell) - 3(p_Q \cdot p_\nu)(p_Q \cdot p_\ell) + (p_Q \cdot p_\ell)(p_q \cdot p_\nu)] + 2G_3 m_Q^2 (p_q \cdot p_\nu)[p_q \cdot p_\ell - 2p_Q \cdot p_\ell] + G_4 m_q^2 [-2m_Q^2 (p_\ell \cdot p_\nu) + (p_\ell \cdot p_\nu)(p_Q \cdot p_q) - (p_Q \cdot p_\nu)(p_q \cdot p_\ell) + 2(p_Q \cdot p_\nu)(p_Q \cdot p_\ell) + (p_Q \cdot p_\ell)(p_q \cdot p_\nu)] + G_4 m_Q^2 [(p_\ell \cdot p_\nu)(p_Q \cdot p_q) - (p_Q \cdot p_\nu)(p_q \cdot p_\ell) + 2(p_q \cdot p_\ell)(p_q \cdot p_\nu) + (p_Q \cdot p_\ell)(p_q \cdot p_\nu)]\} , \quad (2.5)$$

where

$$G_1 = \frac{p_Q \cdot p_q}{m_Q m_q \sinh\theta} \left[L\left(\frac{2\sinh\theta}{e^\omega - e^{-\theta}}\right) - L\left(\frac{2\sinh\theta}{e^\theta - e^{-\omega}}\right) + (\omega - \theta) \ln\left(\frac{\sinh\frac{1}{2}(\omega-\theta)}{\sinh\frac{1}{2}(\omega+\theta)}\right) + \theta(\omega - 2\omega_\lambda)\right] + 2\omega_\lambda - \tfrac{3}{2}\omega + \tfrac{9}{4} ,$$

$$G_2 = \frac{1}{m_q m_Q} \frac{\theta}{\sinh\theta} ,$$

$$G_3 = \frac{1}{2m_q m_Q \sinh\theta}\left(\theta + \frac{\omega\sinh\theta - \theta\sinh\omega}{\cosh\omega - \cosh\theta}\right) ,$$

$$G_4 = \frac{1}{(m_Q^2 + m_q^2 - 2p_q\cdot p_Q)}\left(1 + \omega + \frac{\theta\sinh\theta - \omega\sinh\omega}{\cosh\omega - \cosh\theta} + \frac{\theta\, e^{-\omega}}{\sinh\theta}\right) ,$$

$$G_5 = \frac{\omega\sinh\omega - \theta\sinh\theta}{2(\cosh\omega - \cosh\theta)} + \tfrac{1}{2}\omega - \tfrac{3}{4} ,$$

$$\omega_\lambda = \ln(\lambda/m_q), \qquad \omega = \ln(m_Q/m_q), \qquad \cosh\theta = (p_Q\cdot p_q)/m_Q m_q . \tag{2.6}$$

The expression (2.5) is obtained by doing the loop integration in the virtual bremsstrahlung diagram [4] and is valid in the small λ limit.

The analogous formulae for the semileptonic decay of a charge $(-\frac{1}{3})$ quark,

$$Q(-\tfrac{1}{3}) \to q(+\tfrac{2}{3})\, \ell^- \nu_\ell ,$$

can be obtained from expressions (2.2)–(2.6) by simply replacing $(p_\ell \leftrightarrow p_\nu)$. We have decided to give the expressions (2.2)–(2.6) here, despite their length (and age!), since they can be fed into a numerical integration programme to calculate any desired semileptonic decay distribution to $O(\alpha_s)$. The expressions found in the literature [4] for μ-decay are of little use in calculating $(1/\Gamma)\, d\Gamma/dE_\ell$. We emphasize that both the bremsstrahlung contribution and the virtual gluon corrections are separately divergent and depend on λ. However, the sum of the two is convergent and independent of λ. The one gluon corrected decay rate Γ_{SL} is [1,2,4]:

$$\Gamma_{SL} = \Gamma_{SL}^{(0)}\left(1 - \frac{2}{3}\frac{\alpha_s(m_Q^2)}{\pi} f(r)\right) , \tag{2.7}$$

where $\Gamma_{SL}^{(0)}$ is the zeroth-order rate corresponding to (1.1):

$$\Gamma_{SL}^{(0)} = \frac{G_F^2}{192\pi^3} m_Q^5\, g(r) , \tag{2.8}$$

with $r = m_q/m_Q$ and

$$g(r) = 1 - 8r^2 + 8r^6 - r^8 - 24r^4 \ln r . \tag{2.9}$$

The function $f(r)$ can be evaluated numerically by performing the phase-space integration indicated in (2.1), or it can be obtained from the μ-decay results [4]. We have plotted $g(r)$ and $f(r)$ for $0 \geqslant r \geqslant 0.5$, which covers the range of current interest.

The lepton energy distribution can be expressed likewise as:

$$\frac{1}{\Gamma}\frac{d\Gamma}{dX_\ell} = H(X, r_\ell) + \frac{2\alpha_s(m_Q^2)}{3\pi} h(r, X_\ell) , \tag{2.10}$$

with

$$X_\ell = 2E_\ell/m_Q ,$$

where the zeroth-order lepton energy distribution is given by [8]:

$$H(r, X_\ell) \equiv \frac{1}{\Gamma}\frac{d\Gamma^{(0)}}{dX_\ell} = \frac{12X_\ell^2(1-X_\ell-r^2)^2}{(1-X_\ell)} , \qquad (2.11)$$

$$\text{for } Q(+\tfrac{2}{3}) \to q(-\tfrac{1}{3})\,\ell^+\nu_\ell ,$$

$$= 2X_\ell^2\frac{(1-X_\ell-r^2)^2}{(1-X_\ell)^3}\,[(1-X_\ell)(1-r^2-X_\ell)$$

$$+ (2-X_\ell)(1+2r^2-X_\ell)] , \quad \text{for } Q(-\tfrac{1}{3}) \to q(+\tfrac{2}{3})\,\ell^-\bar{\nu}_\ell . \qquad (2.12)$$

The function $h(r, X_\ell)$ is calculated numerically using eqs. (2.1)–(2.6) with the help of a numerical integration programme. We have verified that $h(r, X_\ell)$ numerically satisfies the normalization condition:

$$\int_0^1 h(r, X_\ell)\, dX_\ell = -f(r) .$$

The function $2(\alpha_s(m_Q^2)/3\pi)\, h(r, X_\ell)$ is shown for charm semileptonic decay for $r = 0.28$ (corresponding to $m_c = 1.8$ GeV, $m_s = 0.5$ GeV) in fig. 3 *, which also shows the zeroth-order and the total $(1 + O(\alpha_s))$ corrected lepton energy spectrum. As can be seen from fig. 3 the shape of the normalized lepton energy distribution from the free quark decay is very stable with respect to the $O(\alpha_s)$ corrections though the rate Γ_{SL} changes substantially. In particular, we do not find the appreciable softening of the charm lepton spectrum found in ref. [1].

Before we make a comparison with charm data, we would like to include the non-perturbative confining effects of Q inside the meson $(Q\bar{q})$. These non-perturbative effects are expected to become unimportant for heavier quarks (say top) but should still be taken into account in charm decays. The most important aspect of the confining force, *vis à vis* decay rates and distributions, is the description of the Q-momentum distribution inside the meson $Q\bar{q}$. We make a simple ansatz for $|p_Q|$ by assuming it to have a Gaussian distribution. Thus, the probability of finding Q with momentum between $|p_Q|$ and $|p_Q| + d|p_Q|$ is:

$$D(|p_Q|)\, d|p_Q| = \frac{\sqrt{2}}{\sqrt{\pi}\,\Delta P}\exp(-\tfrac{1}{2}(|p_Q|/\Delta P)^2)\, d|p_Q| . \qquad (2.13)$$

* In order to compare our calculations with the experimental data on lepton energy spectrum from charm semileptonic decays, we have plotted the quantity $h(r, E_\ell)$ which is simply related to $h(r, X_\ell)$ by $h(r, E_\ell) = \frac{1}{2}m_Q h(r, X_\ell)$. The relative normalization of the order $O(\alpha_s)$ lepton energy spectrum is fixed with respect to the zeroth order by the factor $2(\alpha_s(m_Q^2)/3\pi)\, f(r)$.

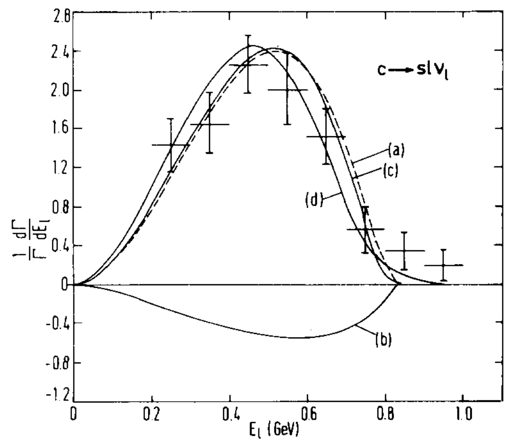

Fig. 3. Inclusive lepton energy spectrum from charm quark decay. (a) Free-quark model result; (b) $O(\alpha_s)$ contribution from one gluon corrections; (c) one gluon corrected spectrum; (d) combined effect of c-quark Fermi motion and one-gluon corrections. The curves (a)–(c) correspond to $m_c = 1.8$ GeV, $m_s = 0.5$ GeV and (9) to $m_c = 1.7$ GeV, $m_s = 0.5$ GeV and $\Delta P_c = 0.6$ GeV. Data points are from ref. [6].

The lepton energy distribution can now be obtained by folding the decay distributions with $D(|p_Q|)$ and integrating over $d|p_Q|$:

$$\frac{1}{\Gamma}\frac{d\Gamma}{dX_\ell} = \int_0^\infty D(|p_Q|)\, d(|p_Q|) \int_{y_\ell^{min}}^{y_\ell^{max}} \frac{m_Q}{2|p_Q|y_\ell} G(r, y_\ell)\, dy_\ell , \tag{2.14}$$

where

$$y_\ell = (p_Q \cdot p_\ell)/m_Q ,$$

$$y_\ell^{min} = \tfrac{1}{2}X_\ell(E_Q - |p_Q|) ,$$

$$y_\ell^{max} = \min[\tfrac{1}{2}X_\ell(E_Q + |p_Q|), \tfrac{1}{2}m_Q(1 - r^2)] ,$$

$$r = m_q/m_Q . \tag{2.15}$$

The analytic expression for $G(r, y_\ell)$ with the inclusion of gluon radiative corrections is too cumbersome to obtain. For the case of the free quark decay it is however simply given by $H(r, y_\ell)$ (eq. (2.11) or (2.12)). We have instead made a Monte Carlo calculation of the combined effect of the Fermi motion and the $O(\alpha_s)$ correction by using expressions (2.1)–(2.6) with the distribution $D(|p_Q|)$. The resulting distribution is then added to the Fermi motion corrected zeroth-order distribution, obtained by substituting (2.11) (or (2.12)) in (2.14).

The resulting lepton energy spectra for the charm, bottom and top semileptonic decays are shown in figs. 3, 4 and 5, respectively. The effects of gluon radiative corrections and the Fermi motion on the lepton energy spectrum vanish as m_Q increases. The predictions of the simple quark-parton model thus become very dependable for $(1/\Gamma)\, d\Gamma/dE_\ell$ for heavier quarks. The change in the decay rate due to Fermi motion is negligible even for charm decay.

We are now in a position to make a comparison of our analysis with charm semileptonic data. We have used the data on inclusive lepton spectrum from the DELCO experiment at SPEAR [6], obtained at $\psi''(3.77)$. Since the D's produced in the decay of $\psi''(3.77)$ are almost at rest ($|p_D| \simeq 0.25$ GeV), the comparison is free of ambiguities due to the $c \to D$ fragmentation. We have attempted two χ^2 fits; one with only the QCD corrections treating m_c and m_s as free parameters, and the other by also including the effect of the charm quark Fermi motion. In the latter fit we fix m_s (= 500 MeV) and treat m_c and ΔP_c as free parameters. Since the experimental errors are still large, we have kept all solutions having a χ^2/d.f. ratio of 10/7 or less. The allowed parameter regions are shown in fig. 6. The solutions corresponding to (i) $p_c = 0$, $m_c = 1.8$ GeV, $m_s = 0.5$ GeV and (ii) $\bar{p} = 0.6$ GeV, $m_c = 1.7$ GeV, $m_s = 0.5$ GeV are compared with charm data. We remark that to the extent that the radiative corrections are effective, the peak of the zeroth-order lepton energy spectrum is shifted somewhat towards smaller values of E_ℓ and the high-energy tail is somewhat depleted. These effects, though in the right direction, are not sufficient to exactly describe the general trend of charm lepton energy data [9] which show a lower peak and an extended tail. The Fermi motion induces the desired softening and is noticeable in the case of charm, indicating that the non-perturbative effects are still not entirely negligible in charm decays. Considering the present uncertainty in the data we find $1.8 \text{ GeV} \geqslant m_c \geqslant 1.6$ GeV to be an acceptable range for the charm quark mass, leading to a lifetime estimate of 8 ×

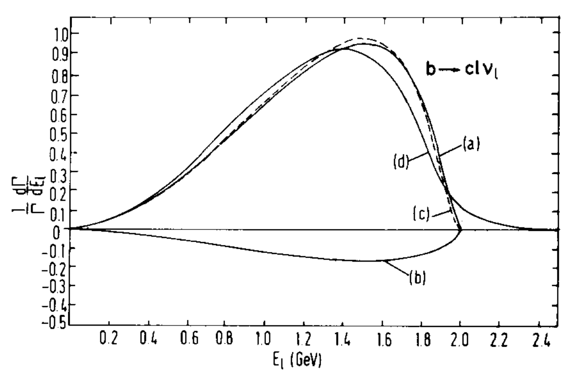

Fig. 4. Inclusive lepton energy spectrum from $b \to c\ell\nu_\ell$. (a)–(d) are the same as in fig. 3. We have assumed $m_b = 4.5$ GeV, $m_c = 1.5$ GeV and for (d) $\Delta P_b = 1.0$ GeV. The curve corresponding to $\Delta P_b = 0.5$ GeV is hardly distinguishable from curve (c).

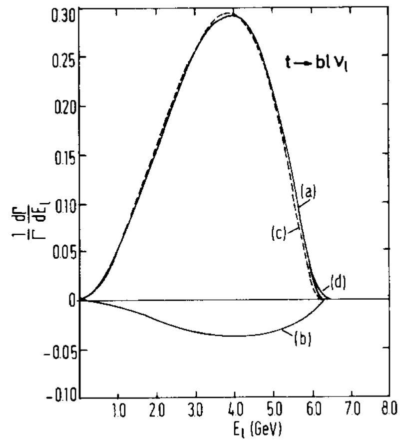

Fig. 5. Inclusive lepton energy spectrum from $t \to b\ell\nu_\ell$. (a)–(d) are the same as in fig. 3. We have assumed m_t = 14.0 GeV, m_b = 4.5 GeV and ΔP_t = 1.0 GeV.

10^{-13} sec $\geqslant \tau_c \geqslant 3.78 \times 10^{-13}$ sec. This prediction can be compared to an independent measurement of τ_D to test the QCD semileptonic rate calculation. The lepton energy spectrum and semileptonic branching ratio thus provide reliable estimates for m_Q and $\tau_{(Q\bar{q})}$, modulo mixing angles. The change in τ due to the mixing angles should be rather small for the decays of charm and top mesons, though it renders the possibility of an absolute measurement of τ_B through semileptonic decay measurements rather intractable.

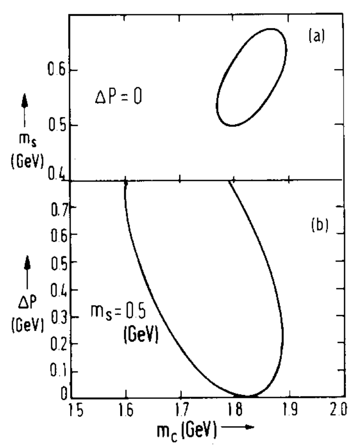

Fig. 6. χ^2 fit of the DELCO data [6] with the QCD corrected lepton energy spectrum: (a) acceptable region in the $(m_c - m_s)$ plane with no Fermi motion; (b) acceptable region in the $(m_c - \Delta P_c)$ plane with a Gaussian charm quark Fermi motion (m_s = 0.5 GeV).

3. Hard gluon bremsstrahlung in semileptonic decays

In this section we study the production of a single gluon in the semileptonic process

$$Q \to q\ell\nu_\ell + G\,. \tag{3.1}$$

As is clear from sect. 2, a part of the process (3.1), with soft collinear gluon emission, belongs to the radiative $O(\alpha_s)$ correction to the process (1.1). However, for very heavy quarks there will be a non-zero probability of observing (3.1) as an independent process, with the emission of a hard non-collinear gluon. The decay probability of (3.1) is already given in the last term of eq. (2.1) with the $|M_2|^2$ given in eqs. (2.3) and (2.4). Since, we will put large cuts on X_G and $\cos\theta_{qG}$, to define the hard gluon process (3.1) one could set $\lambda = 0$. To get the rate for (3.1), one could trivially do the integrals over the lepton variables in (3.2) using the formula

$$\int d^3p_\ell\, d^3p_\ell (E_\ell E_{\nu_\ell})^{-1}\delta^4(X - p_\ell - p_{\nu_\ell})(p_\ell)_\mu (p_{\nu_\ell})_\nu$$
$$= \tfrac{1}{6}\pi(2X_\mu X_\nu + X^2 g_{\mu\nu})\;, \tag{3.2}$$

with

$$X_\mu = (p_Q - p_q - p_G)_\mu\,.$$

Three of the four remaining angular integrals can be done trivially leading to the three-dimensional integral

$$d\Gamma = \frac{1}{192 m_Q}\,\frac{1}{(2\pi)^5}\,|p_q| E_G |\widetilde{M}_2|^2\, dE_G\, dE_q\, d\cos\theta_{qG}\,, \tag{3.3}$$

where $|\widetilde{M}_2|^2$ can be obtained from eqs. (2.3) and (2.4) by substituting: $(p_\ell)_\mu (p_{\nu_\ell})_\nu \to 2X_\mu X_\nu + X^2 g_{\mu\nu}$. We have done the remaining integrals in (3.4) numerically by putting cuts on $\epsilon = 2E_G/m_Q$ and $\delta \equiv 1 - \cos\theta_{qG}$. Defining:

$$\Gamma_{(\epsilon,\delta)}(Q \to q\ell\nu_\ell + \text{Gluon})$$
$$= 2(\alpha_s(m_Q^2/3\pi)\Gamma_{SL}^{(0)} F(\epsilon,\delta,r)\,, \tag{3.4}$$

the function $F(\epsilon,\delta,r)$ is plotted in fig. 2 for $0.5 \geqslant r \geqslant 0$, with the cuts $\epsilon = \delta = 0.2$ and $\epsilon = \delta = 0.4$.

Since the total $1 + O(\alpha_s)$ semileptonic rate, Γ_{SL}, is given by (2.7), we could get the decay rate for the process

$$Q \to q\ \text{jet} + \ell\nu_\ell$$

by subtracting $\Gamma_{(\epsilon,\delta)}$ from Γ_{SL}. Thus,

$$\Gamma_{(\epsilon,\delta)}(Q \to q\ \text{jet} + \ell^\pm\nu_\ell)$$
$$= \Gamma_{SL}^{(0)}\,[1 - 2(\alpha_s(m_Q^2)/3\pi)(f(r) + F(\epsilon,\delta,r))]\,. \tag{3.5}$$

The expression (3.5) is the analogue of the Sterman-Weinberg 2-jet cross section in e^+e^- [7] for the semileptonic decay of a heavy quark. $\Gamma_{SL}^{(0)}$ drops out from the ratio of (3.4) and (3.5), thus leading to a relative rate estimate for the process (3.1) (with respect to (1.1)) which depends on m_Q only through $\alpha_s(m_Q^2)$.

$$R(\epsilon,\delta) \equiv \frac{\Gamma_{(\epsilon,\delta)}(Q \to q + \ell^\pm \nu_\ell + G)}{\Gamma_{(\epsilon,\delta)}(Q \to q\,\mathrm{jet} + \ell^\pm \nu_\ell)} \tag{3.6}$$

$$= \frac{2}{3\pi}\,\frac{\alpha_s(m_Q^2)F(\epsilon,\delta,r)}{1-2(\alpha_s(m_Q^2)/3\pi)(f(r)+F(\epsilon,\delta,r))}\,.$$

Presented in table 1 are some values of the quantity $R(\epsilon, \delta)$ corresponding to various cuts on (ϵ, δ) for the expected decay rate of the top quark. Since the semileptonic

Table 1
The ratio

$$R(\epsilon,\delta) = \frac{\Gamma(Q \to q + \ell\nu_\ell + G)}{\Gamma(Q \to q\,\mathrm{jet} + \ell\nu_\ell)}$$

for various choice of cuts on $\epsilon = 2E_G/m_Q$ and $\delta = 1 - \cos\theta_{qG}$

m_Q (GeV)	m_q (GeV)	ϵ	δ	$R_{(\epsilon,\delta)}$ (%)
14.0	0	0.3	0.3	13.8
			0.0	4.5
		0.4	0.4	8.0
			0	3.4
	4.5	0.3	0.3	0.95
			0	0.6
		0.4	0.4	0.45
			0	0.33
27.0	0	0.2	0.2	22.1
			0	5.3
		0.3	0.3	10.8
			0	3.6
	4.5	0.2	0.2	5.9
			0	2.0
		0.3	0.3	2.7
			0	1.2

The choice of m_Q is motivated by theoretical predictions and fits for the top quark mass [12].

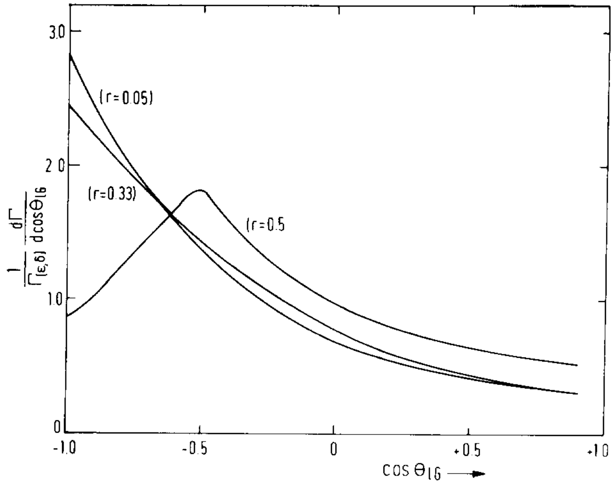

Fig. 7. The distribution $(1/\Gamma_{(\epsilon,\delta)})\, d\Gamma/d \cos\theta_{G\ell}$ from the hard gluon process $Q \to q\ell\nu_\ell + G$ for $r = 0.05, 0.33$ and 0.5, assuming $\epsilon = \delta = 0.4$.

branching ratio, $\Gamma_{SL}/\Gamma_{total}$, in the decays of heavy quarks is expected to be around the free-quark decay model value ~40%, we expect the process (3.1) to show up in top decays at a measurable rate.

Encouraged somewhat by this result, we calculate the distributions $(1/\Gamma_{(\epsilon,\delta)})\, d\Gamma/$

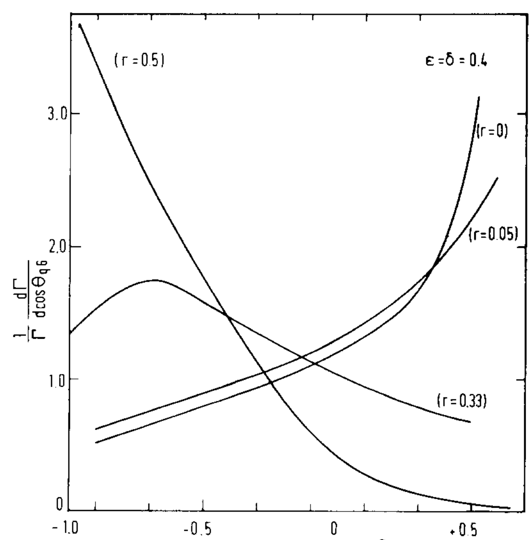

Fig. 8. The distribution $(1/\Gamma_{(\epsilon,\delta)})\, d\Gamma/d \cos\theta_{qG}$ from the hard gluon process $Q \to q\ell\nu_\ell + G$ for $r = 0, 0.05, 0.33$ and 0.5, assuming $\epsilon = \delta = 0.4$.

d cos $\theta_{G\ell}$ and $(1/\Gamma_{(\epsilon,\delta)})$ dΓ/d cos θ_{qG} for the process (3.1). They are shown in figs. 7 and 8 respectively for some representative values of r with the cuts $\epsilon = \delta = 0.4$. These distributions, while influenced somewhat by non-perturbative effects, are expected to hold qualitatively. They serve to show that if m_q is not small, as is expected in most decay schemes, then the distribution dΓ/d cos θ_{qG} does not peak around $\theta_{qG} \simeq 0$. In fact there is a measurable probability (see fig. 8 and table 1) of observing the gluon in (3.1) with cos $\theta_{Gq} \geqslant 90°$.

The cleanest signature of the process (3.1) is:

$$e^+e^- \to t\bar{t} \to (\text{q jet}) + (\bar{\text{q}}\text{ jet}) + \ell^+\ell^- + \text{gluon jet}, \tag{3.7}$$

which should have a branching ratio of $2(\Gamma_{SL}/\Gamma_{total})^2 R_{\epsilon,\delta} \simeq 0.3 R_{\epsilon,\delta}$. The process (3.7) can be distinguished from the "ordinary" QCD process:

$$e^+e^- \to t\bar{t}G \to \text{q jet} + \bar{\text{q}}\text{ jet} + \ell^+\ell^- + \text{gluon jet}, \tag{3.8}$$

if one is near the threshold of the $t\bar{t}$ production.

4. Discussion

In the preceding sections, we have investigated the consequences of one gluon corrections to the semileptonic decays of heavy quarks. Our estimates of $\Gamma_{SL}/\Gamma_{SL}^{(0)}$ agree with previous estimates [1,2] but we disagree in the treatment of radiative corrections to the lepton energy distribution presented in ref. [1]. We find the distribution $(1/\Gamma)$ dΓ/dE_ℓ to be rather stable with respect to the one gluon QCD corrections. The reason for this can be traced back to the fact that the virtual gluon radiative corrections dominate the $O(\alpha_s)$ corrections (recall $\Gamma_{SL}/\Gamma_{SL}^{(0)} < 1$), which act as form factors for the quark transition Q → q. However, it is known from the studies in $D \to K\ell\nu_\ell, K^*\ell\nu_\ell$ that the form factors, while changing the hadron energy distribution appreciably do not influence the shape of the lepton spectrum [10] ★. We have also tried to study the effects of the non-perturbative confining force on the shape of the lepton energy spectrum. The non-perturbative effects for charm decays are appreciable but become increasingly unimportant for bottom and top semileptonic decays. The semileptonic branching ratio and the lepton energy distribution provide, within QCD, an estimate of charm meson lifetime, τ_D and we estimate it to lie in the range 8×10^{-13} sec $\geqslant \tau_D \geqslant 3.78 \times 10^{-13}$ sec, which is based on a χ^2 fit of charm data [6].

We have then proceeded to study the semileptonic decay of heavy quarks involving the emission of a hard non-collinear gluon. Rates and the decay distributions $(1/\Gamma)$ dΓ/d cos $\theta_{G\ell}$ and $(1/\Gamma)$ dΓ/d cos θ_{qG} are presented. Depending on m_Q, this process may have a branching ratio of several tenths to a few percent. The cleanest signal of a gluon jet would be in dilepton processes: $e^+e^- \to \ell^+\ell^-$ + q jet + $\bar{\text{q}}$ jet +

★ For the exclusive channel decays $D \to K\ell\nu_\ell$, $K^*\ell\nu_\ell$, $F \to \eta\ell\nu_\ell$, $\phi\ell\nu_\ell$, see ref. [11].

gluon jet, where the gluon jet will mostly consist of pions. This necessarily involves a high statistics dilepton experiment above the top quark threshold. The detection of (3.1) would be a nice and perhaps the only direct test that QCD is the right frame work to study weak decays.

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