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DISCRIMINATIVE DEEP INELASTIC TESTS OF STRONG INTERACTION FIELD THEORIES

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It is demonstrated that recent measurements of $\int_0^1 F_2(x, Q^2)\,dx$ eliminate already all strong interaction field theories except QCD A detailed study of scaling violations of $F_2(x, Q^2)$ in QCD shows their insensitivity to the gluon content of the hadron at presently measured values of Q^2

1. Introduction

Recently, much new and accurate data on the structure functions of the nucleon were accumulated [1–3] in high-energy lepton-hadron deep inelastic scattering experiments. The Q^2 dependence of the structure functions was found to be compatible with the predictions of QCD. Comparison of the new data with other conventional (fixed-point) field theories of the strong interactions was, however, not seriously undertaken. A first attempt in this direction was started in ref. [4] for the non-singlet structure function $F_3^{\nu N}$ and it was stated that only QCD is compatible with the observed Q^2 dependence. This analysis was repeated in ref. [5] where it was found that fixed-point theories with smaller fixed coupling constants α^* than those taken in ref. [4] *are also compatible* with the observed Q^2 dependence of $F_3^{\nu N}$ at $Q^2 \geqslant 5$ GeV^2. Such results rest of course on the (so far unproven) assumption that there exists a fixed point coupling g^* as $Q^2 \to \infty$, i.e., $\beta(g^*) = 0$, such that the effective coupling constant $\alpha^*/4\pi << 1$, a necessary requirement to get approximate scaling in such theories.

Fixed-point theories differ from QCD mainly in their *singlet* mixing properties [6,7] which explains why a study of the non-singlet structure function $F_3(x, Q^2)$ over a *limited* range of Q^2 does not suffice [5] to eliminate these theories as possible candidates for describing fundamental strong interactions. For this purpose one must turn to the structure functions $F_2(x, Q^2)$ which contain singlet components.

This analysis was undertaken in refs. [6,7] for the older SLAC-MIT [8] and Fermilab [9] data. It was found that only asymptotically free (AF) theories survived the test of comparison with the data on scaling violations.

It is interesting to see how fixed-point theories compare with the recent and more accurate data [1–3] on $F_2(x, Q^2)$. Especially, the new data on $\int_0^1 F_2(x, Q^2)\, dx$ enable one to eliminate the fixed-point theories by purely *qualitative* arguments in contrast to the detailed quantitative elimination undertaken in refs. [6,7]. In sect. 3 we shall concentrate just on the information obtainable from studying the lowest moment of F_2, i.e., the Q^2 dependence of the area under F_2. We will see that the recent measurements [1–3] of $\int_0^1 F_2(x, Q^2)\, dx$, which *decreases* for increasing Q^2, already eliminate all fixed-point field theories. In sect. 4 the full x and Q^2 dependence of the data will be compared with the predictions of QCD and their sensitivity to the gluon distribution in the hadron will be tested.

2. Vector and scalar gluon theories

These are extensively discussed in ref. [7] and the reader is referred to this paper for details. Here we only recapitulate and extend those parts essential for our present analysis. The notation of ref. [7] will be followed throughout. The anomalous dimensions for vector gluon theories are given by

$$\gamma^{\rm F}_{\rm FF} = \frac{\alpha}{2\pi} C_2(R)\left[1 - \frac{2}{n(n+1)} + 4\sum_{j=2}^{n}\frac{1}{j}\right],$$

$$\gamma^{\rm V}_{\rm VV} = \frac{\alpha}{2\pi}\left\{C_2(G)\left[\tfrac{1}{3} - \frac{4}{n(n-1)} - \frac{4}{(n+1)(n+2)} + 4\sum_{j=2}^{n}\frac{1}{j}\right] + \tfrac{4}{3}T(R)\right\}, \qquad (2.1)$$

$$\gamma^{\rm F}_{\rm VV} = -\frac{\alpha}{2\pi}\frac{4(n^2+n+2)}{n(n+1)(n+2)}T(R),$$

$$\gamma^{\rm V}_{\rm FF} = -\frac{\alpha}{2\pi}\frac{2(n^2+n+2)}{n(n^2-1)}C_2(R),$$

with $\alpha = g^2/4\pi$ and where $C_2(R)$, $T(R)$ and $C_2(G)$ are as follows.

(i) For QCD, i.e., eight colored vector gluons and three colored quarks (of each flavor, the number of flavors being f):

$$C_2(R) = \tfrac{4}{3}, \qquad T(R) = \tfrac{1}{2}f, \qquad C_2(G) = 3.$$

(ii) For non-colored (Abelian) vector gluons and three colored quarks.

$$C_2(R) = 1\,, \quad T(R) = 3f\,, \quad C_2(G) = 0\,.$$

(iii) For non-colored (Abelian) vector gluons and non-colored quarks:

$$C_2(R) = 1\,, \quad T(R) = f\,, \quad C_2(G) = 0\,.$$

For scalar gluons one has [10] ★

$$\begin{aligned}
\gamma^{\mathrm{F}}_{\mathrm{FF}} &= \frac{\alpha}{4\pi} C_2(R)\left[1 - \frac{2}{n(n+1)}\right], \\
\gamma^{\phi}_{\phi\phi} &= \frac{\alpha}{\pi} T(R)\,, \\
\gamma^{\mathrm{F}}_{\phi\phi} &= -\frac{2\alpha}{\pi} T(R)\frac{1}{n}\,, \\
\gamma^{\phi}_{\mathrm{FF}} &= -\frac{\alpha}{2\pi} C_2(R)\frac{1}{n+1}\,,
\end{aligned} \tag{2.2}$$

where $C_2(R)$ and $T(R)$ are exactly as before in the corresponding color situations (i)-(iii).

The crucial role in discriminating between different field theories is played by the flavor-singlet part of structure functions, which is uniquely fixed by the well-known mixing of the fermionic and gluonic singlet Wilson operators. This comes about by diagonalizing the singlet anomalous dimension matrix

$$\hat{\gamma}(n) = \begin{pmatrix} \gamma^{\mathrm{F}}_{\mathrm{FF}} & \gamma^{\mathrm{V}}_{\mathrm{FF}} \\ \gamma^{\mathrm{F}}_{\mathrm{VV}} & \gamma^{\mathrm{V}}_{\mathrm{VV}} \end{pmatrix},$$

by $\hat{\gamma} = \gamma_- \hat{P}^- + \gamma_+ \hat{P}^+$ with

$$\gamma_\pm = \tfrac{1}{2}\left[\gamma^{\mathrm{F}}_{\mathrm{FF}} + \gamma^{\mathrm{V}}_{\mathrm{VV}} \pm \sqrt{(\gamma^{\mathrm{V}}_{\mathrm{VV}} - \gamma^{\mathrm{F}}_{\mathrm{FF}})^2 + 4\gamma^{\mathrm{F}}_{\mathrm{VV}}\gamma^{\mathrm{V}}_{\mathrm{FF}}}\right], \tag{2.3}$$

and where the projection operators are given by

$$\hat{P}^- = \begin{pmatrix} p^-_{11} & p^-_{12} \\ p^-_{21} & 1 - p^-_{11} \end{pmatrix}$$

and $\hat{P}^+ = 1 - \hat{P}^-$ with

$$p^-_{11} = \frac{\gamma^{\mathrm{F}}_{\mathrm{FF}} - \gamma_+}{\gamma_- - \gamma_+}\,, \qquad p^-_{21} = \frac{\gamma^{\mathrm{F}}_{\mathrm{VV}}}{\gamma_- - \gamma_+}\,, \qquad p^-_{12} = \frac{\gamma^{\mathrm{V}}_{\mathrm{FF}}}{\gamma_- - \gamma_+}\,, \tag{2.4}$$

★ Note that the anomalous dimensions for scalar gluon theories calculated by Bailin and Love [11] are wrong! Although these expressions have been adopted for the analyses in refs. [6,7], the results obtained there for the scalar gluons remain practically unchanged.

and similarly for scalar gluon theories. The Q^2 dependence of the fermionic singlet component of F_2,

$$x\Sigma(x, Q^2) \equiv x\sum_q [q(x, Q^2) + \bar{q}(x, Q^2)] , \tag{2.5}$$

where the sum runs over all quark flavors f, is predicted to be [7]

$$\Sigma_n(Q^2) = [\alpha_n \Sigma_n(Q_0^2) + \beta_n G_n(Q_0^2)]\, \mathrm{e}^{-sa_-(n)} + [(1-\alpha_n)\Sigma_n(Q_0^2) - \beta_n G_n(Q_0^2)]\, \mathrm{e}^{-sa_+(n)} \tag{2.6}$$

where for brevity we have defined $\alpha_n \equiv p_{11}^-(n)$ and $\beta_n \equiv p_{21}^-(n)$, which govern the signlet mixing to leading order in α_s, and the moments are defined by

$$G_n(Q^2) \equiv \int_0^1 \mathrm{d}x x^{n-1} G(x, Q^2) ,$$

etc., with $G(x, Q^2)$ being the gluon distribution. For QCD the renormalization group exponents in eq. (2.6) are given by

$$a_i = \gamma_i/(\alpha b) , \qquad s = \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} , \tag{2.7}$$

with $\alpha b = (\alpha/2\pi)(11 - \frac{2}{3}f)$ and Λ being determined by experiment ($\Lambda \simeq 0.5$ GeV). For an asymptotically non-free fixed-point theory these exponents read

$$a_i = \tfrac{1}{2}\gamma_i , \qquad s = \ln \frac{Q^2}{Q_0^2} , \tag{2.8}$$

where now the value of the UV finite fixed point $\alpha = \alpha^*$, appearing in γ_i, has to be determined by experiment. The flavor non-singlet (NS) component, governed by $\gamma_{\mathrm{FF}}^{\mathrm{F}}$ alone, is easy to calculate since it does not mix with the gluons, and can be found for example in ref. [7].

One of the important advantages of the recent deep inelastic neutrino experiments [2,3] is that they can measure directly the singlet component in eqs. (2.5) and (2.6) since

$$F_2^{\nu\mathrm{N}}(x, Q^2) = x\Sigma(x, Q^2) \tag{2.9}$$

(above charm threshold and always assuming $s = \bar{s}$, $c = \bar{c}$), whereas deep inelastic e (or μ) scattering off nucleons measures in addition the NS part, as for example

$$F_2^{\mu\mathrm{p}}(x, Q^2) = \tfrac{5}{18}\, x\Sigma(x, Q^2) + \tfrac{1}{6}x[u + \bar{u} - d - \bar{d} - s - \bar{s} + c + \bar{c}] \tag{2.10}$$

with $u = u(x, Q^2)$ etc., and where the Q^2 dependence of the NS expression in square-brackets is determined solely by $a_{\mathrm{NS}} = \gamma_{\mathrm{FF}}^{\mathrm{F}}/\alpha b$

3. The lowest moment of F_2

According to eq. (2.6) the Q^2 dependence of the lowest ($n = 2$) moment of the singlet component is given by

$$\Sigma_2(Q^2) = \alpha_2 + [\Sigma_2(Q_0^2) - \alpha_2]\, e^{-sa_+(2)}\,, \tag{3.1}$$

where we have used $a_-(2) = 0$, $G_2 = 1 - \Sigma_2$ and $\alpha_2 = \beta_2$. This quantity, being the total fractional momentum carried by the fermionic constituents in the nucleon, is experimentally directly measured in neutrino scattering on matter

$$\int_0^1 F_2^{\nu N}(x, Q^2)\, dx = \Sigma_2(Q^2)\,, \tag{3.2}$$

whereas for e(μ) p processes we have

$$\int_0^1 F_2^{\mu p}(x, Q^2)\, dx = \tfrac{5}{18}\, \Sigma_2(Q^2)$$

$$+ \tfrac{1}{6}[u_2(Q_0^2) + \bar{u}_2(Q_0^2) - d_2(Q_0^2) - 2s_2(Q_0^2) + 2c_2(Q_0^2)]\, e^{-sa_{NS}(2)}\,. \tag{3.3}$$

At moderate $Q^2 \simeq 2$–$4\ \mathrm{GeV}^2$, corresponding to our input Q_0^2, $\Sigma_2(Q_0^2) \simeq 0.52$ [2,3] and hence, since $a_+ > 0$, $\Sigma_2(Q^2)$ is an increasing or decreasing function of Q^2 depending on whether α_2 is larger or smaller than $\frac{1}{2}$, respectively. Substituting the different possible values of $C_2(R)$ and $T(R)$ into eqs. (2.1) and (2.2), it turns out that $\alpha_2 < \frac{1}{2}$ *only* for QCD where $\alpha_2 = \frac{3}{7}$. Note that, although for $n = 2$ we have

$$\alpha_2 = \frac{\gamma_{VV}^{V}(2)}{\gamma_{FF}^{F}(2) + \gamma_{VV}^{V}(2)}\,, \tag{3.4}$$

this expression is *not* sensitive to the triple-gluon coupling since the coefficient of

Table 1
Values for $\alpha_2 \equiv p_{11}^{-}(n = 2)$, assuming always four flavors ($f = 4$)

		α_2
colored vector gluons and quarks	QCD	$\frac{3}{7}$
non-colored (Abelian) vector gluons	colored quarks	$\frac{6}{7}$
	non-colored quarks	$\frac{2}{3}$
colored (non-Abelian) scalar gluons and quarks		$\frac{9}{10}$
non-colored (Abelian) scalar gluons	colored quarks	$\frac{72}{73}$
	non-colored quarks	$\frac{24}{25}$

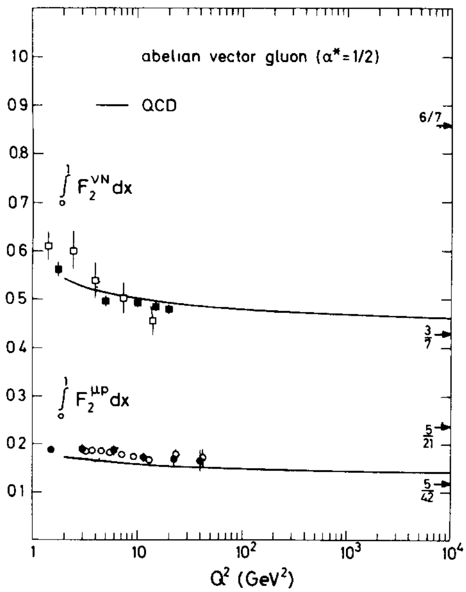

Fig. 1 Comparison of the Q^2 evolution of the area under F_2, predicted by vector gluon theories according to eqs (3 2) and (3.3), with the μp data of ref. [1] (•) and ref. [12] (○), and with the νN data of ref. [3] (■) and ref. [2] (□).

$C_2(G)$ in $\gamma^V_{VV}(2)$ vanishes. It is a unique feature of *all* other presently known field theories that $\alpha_2 > \frac{1}{2}$, as it is summarized in table 1, which forces $\int_0^1 F_2(x, Q^2)\, dx$ to *increase* with Q^2. Since $\int_0^1 F_2(x, Q^2)\, dx$ is experimentally observed [1–3] to *decrease* with Q^2, all theories except QCD are already excluded on the basis of this single *qualitative* observation.

In fig. 1 we compare the data [1–3] for $\int_0^1 F_2(x, Q^2)\, dx$ with the predictions of QCD and of the Abelian vector field theory, for which we have taken the fixed point α^* to be 0.5 in agreement with an analysis [5] of the moments of $F_3^{\nu N}$. The input for the small NS contribution in eq. (3.3) can be easily estimated from the e(μ) p and e(μ) n measurements [8,12] and the νN data [2,3] to be $u_2 + \bar{u}_2 - d_2 - \bar{d}_2 - 2s_2 + 2c_2 \simeq 0.12$ at $Q_0^2 \simeq 4$ GeV^2. We clearly see how the data eliminate the Abelian vector theory where $\alpha_2 = \frac{6}{7}$ (see table 1); the prediction of scalar gluon theories is in even worse agreement with the data since their values for α_2 are always larger than $\frac{6}{7}$.

4. Scaling violations in $F_2(x, Q^2)$ and their sensitivity to the gluon distribution

Besides confirming QCD it was also attempted in refs. [1–3] to extract the gluon distribution $G(x, Q^2)$ in the hadron form the observed scaling violations of

$F_2(x, Q^2)$. For this to be a reliable method, the predicted scaling violations must be sensitive to $G(x, Q^2)$. To check this sensitivity we have calculated the scaling violations once with the standard gluon distribution $xG(x, Q_0^2 \simeq 4) = 2.6(1-x)^5$ and once with $G(x, Q_0^2) = 0$. This latter choice obviously violates the energy momentum sum rule and is intended only as a check on the abovementioned sensitivity to $G(x, Q_0^2)$. For the quark distribution we took at $Q_0^2 = 4$ GeV^2

$$\begin{aligned} x(u_V + d_V) &= 4.546x^{0.624}(1-x)^{2.657}\,, \\ xd_V &= 2.715x^{0.773}(1-x)^{3.7}\,, \\ xs &= 0.17(1-x)^7\,, \\ xc &= 0.05(1-x)^{30}\,, \end{aligned} \tag{4.1}$$

which result from a fit to the data [1,3,8,13] at $Q_0^2 \simeq 4$ GeV^2 and $x \gtrsim 0.04$, assuming $\bar{u} = \bar{d} = s$ and $u = u_V + \bar{u}$, etc. We have deliberately avoided the region $x < 0.04$ in order to avoid any sensitive dependence [14] on the charmed sea distribution. The negligibly small charm distribution, which has been included in eq. (4.1), results from the lowest two moments predicted by the virtual Bethe-Heitler process [14,15], i.e., $c_2(Q_0^2) = 1.6 \times 10^{-3}$ and $c_4(Q_0^2) = 2.9 \times 10^{-6}$ corresponding to $m_c = 1.25$ GeV. To further make sure that the results do not sensitively depend on our standard input gluon distribution chosen, we have repeated the calculations using a broad gluon $xG(x, Q_0^2) = 0.88(1+9x)(1-x)^4$ as suggested by the Caltech group [16]:

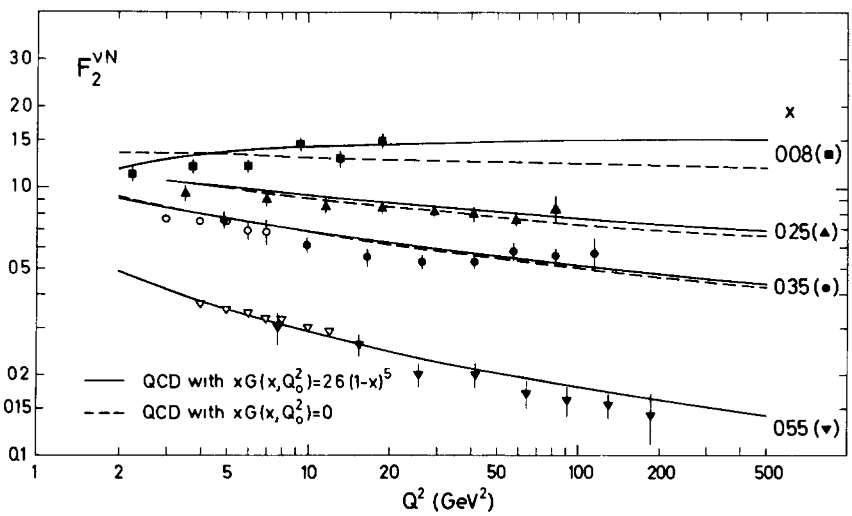

Fig. 2 Predictions of scaling violations in QCD for standard gluon input distribution (full lines) and zero gluon input distribution (dashed lines) as compared with neutrino data [3] (solid points) and ed data [8] (open points) multiplied by $\frac{9}{5}$.

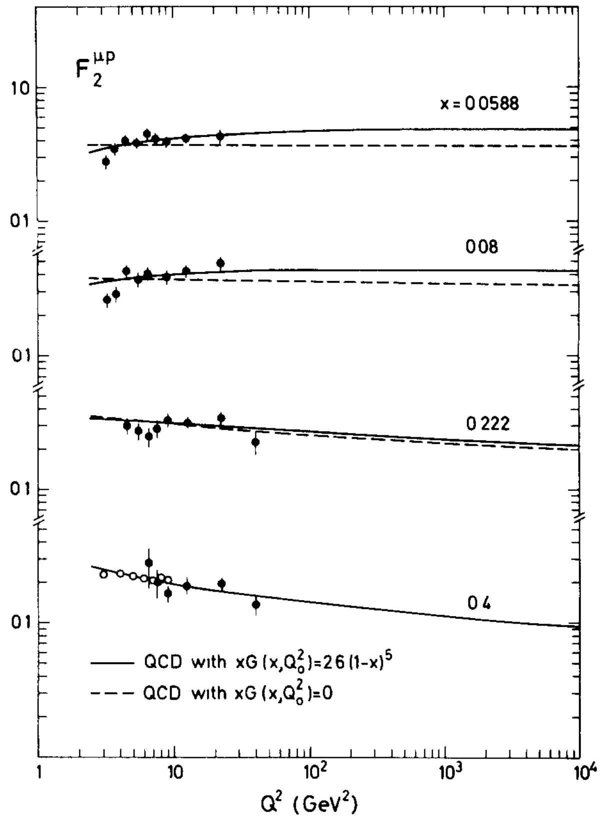

Fig. 3. Comparison of the predictions for scaling violations with the 219 GeV μp data (•) of refs [1,13] and with the ep data (○) of ref. [8] The theoretical curves are as in fig. 2.

within a few percent our predictions remain unchanged. The full Q^2 dependence of $F_2(x, Q^2)$ is then obtained by using the standard Mellin inversion techniques as described for example in ref. [7].

As one can see from figs. 2 and 3 the scaling violations with the standard gluon distribution (full lines) do not differ significantly (i.e., by less than a standard deviation) from the ones with a zero input gluon distribution (dashed lines). A distinction can be made only in the *small x* region at high values of Q^2, i.e., $Q^2 \gtrsim 50$ GeV^2. Thus any moment analysis of F_2 with $n \gtrsim 3$ for determining the gluon distribution is rendered meaningless.

5. Conclusions

To summarize, we have shown that recent measurements of $\int_0^1 F_2(x, Q^2)\,dx$ which *decreases* for increasing values of Q^2 already eliminate *all* strong interaction field theories except QCD. This should be contrasted with the information extracted from measurements of $F_3(x, Q^2)$ which, at present energies, can *not* be used to distinguish between the different field theories of the strong interactions [5]. Furthermore we have shown that attempts [1–3] to extract the gluon distribution in the hadron from the measured Q^2 dependence of $F_2(x, Q^2)$ are misleading since the scaling violations presently observed are rather insensitivie to the gluon distribution. Only precision measurements in the small x region, not accessible to any moment analysis, at higher values of Q^2 could shed further light on $G(x, Q_0^2)$.

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