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[4]I will ignore the restriction that $\vec{p}$ must also lie outside the horn of the antiquark, since in this diagram there is no divergence as the quark and antiquark become collinear.

[5]This is not true graph by graph, unless one works in a physical gauge such as axial gauge—see R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer, and G. G. Ross, Harvard University Report No. HURP-78/A045 (to be published); D. Amati, R. Petronzio, and G. Veneziano, Nucl. Phys. B140, 154 (1978), and B146, 29 (1978); S. Libby and G. Sterman, Phys. Rev. D 18, 3252 (1978). (Cross sections are, of course, gauge invariant.)

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Measuring the Triple-Gluon Vertex

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It is proposed to measure the deep-inelastic longitudinal structure function $F_L(x,Q^2)$, or moments thereof, in order to determine the Q^2 dependence of the gluon distribution. In contrast to $F_2(x,Q^2)$, the theoretical predictions for the Q^2 evolution of the moments of the gluon density depend critically on the gluon self-couplings and thus provide us with a sensitive test of the Yang-Mills structure of quantum chromodynamics.

Most of the "direct" tests of quantum chromodynamics (QCD) done or suggested so far are mainly sensitive to the structure of the quark-gluon coupling, but not to the non-Abelian self-couplings of colored vector gluons. These tests include the well-known scaling violations in electroproduction and neutrinoproduction, jets in e^+e^- annihilation, Drell-Yan processes, and many others. Although present deep-inelastic scattering data eliminate[1,2] already all strong-interaction (fixed-point) field theories, present high-statistics experiments are insensitive[2] to the gluon content of the nucleon and thus to the triple-gluon vertex of asymptotic freedom. This is not too surprising, however, since the triple-gluon coupling enters the Q^2 evolution of $F_2(x,Q^2)$, for example, only in a very indirect way via the singlet Wilson operator mixing [the term proportional to $C_2(G)$ in the gluonic anomalous dimension $\gamma_{VV}{}^V(n)$ —we follow closely the notation of Ref. 1]. Or, in the language of Altarelli and Parisi,[3] the Q^2 evolution of $F_2(x,Q^2)$ is not directly proportional to the gluon→gluon decay probability P_{gg}. This is, of course, in contrast to the Q^2 development of the gluon distribution $G(x,Q^2)$ itself. Thus, although the measured effects of the breakdown of scaling require[2] a fundamental strong-interaction theory to have colored quarks *and colored* vector gluons, the (local) gauge structure, i.e., possible self-couplings of the gluons, has not been seen directly or tested at all so far. Needless to say, gluon self-couplings are essential for asymptotic freedom and are theoretically required in order to render a non-Abelian vector-gluon theory renormalizable.

Possible direct tests of the gluon self-couplings suggested so far either are exceedingly small,[4] and therefore may be totally masked by nonperturbative effects due to final-state interactions, or[5] are experimentally inaccessible in the near future. For this latter test of the three-gluon ($3g$) vertex it has been proposed[5] to look for the Q^2 evolution of gluon jets in heavy quarkonium decay, $e^+e^- \to Q\overline{Q} \to 3g \to h + \text{anything}$, where the gluon-decay function $D_g^h(z, Q^2)$ could be measured experimentally. This requires, however, detailed measurements of $D_g^h(z, Q^2)$ for $0.3 \lesssim z \lesssim 1$ and $100\ \text{GeV}^2 \lesssim Q^2 \lesssim 1000\ \text{GeV}^2$, in order to delineate the triple-gluon contribution.

In this note I propose to study the Q^2 evolution of the gluon distribution $G(x, Q^2)$ itself, and to determine experimentally this distribution by measuring the longitudinal structure function $F_L(x, Q^2)$ in deep-inelastic electroproduction (muoproduction) or neutrinoproduction. Measurements of this latter quantity should become available in the near future.[6] The idea is the following: In order to avoid any ambiguities on the theoretically ill-understood x dependence of $G(x, Q_0^2)$ at a certain input momentum Q_0^2, I discuss only moments of structure functions, which weight the large-Bjorken-x region and are partly already known experimentally.[7-9] Of course, our subsequent discussion applies to the full x dependence as well. To leading order in perturbation theory, the nth moment of the longitudinal structure function $F_L \equiv F_2 - 2xF_1$ is given by

$$F_{L,n-1}(Q^2) = C_n^q F_{2,n-1}(Q^2) + aC_n^g G_n(Q^2), \tag{1}$$

where C_n^q and C_n^g are the moments of the longitudinal projections of the fundamental processes[10] $Vq \to gq$ and $Vg \to q\overline{q}$ ($V = \gamma^*$ or W), respectively,

$$C_n^q = \frac{4\alpha_s(Q^2)}{3\pi(n+1)}, \quad C_n^g = \frac{2\alpha_s(Q^2)}{\pi(n+1)(n+2)}, \tag{2}$$

and, in the four-quark model, $a = \sum_q e_q^2 = \frac{10}{9}$ for electroproduction and $a = 4$ for ν and $\overline{\nu}$ scattering on matter, and $\alpha_s(Q^2) = 12\pi/25\ln(Q^2/\Lambda^2)$ with $\Lambda \simeq 0.5$ GeV. The moments of the structure functions are defined by $G_n(Q^2) \equiv \int_0^1 dx\, x^{n-1} G(x, Q^2)$. From Eq. (1) we see that good data on $F_{L,n-1}(Q^2)$, together with the experimental knowledge of[7-9] $F_{2,n-1}(Q^2)$, $n = 2, 3, \ldots$, can be translated into a reasonable knowledge of the gluon density

$$G_n(Q^2) = \frac{\pi(n+1)(n+2)}{2a\alpha_s(Q^2)} F_{L,n-1}(Q^2) - \frac{2(n+2)}{3a} F_{2,n-1}(Q^2). \tag{3}$$

It should be noted that, for this case, $F_L(x, Q^2)$ needs only to be measured accurately for $x \gtrsim 0.3$, since $n > 2$ moments are sensitive mainly to the large-x region of structure functions. Furthermore, as we shall see, measurements in the region $10\ \text{GeV}^2 \lesssim Q^2 \lesssim 100\ \text{GeV}^2$ will be required in order to clearly pin down the triple-gluon coupling. At these large values of Q^2, nonperturbative contributions to F_L can be safely neglected[11] since they are of the order k_T^2/Q^2 or m^2/Q^2, with k_T being the intrinsic transverse momentum of partons and m some typical hadronic mass scale.

Theoretically the Q^2 evolution of the moments of the gluon distribution is predicted by the renormalization group to be[1]

$$\frac{G_n(Q^2)}{G_n(Q_0^2)} = \left[1 - \alpha_n + \frac{\alpha_n}{\beta_n}(1+\alpha_n)\rho_n(Q_0^2)\right] e^{-sa_-(n)} + \left[\alpha_n - \frac{\alpha_n}{\beta_n}(1-\alpha_n)\rho_n(Q_0^2)\right] e^{-sa_+(n)}, \tag{4}$$

where $\alpha_n = (\gamma_{FF}^F - \gamma_+)/(\gamma_- - \gamma_+)$ and $\beta_n = \gamma_{VV}^F/(\gamma_- - \gamma_+)$ with the anomalous dimensions in the well-known notation as given, for example, in Ref. 1. The eigenvalues $\gamma_\pm$ of the singlet anomalous dimension matrix are simply related to the renormalization group exponents in Eq. (4) for an asymptotically free QCD by[1]

$$a_i = \frac{\gamma_i}{\alpha_s b}, \quad s = \ln\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}, \tag{5}$$

with $2\pi b = 11 - 2f/3$ and f being the number of flavors. Other renormalizable (finite-fixed-point) field theories, such as Abelian vector gluon ($\overline{\psi}\gamma_\mu\psi A^\mu$), non-Abelian scalar gluon ($\overline{\psi}\lambda_a\psi\varphi_a$), and Abelian scalar gluon ($\overline{\psi}\psi\varphi$) theories, are already eliminated[1,2] by present measurements of scaling violations in $F_2(x, Q^2)$.

The Q^2 evolution of the gluon distribution is now uniquely predicted by Eq. (4) provided we know the input wave functions at Q_0^2, i.e.,

$$\rho_n(Q_0^2) \equiv \Sigma_n(Q_0^2)/G_n(Q_0^2), \tag{6}$$

with the fermionic flavor-singlet component

$$\Sigma(x, Q^2) \equiv \sum_q [q(x, Q^2) + \overline{q}(x, Q^2)];$$

the sum on q runs over all quark flavors f, being measured directly by[8,9] $F_2^{\nu N}(x, Q^2) = x\Sigma(x, Q^2)$ above charm threshold, and I have taken $s \simeq \overline{s}$ and $c \simeq \overline{c}$ for the small strange and charmed parton

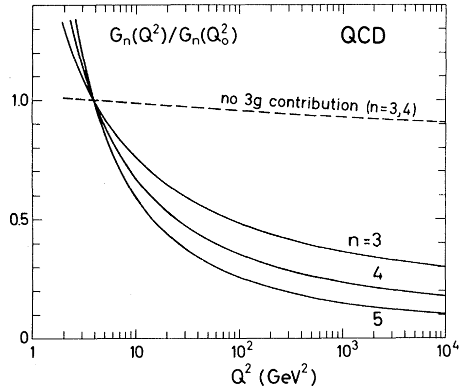

FIG. 1. Predictions for the Q^2 evolution of gluon moments according to Eq. (4). The dashed curve for $n=3$, 4 demonstrates the sensitivity of these predictions to the gluon self-couplings: It represents the contribution with the triple-gluon coupling turned off [$C_2(G)=0$ in $\gamma_{VV}{}^V$]. This contribution is similar but slightly larger for $n=5$.

distributions. In order to avoid any theoretical prejudices as far as the input gluon distribution $G(x, Q_0^2)$ is concerned, I only use the lowest moments $G_n(Q_0^2)$ determined experimentally[8,9] from the Q^2 variation of the $n=2, 3, \ldots$ moments $F_{2,n-1}{}^{\nu N}(Q^2)$. It should be noted that such a determination is insensitive[2] to the triple-gluon vertex, which enters the Q^2 evolution predicted by the renormalization group only in a very indirect way (via the singlet mixing). The input quantities in Eq. (6) are then estimated to be[8,9]

$$\rho_3 = 1.08 \pm 0.15, \quad \rho_4 = 2 \pm 0.5, \quad \rho_5 = 1.6 \pm 0.4, \quad (7)$$

at $Q_0^2 \simeq 4$ GeV^2. The $n=2$ moments are not very interesting for my purpose since they refer just to energy-momentum conservation.

Figure 1 shows the predictions for the Q^2 evolution of the lowest gluon moments as predicted by Eq. (4). It can be seen that the effects due to the triple-gluon coupling, the term proportional to the group invariant $C_2(G)$ in the purely gluonic anomalous dimension[1] $\gamma_{VV}{}^V$, are enormous [this is in contrast[2] to the fermionic-singlet quantity $\Sigma(x, Q^2)$ measured by $F_2{}^{\nu N}(x, Q^2)$]. Already at $Q^2 \simeq 50$–200 GeV^2, the predictions in Fig. 1 are entirely dominated by the triple-gluon vertex as can be seen from the difference between the solid and dashed curves in Fig. 1, the latter being the result without the contribution stemming from the triple-gluon coupling [$C_2(G)=0$ in $\gamma_{VV}{}^V$]. Thus, in contrast to $F_2(x, Q^2)$, measurements of the Q^2 evolution of $F_L(x, Q^2)$ and, therefore, of $G(x, Q^2)$ provide us with *sensitive* tests of the Yang-Mills structure (gluon self-couplings) of QCD. At $Q^2 \simeq 10^4$ GeV^2, a region appropriate for future *ep* colliding-beam facilities, the effects due to the triple-gluon coupling become as large as one order of magnitude. These results depend rather weakly on the poorly known higher input moments in Eq. (7): If we take the extreme values of $\rho_n(Q_0^2)$ as allowed by experiment in Eq. (7), the results shown in Fig. 1 differ by less than 3%.

Thus by measurement, in addition to $F_2(x, Q^2)$, of the lowest moments of $F_L(x, Q^2)$ up to $Q^2 \simeq 50$–100 GeV^2, which should be feasible[6] with the μ-beam and neutrino experiments at CERN in the near future, the Q^2 dependence of the moments of gluon distributions will provide us with clean and sensitive tests of the Yang-Mills structure of QCD, i.e., of the gluon self-couplings which are so very essential for asymptotic freedom.

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