

## Chiral U(1) Symmetry and Weak Neutral Currents at High Energy

T. C. Yang

Department of Physics and Astronomy, University of Massachusetts, Amherst, MA, USA  
and Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg, Federal Republic of Germany

Received 23 July 1979; and in revised form 30 October 1979

**Abstract.** If chiral U(1) symmetry is a gauge symmetry, CP is automatically conserved despite the instanton effects, and the weak neutral currents have a definite structure. A realistic  $SU(2)_L \otimes U(1) \otimes U(1)_R$  model contains an axion which is consistent with present data. Furthermore the neutrino interactions to lowest order are identical to the Weinberg-Salam model. Implications for the chiral U(1) currents are discussed.

### 1. Introduction

Present experiments on neutral currents are either probed by neutrinos or electrons. Neutrino experiments in the last decade have measured various elastic and inelastic cross sections. The results can be used to deduce the effective neutral current couplings between the neutrino and hadrons [1]. They agree well with the Weinberg-Salam model [2] within experimental errors. The electron neutral current couplings, on the other hand, could not be deduced from the data in a model independent way for lack of information [3]. In fact, depending on the experimental data one uses, different conclusions can be drawn.

The strong interactions are generally believed to be mediated by SU(3) color gluons. At some very high energy scale, the strong, electromagnetic and weak interactions could be unified and the theory has only one coupling constant. The grand unified theory is more predictive. For example, the SU(5) theory gives a value for the Weinberg angle in agreement with data [4], and the O(10) theory predicts further the  $t$  quark mass [5]. SU(5) symmetry breaking assumes that at some very high scale, the symmetry is broken to  $SU(3)_{\text{color}} \otimes SU(2) \otimes U(1)$ . The theory thus suggests that between the present energy and the super high energy scale, the world is adequately described by just three

interactions as seen by the  $SU(3)_{\text{color}} \otimes SU(2) \otimes U(1)$  symmetries. Whether this prediction is right or not obviously has significant consequences.

In view of the above striking predictions, one might investigate the possibility that the neutral current phenomena as presently known could be a low energy phenomena. When probed at high energies, the weak currents could be more complicated. After all, the confirmation of the Weinberg-Salam model has to wait for the discovery of the  $W$  and  $Z$  boson at the predicted mass. Before such experimental evidence is found, it seems appropriate to ask what could be the other gauge symmetries which reduce to the Weinberg-Salam model at low energy<sup>1</sup>. The simplest candidate is the  $SU(2) \otimes U(1) \otimes U'(1)$  symmetry [6, 7]. Unfortunately there are many models in the literature, most of them are either unnatural in embedding the experimental constraints or have arbitrary assignment of the  $U'(1)$  hypercharges. The reason is that the  $U'(1)$  symmetry, unlike the electromagnetic U(1) symmetry, does not restrict the hypercharges of the representations, thus in order to agree with the data one finds a certain relation between the hypercharges and the parameter of the theory, which cannot always be met in a "natural" way. A recent analysis has suggested a "natural" model to account for the suppression of parity violation in atomic bismuth [6]. The model predicts a decreasing  $y$  distribution for polarized-electron deuterium scattering which is not supported by recent data from SLAC [8].

We report here a gauge  $SU(2) \otimes U(1) \otimes U(1)_R$  model. Under the chiral  $U(1)_R$  symmetry, the hypercharges of the fermion representations are determined relative to that of the Higgs mesons thus the theory is free of the above mentioned arbitrariness and unnaturalness.

The weak neutral currents consist of a piece belonging to  $SU(2)_L \otimes U(1)$  and a piece belonging to  $U(1)_R$

<sup>1</sup> The answer is partially given by Georgi and Weinberg [18]. Here we are interested in realistic models

symmetry. Naively, one might conclude that since the neutrino is left-handed, it interacts only with the  $SU(2)_L \otimes U(1)$  piece and therefore the neutrino interaction is identical to the Weinberg-Salam model. Technically, one must ask whether this can be done naturally, for if the  $Z$  boson mixes with the chiral gauge boson  $Z_R$ , then the mixing is not naturally small. Georgi and Weinberg have considered the case of  $SU(2) \otimes U(1) \otimes G$  symmetry [18]. They found that if the Higgs bosons are either neutral to  $SU(2) \otimes U(1)$  or neutral to  $G$ , then the neutrino interactions reduce to the Weinberg-Salam model at low energy. Their theorem is not useful here, since such Higgs bosons do not have Yukawa coupling to  $\psi_L$  and  $\psi_R$  (which transform under  $SU(2)_L \otimes U(1) \otimes U(1)_R$  as  $(1/2, 1, 0)$  and  $(0, 2, 1)$  respectively), therefore the quarks remain massless. In order that quarks are massive after spontaneous symmetry breaking, we must have doublets of Higgs bosons which transform as  $(1/2, 1, 1)$  under the gauge symmetries (i.e. nontrivially). Fortunately, we find that the Lagrangian possesses a chiral “hypercharge conjugation” symmetry such that the  $Z$  and  $Z_R$  bosons decouple from each other naturally. We have thus demonstrated an example of  $SU(2) \otimes U(1) \otimes G$  which has nontrivial Higgs representations but still contains identical neutrino interactions as the Weinberg-Salam model.

This paper is organized as follows. In Sect. 2, we review the properties of chiral U(1) symmetry in relation to the strong interaction CP problem. In Sect. 3 the  $SU(2)_L \otimes U(1) \otimes U(1)_R$  model is presented. We discuss the question of uniqueness, anomaly cancellation and finally the discrete and global chiral U(1) symmetries. In Sect. 4, we investigate the weak neutral current phenomenology considering first low energy data and then high energy experiments. A brief summary is given in Sect. 5.

## 2. Chiral U(1) Symmetry

The reason we are interested in a chiral U(1) symmetry is rooted in the CP invariance problem of non-abelian color interaction (QCD). The perturbative color gluon theory (QCD) has many nice features, including the fact that CP violation is naturally suppressed at the observed level [9]. However, when the non-perturbative effects of the instantons are included, it was found that QCD no longer is CP invariant [10], unless one of the quark is massless, which does not seem to agree with nature [11]. Since CP is experimentally observed to be a very good symmetry of the strong interactions, it suggests that other mechanisms are responsible for CP invariance in strong interactions. An attractive suggestion was made by Peccei and Quinn [12] who showed

that if the Lagrangian possesses a global chiral U(1) symmetry as did the  $SU(2) \otimes U(1)$  model with two Higgs doublets, the unpleasant CP violating interaction (phase) is absent (rotated away) when the fermion states have real masses after spontaneous symmetry breaking.

The chiral U(1) symmetry is also interesting in other respects. For example, in order to conserve flavor in neutral currents, the Yukawa couplings of quarks to two Higgs bosons is chiral U(1) invariant. One finds two charged Higgs bosons with couplings to quarks proportional to their masses. If the charged Higgs bosons are light enough, they could be discovered in heavy particle decays [13].

How the Peccei-Quinn mechanism is embedded in of course model dependent. Naturally the axion property varies from model to model. (When the global chiral U(1) symmetry is intrinsically broken by instantons, a Goldstone boson called an axion is present [14].) A light axion with mass  $< 2 \text{ MeV}$  in the  $SU(2) \otimes U(1)$  model seems to be experimentally ruled out [15]. It was pointed out that a heavier axion in a  $SU(2) \otimes U(1) \otimes U(1)_{\text{chiral}}$  model does not have such difficulties with the present data [16]. Here chiral U(1) is a gauge symmetry.

What is the function of a *gauge* chiral U(1) symmetry? Recall the Peccei-Quinn mechanism. Because of the instantons, the vacuum is degenerate and depends on a parameter  $\theta$ . The Higgs potential also depends on  $\theta$  since Higgs bosons couple to quarks via Yukawa couplings. It turns out that if the Lagrangian is chiral U(1) invariant, at the minimum of the Higgs potential, one has  $\arg(\det m) = -\theta$ ,  $m$  being the quark mass matrix. Therefore the net CP violating phase  $\bar{\theta} = \arg(\det m) + \theta = 0$ . This means that CP is conserved for any value of  $\theta$ . Now in the  $SU(2)_L \otimes U(1) \otimes U(1)_{\text{chiral}}$  model, we have a gauge  $U(1)_{\text{chiral}}$  symmetry as well as a global chiral U(1) symmetry [the latter is induced and cannot be the same as the gauge  $U(1)_{\text{chiral}}$  symmetry]. The global chiral U(1) symmetry determines the property of the axion and conserves CP symmetry automatically as indicated above. The gauge  $U(1)_{\text{chiral}}$  symmetry on the other hand generates a chiral U(1) weak current. The chiral U(1) weak current may be experimentally required as we shall see below. The points we wish to make are: (1) The chiral U(1) weak currents are theoretically motivated because of the axion problem. (2) The weak currents must commute with the global chiral U(1) currents. In other words, the weak currents are not arbitrary.

Although CP can also be conserved by assuming that for some reason, the  $\theta = 0$  vacuum is the chosen ground state, but in order to agree with the observed level of CP violation, one must further check that CP is softly broken [17]. The Peccei-Quinn mechanism on the other hand guarantees that CP is conserved for

**Table 1.** Hypercharges of the fermion representation

	$Q$	$y_L$	$y_R$	$h_L$	$h_R$
$p_i$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	0	1
$n_i$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$	0	1
$t_i$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	0	-1
$b_i$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$	0	-1
$\nu_i$	0	-1	0	0	1
$e_i$	-1	-1	-2	0	1
$\nu'_i$	0	-1	0	0	-1
$E_i$	-1	-1	-2	0	-1

whatever the vacuum state one chooses. The question concerning the Peccei-Quinn proposal has been whether it can be demonstrated in a realistic model. Toward this goal, the  $SU(2)_L \otimes U(1) \otimes U(1)_R$  model can be considered realistic in that (1) the quarks are massive, (2) the axion is consistent with the data, and (3) the neutral current interactions are phenomenologically satisfactory. The major results of this paper are that the neutrino interactions in this model are identical to the Weinberg-Salam model and the model also predicts a flat  $y$  distribution for polarized electron deuterium scattering in agreement with recent SLAC data [8]. These points will become clear after the  $SU(2)_L \otimes U(1) \otimes U(1)_R$  invariant Lagrangian is given in the next section.

### 3. The $SU(2)_L \otimes U(1) \otimes U(1)_R$ Model

#### A. Anomaly-free Lagrangian

Chiral U(1) symmetry can be best seen from the Yukawa coupling

$$\mathcal{L}^{Yuk} = \Gamma_{ij}^1(\bar{p}\bar{n})_{iL}\varphi_1 n_{jR} + \Gamma_{ij}^2(\bar{p}\bar{n})_{iL}\tilde{\varphi}_2 p_{jR} + \text{H.c.}, \quad (1)$$

$$\tilde{\varphi}_2 = \tau_1 \varphi_2^*$$

which is  $SU(2)_L$  invariant, but also invariant under the following chiral U(1) symmetry

$$\begin{pmatrix} p_i \\ n_i \end{pmatrix} \rightarrow \exp(i(\alpha + \beta\gamma_5)) \begin{pmatrix} p_i \\ n_i \end{pmatrix}, \quad (2)$$

$$\varphi_1 \rightarrow \exp(i2\beta)\varphi_1, \quad \varphi_2 \rightarrow \exp(-i2\beta)\varphi_2.$$

The following combinations are familiar: (1)  $\alpha \neq 0, \beta = 0$  is the ordinary U(1) symmetry. (2)  $\alpha = 0, \beta \neq 0$  is the Peccei-Quinn  $U(1)_A$  symmetry. (3)  $\alpha = \beta$  is the  $U(1)_R$  symmetry considered by Wilczek. (4)  $\alpha = -\beta$  corresponds to a  $U(1)_L$  symmetry. Let us now include heavy quarks ( $t_i, b_i$ ) for the purpose of cancelling triangle anomalies, and two Higgs bosons  $\varphi_3$  and  $\varphi_4$  which couple to the heavy quarks. In addition, we allow a singlet Higgs boson  $\varphi_5$ . The Lagrangian can now be

written as

$$\begin{aligned} \mathcal{L} = & \sum_i \bar{\psi}_i \gamma_\mu D_\mu^i \psi_i - \sum_i |\mathcal{D}_\mu^i \varphi_i|^2 \\ & + \Gamma_{ij}^1(\bar{p}\bar{n})_{iL}\varphi_1 n_{jR} + \Gamma_{ij}^2(\bar{p}\bar{n})_{iL}\tilde{\varphi}_2 p_{jR} \\ & + \Gamma_{ij}^3(\bar{t}\bar{b})_{iL}\varphi_3 b_{jR} + \Gamma_{ij}^4(\bar{t}\bar{b})_{iL}\tilde{\varphi}_4 t_{jR} + \text{H.c.} \\ & + V(\varphi), \end{aligned} \quad (3)$$

where

$$\mathcal{D}_\mu^{i=5} = \partial_\mu - i\frac{g''}{2}A_\mu$$

$$\mathcal{D}_\mu^i = \partial_\mu - i\frac{g'}{2}\boldsymbol{\tau} \cdot \mathbf{W}_\mu$$

$$-i\frac{g'}{2}B_\mu \mp i\frac{g''}{2}A_\mu \quad \text{for } i = \begin{matrix} 1, 4 \\ 2, 3 \end{matrix}$$

$$\begin{aligned} D_\mu^i = & \partial_\mu - i\frac{g}{2}\boldsymbol{\tau} \cdot \mathbf{W}_\mu L - i\frac{g'}{2}B_\mu(y_L^i L + y_R^i R) \\ & - i\frac{g''}{2}A_\mu(h_L^i L + h_R^i R) \end{aligned} \quad (3.1)$$

$$\begin{aligned} V(\varphi) = & \sum_1^5 a_\alpha |\varphi_\alpha|^2 + \sum_1^5 b_\alpha |\varphi_\alpha|^4 \\ & + \sum_{\alpha \neq \beta} c_{\alpha\beta} (\varphi_\alpha^* \boldsymbol{\tau} \varphi_\alpha)(\varphi_\beta^* \boldsymbol{\tau} \varphi_\beta) \\ & + \sum_{\alpha \neq \beta} d_{\alpha\beta} (\varphi_\alpha^* \varphi_\alpha)(\varphi_\beta^* \varphi_\beta) \\ & + e(\varphi_4^* \varphi_1)(\varphi_3^* \varphi_2) + f(\varphi_3^* \varphi_1)(\varphi_4^* \varphi_2) + \text{H.c.} \end{aligned}$$

The notations used above are  $L = \frac{1}{2}(1 - \gamma_5)$ ,  $R = \frac{1}{2}(1 + \gamma_5)$ .  $W_\mu$ ,  $B_\mu$ , and  $A_\mu$  are the  $SU(2)_L$ ,  $U(1)$ , and  $U(1)_{\text{chiral}}$  gauge bosons respectively.  $y_L$  and  $y_R$  are the U(1) hypercharge assigned by the standard equation  $Q = I_3 + \frac{1}{2}(y_L + y_R)$ . They are listed in Table 1. For the  $U(1)_{\text{chiral}}$  symmetry, if we adopt the Peccei-Quinn  $U(1)_A$ , we have  $h_L = -h_R = \frac{1}{2}$  for ( $p_i, n_i$ ) and  $-\frac{1}{2}$  for ( $t_i, b_i$ ). Equation (3) reduces to the Lagrangian considered previously in [16].

We consider here a more interesting case  $U(1)_{\text{chiral}} = U(1)_R$ , hence  $h_2 = 0$ .

The hypercharge  $h_R$  can be easily deduced from the Yukawa coupling in (3) and are included also in Table 1. Together with the quarks, we also give the lepton representations, the generalization to leptons being straightforward.

For  $U(1)_R$  symmetry, the Yukawa Lagrangian allows terms which couple heavy to light quarks [but not for  $U(1)_A$ ], namely<sup>2</sup>

$$\begin{aligned} & \bar{\Gamma}_{ij}^1(\bar{p}\bar{n})_{iL}\varphi_3 b_{jR} + \bar{\Gamma}_{ij}^2(\bar{p}\bar{n})_{iL}\tilde{\varphi}_4 t_{jR} \\ & + \bar{\Gamma}_{ij}^3(\bar{t}\bar{b})_{iL}\varphi_1 n_{jR} + \bar{\Gamma}_{ij}^4(\bar{t}\bar{b})_{iL}\tilde{\varphi}_2 p_{jR} + \text{H.c.} \end{aligned} \quad (3.2)$$

2 This allows mass mixing between heavy and light quarks

We note that (3) has the following properties:

(1) The chiral hypercharges of the fermion representations relative to that of the Higgs bosons are fixed by the chiral transformations (2). Without losing generality, we assign the chiral hypercharges of  $\varphi_1, \varphi_2$  to be  $\pm 2$ . [Note that the magnitude of  $\beta$  in (2) has no physical meaning.] The chiral hypercharges of the fermions are then uniquely fixed to be  $h_L=0, h_R=\pm 1$  by (2).

(2) Because  $\sum h_L=0, \sum h_L^3=0,$  and  $\sum h_R=0, \sum h_R^3=0,$  the model is anomaly free. In particular, the anomalies associated with the chiral gauge boson and the two color gluons are canceled between light and heavy quarks.

### B. Neutrino Interactions

As we promised, we now show that the quarks have masses after the Higgs mechanism and that the neutrino interactions in this model is identical to the Weinberg-Salam model to the lowest order in  $G_F$ . The former follows from the representation content. For the latter one has to show “naturalness”. Since the Higgs mechanism is by now familiar with us, we need only to point out that the mass mixing terms between  $A_\mu$  and  $W_\mu^3, B_\mu$  are in general nonzero; the mixing terms are proportional to  $\langle \varphi_1 \rangle^2 + \langle \varphi_4 \rangle^2 - \langle \varphi_2 \rangle^2 - \langle \varphi_3 \rangle^2$ , since  $\langle \varphi_1 \rangle, \langle \varphi_4 \rangle$  and  $\langle \varphi_2 \rangle, \langle \varphi_3 \rangle$  have opposite hypercharges under  $U(1)_R$  symmetry. There is available a discrete symmetry which ensures the mixing to be zero at zeroth order, which means corrections are necessarily of higher order and phenomenologically negligible. We now turn our attention to this discrete symmetry.

The covariant gauge coupling in (3) are invariant under a reflection symmetry given by

$$\begin{pmatrix} p_i \\ n_i \end{pmatrix} \rightleftharpoons \begin{pmatrix} t_i \\ b_i \end{pmatrix}, \quad \varphi_1 \rightleftharpoons \varphi_3, \quad \varphi_2 \rightleftharpoons \varphi_4 \quad (4)$$

$$A_\mu \rightleftharpoons -A_\mu.$$

Note that  $(p_i, n_i)$  and  $(t_i, b_i)$  have opposite hypercharge under  $U(1)_R$ . The reflection symmetry (4) is nothing but a “chiral hypercharge conjugation” symmetry. The most general Yukawa coupling and the Higgs potential are given in (3) and (3.2), but with

$$\Gamma_{ij}^1 = \Gamma_{ij}^3, \quad \Gamma_{ij}^2 = \Gamma_{ij}^4 \quad \text{etc.}$$

$$a_1 = a_3, \quad a_2 = a_4, \quad b_1 = b_3, \quad b_2 = b_4.$$

Consequently, one has

$$\langle \varphi_1 \rangle = \langle \varphi_3 \rangle, \quad \langle \varphi_2 \rangle = \langle \varphi_4 \rangle. \quad (5)$$

From above discussion, we find that (5) implies that the mass mixing terms between  $Z_R$  and  $A, W^3$  are zero. Therefore  $Z_R$  decouples from the Weinberg-Salam  $Z$  bosons (to the lowest order). Now we can state that the

effective neutral current neutrino interaction in this model is the same as the Weinberg-Salam model, since the neutrino is left-handed. This result is natural, namely that independent of the parameters of the theory any mixing between  $Z_R$  and  $Z$  is guaranteed to be higher order and therefore naturally small. As remarked earlier, the Higgs bosons have nontrivial representations under both  $SU(2) \otimes U(1)$  and  $U(1)_R$  symmetry and do not belong to the class considered by Georgi and Weinberg [18], otherwise the above result would have been anticipated by their work. To our knowledge, this is a first example of its kind. It also demonstrates that the neutrino data is not a conclusive proof of the Weinberg-Salam model, even within the gauge framework.

### C. Global Chiral U(1) Symmetry and the Axion

We now point out the global chiral  $U(1)$  symmetry of the Peccei-Quinn type which by the Peccei-Quinn mechanism ensures CP invariance for strong interaction QCD theory. The point is that the gauge  $U(1)_{\text{chiral}}$  symmetry cannot be the Peccei-Quinn symmetry, since in order to cancel the anomalies, the quark phase under gauge  $U(1)_{\text{chiral}}$  must sum up to zero. We note that the Lagrangian (3) is invariant under the following two independent chiral  $U(1)$  rotations:

$$\begin{aligned} (p_i, n_i)_R &\rightarrow \exp(i\alpha)(p_i, n_i)_R, \\ \varphi_1 &\rightarrow \exp(i\alpha)\varphi_1, \quad \varphi_2 \rightarrow \exp(-i\alpha)\varphi_2 \\ (t_i, b_i)_R &\rightarrow \exp(i\beta)(t_i, b_i)_R, \\ \varphi_3 &\rightarrow \exp(i\beta)\varphi_3, \quad \varphi_4 \rightarrow \exp(-i\beta)\varphi_4. \end{aligned} \quad (6)$$

One finds that the  $\alpha = -\beta$  case corresponds to the gauge  $U(1)_R$  rotation. The other case  $\alpha = \beta$  corresponds to a chiral rotation where the quark phases are additive. This is the Peccei-Quinn symmetry (a la Wilczek [14]) for our model. The reason that CP is now automatically conserved for any value of  $\theta$  can be seen by studying the Higgs potential (3.1) following [12]. The axion is determined by the global chiral  $U(1)$  rotation  $\alpha = \beta$ , not by the gauge  $U(1)_R$  symmetry (or  $\alpha = -\beta$ ).

The axion interacts weakly with matter through the Yukawa coupling. Its mass and interaction can be obtained from [16], with the replacement of  $Z$  by  $f$ , namely

$$M_a \simeq 8\xi \frac{(M_u M_d)^{1/2}}{(M_u + M_d)} M_\pi f \simeq 100f \text{ MeV} \quad (7)$$

$$\xi_\pi \simeq \xi \frac{8}{3} \frac{M_u - M_d}{M_u + M_d}, \quad \xi_\eta \simeq \frac{f}{\sqrt{3}} \xi,$$

where  $\xi_\pi$  and  $\xi_\eta$  are the mixing parameters with the bare  $\pi$  and  $\eta$  respectively and

$$\xi = \left(\frac{F_\pi}{4}\right) 2^{1/4} G_F^{1/2} \simeq 1.9 \times 10^{-4} \quad (8)$$

$$f = \left(\frac{\lambda_2}{\lambda_1} + \frac{\lambda_1}{\lambda_2}\right) = \left(\frac{\lambda_3}{\lambda_4} + \frac{\lambda_4}{\lambda_3}\right), \quad \lambda_i = \langle \varphi_i \rangle.$$

The properties of this axion were previously examined in [16]. The axion with a mass in the range of 2–40 MeV has a life time in the order of  $10^{-11}$  s or less and were found consistent with present data. Such a relatively heavy axion could possibly be found in an improved experiment on  $K \rightarrow \pi e^+ e^-$  by measuring the recoil momentum of pion as the axion production is perhaps dominated by a two body decay channel  $K \rightarrow \pi a$ .

#### 4. Electron Neutral Currents

##### A. Effective Hamiltonian

The electromagnetic and weak neutral currents can be written as

$$eA_\mu j_\mu^{\text{em}} + \frac{1}{2} \sqrt{g^2 + g'^2} Z_\mu (j_{\mu L}^3 - 2 \sin^2 \theta_w j_\mu^{\text{em}}) + \frac{g''}{2} Z_{R\mu} j_{\mu R}^0 \quad (9)$$

when  $j^{\text{em}}$  is the electromagnetic current and

$$j_{\mu L}^3 = \sum_i \bar{\psi}_i \gamma_\mu \tau_3 \frac{1 - \gamma_5}{2} \psi_i$$

$$j_{\mu R}^0 = \sum_i \bar{\psi}_i \gamma_\mu \frac{1 + \gamma_5}{2} \psi_i$$

are isovector and isosinglet currents. The Higgs mechanism gives

$$\frac{g^2}{8M_w^2} = \frac{g^2 + g'^2}{8M_Z^2} = \frac{G_F}{\sqrt{2}} \quad (10)$$

$$\frac{g''^2}{8M_{Z_R}^2} = \frac{G_F}{\sqrt{2}} \sin^2 \phi,$$

where

$$\sin^2 \phi = \frac{\lambda^2}{\lambda^2 + \langle \varphi_5 \rangle^2}, \quad \lambda^2 = \sum_{i=1}^4 \langle \varphi_i \rangle^2.$$

Here we define  $\phi$  to be a mixing angle of the vacuum expectation values (in contrast to convention). As apparent from (10) this mixing angle measures the effective strength of the  $U(1)_R$  isosinglet currents<sup>3</sup> [in similar fashion as  $\sin^2 \theta_w$  measures the strength of the

<sup>3</sup> The singlet  $\varphi_5$  is here introduced ad hoc, only for phenomenological reasons

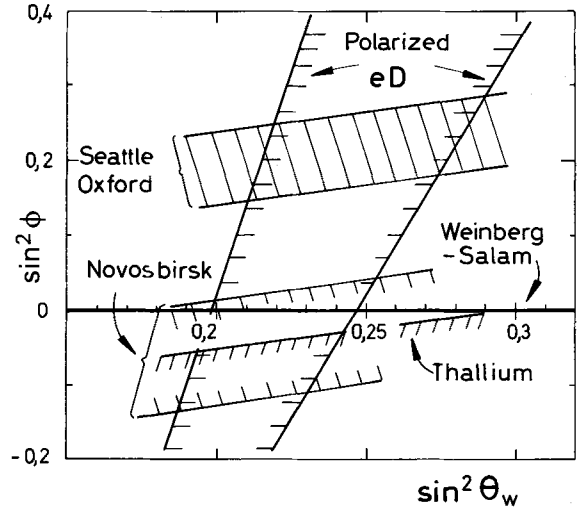


Fig. 1. Allowed region in  $\sin^2 \theta_w$  and  $\sin^2 \phi$  from polarized electron deuterium experiments [8], atomic bismuth [21, 22], and thallium [23] experiments with one standard deviation

$U(1)$  coupling constant]. Since  $Z_R$  and  $Z$  do not mix, one has the following effective neutral current interactions.

$$\mathcal{H}^{\text{NC}} = \frac{G_F}{\sqrt{2}} \{ (j_{\mu L}^3 - 2 \sin^2 \theta_w j_\mu^{\text{em}})^+ \cdot (j_{\mu L}^3 - 2 \sin^2 \theta_w j_\mu^{\text{em}}) + \sin^2 \phi j_{\mu R}^{0+} j_{\mu R}^0 \}. \quad (11)$$

To relate to experiments, we rewrite the parity-violating piece of (11) as

$$\mathcal{H}_{\text{PV}}^{\text{NC}} = \frac{G_F}{\sqrt{2}} \{ C_{1u} \bar{e} \gamma_\mu \gamma_5 e \bar{u} \gamma^\mu u + C_{2u} \bar{e} \gamma_\mu e \bar{u} \gamma^\mu \gamma_5 u + C_{2d} \bar{e} \gamma_\mu \gamma_5 e \bar{d} \gamma^\mu d + C_{2d} \bar{e} \gamma_\mu e \bar{d} \gamma^\mu \gamma_5 d \}, \quad (12)$$

where [19]

$$\begin{aligned} C_{1u} &= \frac{1}{2} - \frac{4}{3} \sin^2 \theta_w + \frac{1}{2} \sin^2 \phi \\ C_{2u} &= \frac{1}{2} - 2 \sin^2 \theta_w + \frac{1}{2} \sin^2 \phi \\ C_{1d} &= -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_w + \frac{1}{2} \sin^2 \phi \\ C_{2d} &= -\frac{1}{2} + 2 \sin^2 \theta_w + \frac{1}{2} \sin^2 \phi. \end{aligned} \quad (13)$$

##### B. Comparison with Data

Parity-violation effects due to neutral currents have recently been measured in two different types of experiments, namely, (1) polarized electron scattering and (2) parity violation experiments in heavy atoms particularly Bi and T. In polarized-electron deuterium scattering, one measures the asymmetry  $A = (\sigma_R - \sigma_L) / (\sigma_R + \sigma_L)$  due to parity violating interactions. The asymmetry can be expressed as

$$A/Q^2 = a_1 + a_2 [1 - (1 - y)^2] / [1 + (1 - y)^2], \quad (14)$$

**Table 2.** Matrix elements of the weak Hamiltonian between  $2S_{1/2}$  and  $SP_{1/2}$  states in hydrogen and deuterium. The energy unit is  $\Delta \simeq 0.013$  Hz. Notation follows standard convention. See [25]

Hydrogen	
$\langle f, 0   \mathcal{H}_{\text{PV}}   \beta, 0 \rangle$	$= -i\Delta(1.125 - 4.5 \sin^2 \theta_w + 1.725 \sin^2 \phi)$
$\langle f, -1   \mathcal{H}_{\text{PV}}   \beta, -1 \rangle$	$= -i\Delta(-0.25(\frac{1}{2} - 2 \sin^2 \theta_w) + 1.27 \sin^2 \phi)$
$\langle e, 0   \mathcal{H}_{\text{PV}}   \beta, 0 \rangle$	$= -i2\Delta(1.25(\frac{1}{2} - 2 \sin^2 \theta_w) + 0.22 \sin^2 \phi)$
Deuterium	
$\langle e, \pm \frac{1}{2}   \mathcal{H}_{\text{PV}}   \beta, \pm \frac{1}{2} \rangle$	$= i\sqrt{2} \Delta(0.45 \sin^2 \phi)$
$\langle f, \frac{1}{2}   \mathcal{H}_{\text{PV}}   \beta, \frac{1}{2} \rangle$	$= -i\Delta(-2 \sin^2 \theta_w + 3.45 \sin^2 \phi)$
$\langle f, -\frac{1}{2}   \mathcal{H}_{\text{PV}}   \beta, -\frac{1}{2} \rangle$	$= -i\Delta(-2 \sin^2 \theta_w + 3 \sin^2 \phi)$
$\langle f, -\frac{3}{2}   \mathcal{H}_{\text{PV}}   \beta, -\frac{3}{2} \rangle$	$= -i\Delta(-2 \sin^2 \theta_w + 2.55 \sin^2 \phi)$

where

$$\begin{aligned} a_1(\text{exp}) &= (-9.7 \pm 2.6) \times 10^{-5} \\ a_2(\text{exp}) &= (4.9 \pm 8.1) \times 10^{-5}. \end{aligned} \quad (15)$$

In terms of the neutral current coupling in (12), one finds [20]

$$\begin{aligned} a_1 &= -\frac{3G_F}{5\sqrt{2}\pi\alpha} \left( C_{1u} - \frac{C_{1d}}{2} \right) \\ a_2 &= -\frac{3G_F}{5\sqrt{2}\pi\alpha} \left( C_{2u} - \frac{C_{2d}}{2} \right). \end{aligned} \quad (16)$$

The experimental data in terms of (16) and (13), gives two constraints on  $\sin^2 \theta_w$  and  $\sin^2 \phi$ . In Fig. 1, we plot the allowed region of  $\sin^2 \theta_w$  and  $\sin^2 \phi$  as determined from the data (15). One sees that for a range of  $\sin^2 \phi$ , the model predicts flat  $y$  distribution consistent with present data.

We can similarly determine the allowed region of  $\sin^2 \theta_w$  and  $\sin^2 \phi$  from the results of atomic bismuth and thallium experiments. In order to interpret the data, we assume standard atomic calculation, and in turn express the experimental results in terms of

$$Q_w = Z(1 - 4\sin^2 \theta_w) - N + 3(Z + N)\sin^2 \phi. \quad (17)$$

The Novosibirsk experiment [21] gives  $Q_w = -140 \pm 40$  which is much larger than the Seattle and Oxford result [22] ( $Q_w = -4 \pm 16$  and  $18 \pm 32$  respectively). The thallium experiment [23] also reported non-zero parity violation effects with however bigger error bars. Before the experimental situation is clarified, one cannot draw any conclusion. Here we use the published results only to indicate the possible conclusions one can draw. As we see from Fig. 1 the Oxford and Seattle results would give  $\sin^2 \phi$  in the neighbourhood of 0.2 whereas the Novosibirsk and thallium would indicate that  $\sin^2 \phi$  is very small. Note that  $\sin^2 \phi$  is theoretically positive whereas the data of the thallium experiment is more consistent with negative values. Since  $\sin^2 \phi$  measures the strength of the chiral U(1)

currents, whether there is any evidence for  $U(1)_R$  currents depends on the final outcome of the atomic experiments. However in view of the uncertainties involved in atomic physics calculations, one still cannot draw any definite conclusion. Nevertheless, we like to emphasize that the question is not whether parity is violated or not but at what level is it experimentally seen.

### C. Implications for Future Experiments

We briefly discuss below one low energy experiment and two high energy experiments which will provide more crucial tests of the chiral  $U(1)_R$  currents.

(1) Parity violation in hydrogen and deuterium. The hydrogen and deuterium experiments are difficult experiments but provide the needed information in order to determine the electron neutral current couplings in a model dependent way. In particular, some of the levels are much sensitive to the chiral U(1) currents, such as the following transitions

$$\langle f, M_F = -1 | \mathcal{H}_{\text{PV}} | \beta, M_F = -1 \rangle \quad \text{for hydrogen}$$

$$\langle e, M_F = \pm \frac{1}{2} | \mathcal{H}_{\text{PV}} | \beta, M_F = \pm \frac{1}{2} \rangle \quad \text{for deuterium.}$$

These matrix elements as well as a few others are listed in Table 2.

(2) Electron and proton colliding experiments. High energy electron and proton colliding beam experiments (LEP) offers an excellent chance to study the weak neutral currents, since at large  $q^2$  (virtual photon momentum squared) the electromagnetic and weak interactions have equal magnitudes. By having polarized electrons, one probes either left-handed or right-handed currents. For example a right-handed electron interacts with the proton with the following form

$$\begin{aligned} & \frac{G_F}{2\sqrt{2}} \left\{ 2\sin^2 \theta_w (j_{\mu L}^3 - 2\sin^2 \theta_w j_{\mu}^{\text{em}}) \frac{M_Z^2}{M_Z^2 - q^2} \right. \\ & \left. + \sin^2 \phi j_{\mu R}^0 \frac{M_{Z_R}^2}{M_{Z_R}^2 - q^2} \right\} \bar{e} \gamma^\mu (1 + \gamma_5) e. \end{aligned} \quad (18)$$

For  $q^2$  near the  $Z_R$  mass, the second term would be non-negligible. The actual effects depend on the  $Z_R$  mass. As an estimate, we note

$$M_{Z_R}^2 = M_Z^2 \left( \frac{g''^2}{g^2 + g'^2} \right) \frac{1}{\sin^2 \phi},$$

and assume  $g' = g''$  [chiral  $U(1) \otimes U(1)$  symmetry], then  $M_{Z_R}^2 = M_Z^2 \frac{\sin^2 \theta_w}{\sin^2 \phi}$ . If  $\sin^2 \phi \sim 0.2$ , then  $M_{Z_R}$  is close to  $M_Z$ , otherwise it is presumably heavier.

(3)  $e^+e^-$  colliding experiments. By measuring the forward-backward asymmetry in muon pairs or hadronic channels, one measures the  $VA$  interference terms between the electromagnetic and weak interactions.

These effects get enhanced at high energies. The prediction of the Weinberg-Salam model is well studied in the literature [24] and we are looking for possible deviations from the Weinberg-Salam model. Since  $Z_R$  is probably heavier than the  $Z$  boson, such deviations are small at low energies and not likely to be picked up experimentally. We may have to wait for the next generation of  $e^+e^-$  colliding facilities in order to see the effects induced by the  $Z_R$  boson.

## 5. Conclusions

We have attacked in this paper two problems in gauge theories of strong, electromagnetic and weak interactions which at first glance are unrelated. One is the CP non-invariance due to instantons. The other concerns the weak neutral currents probed by electrons. Both problems have been of considerable interest recently, the former because of the theoretical difficulty the latter because of the speculations due to experimental uncertainties. These two problems could in fact be related. As Peccei and Quinn have pointed out, the Higgs potential knows about the instantons through the Yukawa couplings with quarks. Furthermore, if the Lagrangian possesses a chiral U(1) symmetry, the strong interactions (QCD) is CP invariant exactly because the Higgs potential depends on  $\theta$ , which is a strong interaction parameter. Because of the instantons, we see that the strong and weak interactions can be influenced by each other. The axion is a strong interaction Goldstone boson, but has a weak interaction mass and couples weakly with matter.

The weak neutral currents are similarly affected by the strong interactions, in turn due to the instantons if the chiral U(1) symmetry is a gauge symmetry. Gauge chiral U(1) symmetry serves two purposes. (1) It induces a global chiral U(1) symmetry to remove the CP violating phase due to the instantons. (2) It avoids the problems faced by the axion in the  $SU(2)\otimes U(1)$  model. However, if chiral U(1) symmetry is a gauge symmetry, one can have a new kind of anomaly, namely, those created by quark loops coupling to the chiral gauge boson and two color gluons. A consistent theory requires the anomalies to be cancelled. Because of these constraints, and the chiral nature of the U(1) symmetry, the particle representations are unique under the gauge symmetries. Therefore the weak neutral currents are also determined.

If one believes that CP should be a symmetry of the strong interaction (QCD) theory, rather than a property of a particular vacuum state, we have argued for the need of a gauge chiral U(1) symmetry, consequently the existence of chiral U(1) weak currents. The interesting point is that the neutrino interaction does not know about the chiral U(1)<sub>R</sub> weak currents, therefore it

provides no test of our model. The electron neutral current data could turn out to support chiral U(1) currents, although the present experimental situation is too confused to conclude either way.

What we have succeeded in this paper are: (1) We have demonstrated a realistic model for the Peccei-Quinn mechanism. (2) We have shown an example in which the Higgs bosons have nontrivial representations under both  $SU(2)\otimes U(1)$  and  $G=U(1)$ , but the neutrino interactions are nonetheless still identical to the Weinberg-Salam model by the virtue of a chiral "hypercharge conjugation" symmetry. This is perhaps the first example of its kind. Note that most examples in the literature belong to the class considered by Georgi and Weinberg.

Finally we remark on the experiments which could bear on the issues discussed in this paper. One area of experiments is parity violation experiments in heavy atoms, where the isoscalar chiral U(1) currents are enhanced by the atomic numbers. If the parity violation effects observed deviate from the Weinberg-Salam prediction, it could be interpreted as evidence for chiral U(1) currents, particularly in view of the fact that polarized electron deuterium data already put severe constraints on the weak neutral current couplings. However, because of the uncertainty in the atomic theory calculations, a more definitive test would have to come from deuterium or hydrogen experiments. We have pointed out above that there exist a few transitions which are sensitive to the isoscalar components of the weak neutral currents. If for some reasons the effective strength of the chiral U(1) currents should be very small, then we would not see it in low energy experiments, but high energy  $ep$  and  $e^+e^-$  colliding experiments could perhaps unveil its existence.

The other area of experiments concerns the detection of the axion. We believe that an improved experiment in  $K\rightarrow\pi ee$  would be sensitive to the axion effects and perhaps doable in the near future.

*Acknowledgement.* I would like to thank Prof. H. Joos and the DESY theory group for the hospitality extended to me at DESY. I also thank Prof. M. Krammer for reading the manuscript.

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