LONGITUDINAL CROSS SECTION AND ASYMMETRIES FOR JETS IN LEPTOPRODUCTION ☆

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We have calculated the longitudinal and other polarization dependent cross sections for jet production in deep inelastic electron–proton scattering up to order α_s of the quark–gluon coupling constant and compared them with estimates of the nonperturbative contributions.

So far most of the theoretical predictions on jet properties based on quantum chromodynamics (QCD) are for e^+e^- annihilation [1]. In leptoproduction previous theoretical work is mainly on one-particle inclusive processes [2]. From this it is known that first order contributions in the QCD coupling $\alpha_s(Q^2)$ lead to angular asymmetries of single produced hadrons relative to the lepton scattering plane. But these predictions depend on poorly known quark and gluon fragmentation functions. Furthermore, nonperturbative effects, in particular the effect of a nonvanishing transverse momentum of the incoming partons, produce similar asymmetries. From this point of view, direct predictions on global jet properties, whether as distributions in the Sterman–Weinberg cut-off variables (ϵ, δ) [3] in the summed transfer momentum of hadrons variable $\pi_T = \Sigma_{\text{hadrons}} |p_T|$ [4] or in the jet variables thrust T and/or spherocity S [5] seem more useful. In particular, by making cuts with these variables, it is easier to eliminate from the distributions unwanted two-jet contributions, which, if dressed by nonperturbative effects, dominate the distributions for small (ϵ, δ), π_T, $1 - T$ or S. These distributions become narrower and narrower with increasing W, where W is the total available energy in the final state. This way one can hope that genuine QCD effects due to gluon emission and absorption become visible if W is large enough.

Recently experimental results for $d\sigma/dT$ and $d\sigma/dS$ have been reported for deep inelastic neutrino scattering for $W \approx 10$ GeV [6]. These data are still in the nonperturbative region and the shape of these distributions can be very well described by known fragmentation models [7]. Data with larger center-of-mass energies W may become available with higher neutrino energies in the near future or with ep storage rings now under discussion [8].

In general, with unpolarized target and beam, the inclusive single jet cross section can be decomposed into four separate terms which can be classified by the polarization of the absorbed virtual photon. This is well known from the more familiar inclusive single particle cross section [9]. These four terms can be disentangled by varying the azimuthal angle of the detected jet and the degree of the polarization of the virtual photon.

In this letter we study these four cross sections as a function of the thrust variable T up to the first order of the quark–gluon coupling $g = (4\pi\alpha_s(Q^2))^{1/2}$. We hope that the separate determination of all four cross sections will give a further handle for the verification of QCD in the spacelike region.

First we consider the three-jet contribution proportional to $\alpha_s(Q^2)$, represented by the diagrams in figs. 1b, c and d. The lowest order diagram, fig. 1a, will be considered later. The three final state jets are the partons, quarks, antiquarks or gluons with momentum p_1 and p_2 and the spectator jet with momentum p_3. We use the final hadronic rest frame $\boldsymbol{P} + \boldsymbol{q} = 0$ and use as variables $W^2 = (P + q)^2$, Q^2 and $x_i = 2p_{i0}/W$ ($i = 1, 2, 3$) with the constraint $x_1 + x_2 + x_3 = 2$ (see fig. 1e for the notation of momenta). As usual we define $y = P_q/P_\ell$ and the angle φ as the

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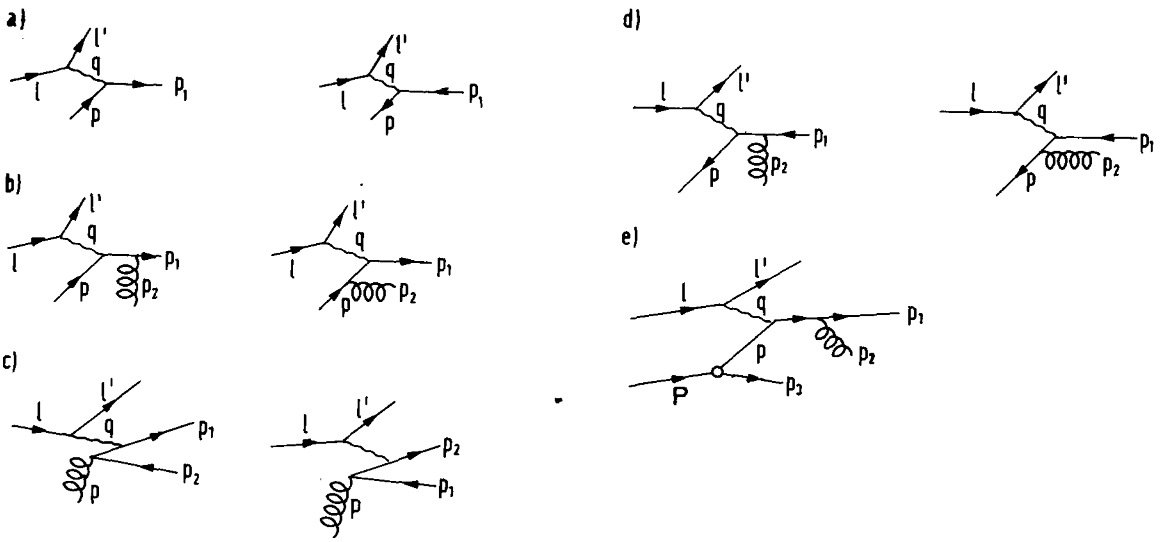

Fig. 1. (a) Zeroth order parton diagrams for lepton–parton scattering. (b, c, d) First order parton diagrams for lepton–parton scattering. (e) Kinematic diagram for three-jet production in lepton–proton scattering.

azimuthal angle of the scattered electron relative to the plane defined by the final parton momenta. The dependence of the cross section on φ and y for electron–nucleon scattering (in the one-photon exchange approximation) is of the following form [‡1]

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}Q^2\,\mathrm{d}W^2\,\mathrm{d}\varphi\,\mathrm{d}x_1\,\mathrm{d}x_2} = \Gamma\left\{\frac{\mathrm{d}^2\sigma_\mathrm{u}}{\mathrm{d}x_1\,\mathrm{d}x_2} + \frac{2(1-y)}{1+(1-y)^2}\,\frac{\mathrm{d}^2\sigma_\mathrm{L}}{\mathrm{d}x_1\,\mathrm{d}x_2} + \frac{2(1-y)}{1+(1-y)^2}\cos 2\varphi\,\frac{\mathrm{d}^2\sigma_\mathrm{T}}{\mathrm{d}x_1\,\mathrm{d}x_2} - \frac{(1-y)^{1/2}(2-y)}{1+(1-y)^2}\cos\varphi\,\frac{\mathrm{d}^2\sigma_\mathrm{I}}{\mathrm{d}x_1\,\mathrm{d}x_2}\right\}, \tag{1}$$

Γ is the well-known equivalent photon spectrum

$$\Gamma = [\alpha W^2/4\pi^2 Q^2(W^2+Q^2)^2]\,(1+(1-y)^2)\,. \tag{2}$$

The term $\mathrm{d}\sigma_\mathrm{u}$ is the cross section for unpolarized transverse virtual photons, $\mathrm{d}\sigma_\mathrm{T}$ is due to the transverse linear polarization of the photon, $\mathrm{d}\sigma_\mathrm{L}$ accounts for the longitudinal polarized photons and $\mathrm{d}\sigma_\mathrm{I}$ describes the interference of the longitudinal and transverse matrix element.

The total cross section is written in the form

$$\mathrm{d}^2\sigma/\mathrm{d}Q^2\,\mathrm{d}W^2\,\mathrm{d}\varphi = \Gamma\sigma_\mathrm{u}\,. \tag{3}$$

The σ_u in zeroth order is

$$\sigma_\mathrm{u}^0 = (\pi e^2 Q_\mathrm{a}^2/W^2) f_\mathrm{a}(x_\mathrm{B}, Q^2)\,. \tag{4}$$

Here $f_\mathrm{a}(x_\mathrm{B}, Q^2)$ is the distribution of parton a inside the nucleon with the fraction of momentum $x_\mathrm{B} = Q^2/(Q^2 + W^2)$ and Q_a is the charge of parton a. The derivation of the three-jet cross section is straightforward. For the diagrams in fig. 1b we obtain [‡2]:

[‡1] We consider ep scattering only. The case of neutrino scattering is studied in a separate paper.

[‡2] These cross sections can be derived from the formulas of Méndez (see ref. [2]) with the substitution $z = 1 - x_{11}/x_3$, where $z = pp_1/pq$.

$$(\sigma_u^0)^{-1}\frac{d^2\sigma_u}{dx_1\,dx_2}=\frac{C_1}{x_{11}x_{13}x_3}\left\{\frac{W^2}{\eta(W^2+Q^2)}x_{13}^2(x_{11}^2+x_{12}^2)+x_{11}x_{13}(x_{12}+x_3)+\frac{Q^2}{W^2}(x_{12}^2+x_3^2)\right\},$$

$$(\sigma_u^0)^{-1}\frac{d^2\sigma_L}{dx_1\,dx_2}=2(\sigma_u^0)^{-1}\frac{d^2\sigma_T}{dx_1\,dx_2}=\frac{4C_1Q^2x_{12}}{\eta(W^2+Q^2)x_3},$$

$$(\sigma_u^0)^{-1}\frac{d^2\sigma_I}{dx_1\,dx_2}=-\frac{4C_1}{x_3}\left(\frac{x_{12}x_{13}Q^2}{x_{11}W^2}\right)^{1/2}\left\{(x_2-x_1)\frac{W^2}{\eta(W^2+Q^2)}+\frac{x_{12}}{x_{13}}\right\}, \tag{5}$$

where $x_{1i}=1-x_i$ $(i=1,2,3)$ and C_1 is a factor universal to all four partial cross sections

$$C_1=[W^2/\eta(W^2+Q^2)x_3][f_a(\eta,Q^2)/f_a(x_B,Q^2)]\,c_1\alpha_s/2\pi, \tag{6}$$

$\eta=[Q^2+(1-x_3)W^2]/(Q^2+W^2)$ is the fraction of the proton momentum carried away by the ingoing parton, i.e. $P=\eta P$. In these formulas we assumed that $p_T=0$. For the diagrams in fig. 1b, where the parton a is the gluon, the corresponding formulae are:

$$(\sigma_u^0)^{-1}\frac{d^2\sigma_u}{dx_1\,dx_2}=\frac{C_3}{x_{11}x_{12}}\frac{W^4x_{13}^2+Q^4}{\eta^2(W^2+Q^2)^2}(x_{11}^2+x_{12}^2),\quad (\sigma_u^0)^{-1}\frac{d^2\sigma_L}{dx_1\,dx_2}=2(\sigma_u^0)^{-1}\frac{d^2\sigma_T}{dx_1\,dx_2}=\frac{8C_3Q^2W^2x_{13}}{\eta^2(W^2+Q^2)^2},$$

$$(\sigma_u^0)^{-1}\frac{d^2\sigma_I}{dx_1\,dx_2}=4C_3\frac{Q^2W^2}{\eta^2(W^2+Q^2)^2}\left(\frac{Q^2x_{13}}{W^2x_{11}x_{12}}\right)^{1/2}(x_1-x_2)\left(\frac{W^2}{Q^2}x_{13}-1\right), \tag{7}$$

C_3 deviates from C_1 just by the colour factors: $C_3=(c_3/c_1)\,C_1$, where $c_3=1/2$, $c_1=4/3$. The contribution of the diagrams in fig. 1c is obtained by making the substitution $x_1\leftrightarrow x_2$. We remark that in the final expression where all three contributions of figs. 1b, c and d are added the normalization is given by eq. (4) with a sum over the quark and antiquark terms.

The two-jet contribution is calculated on the basis of the diagrams in fig. 1a. It is well known that, if there is no transverse momentum of the partons confined within the proton, the contribution of fig. 1a to σ_L, σ_T and σ_I vanishes. To obtain a realistic estimate of the distortion of nonperturbative terms to σ_L and the azimuthal asymmetries we shall incorporate this primordial transverse momentum into the parton model. Let p_T be the transverse momentum (equal to the transverse momentum of the emitted parton). Then we get the following contribution to the various cross sections (considering ingoing and outgoing quarks on zero-mass shell) [10]:

$$(\sigma_u^0)^{-1}\frac{d^2\sigma_L^0}{dx_1\,dx_2}=2(\sigma_u^0)^{-1}\frac{d^2\sigma_T}{dx_1\,dx_2}=\frac{4p_T^2}{Q^2}\,\delta(1-x_1)\delta(1-x_2),\quad (\sigma_u^0)^{-1}\frac{d^2\sigma_I^0}{dx_1\,dx_2}=\frac{4p_T}{(Q^2)^{1/2}}\,\delta(1-x_1)\delta(1-x_2) \tag{8}$$

For increasing Q^2 both terms vanish like $1/Q^2$ and $1/(Q^2)^{1/2}$ compared to $\alpha_s(Q^2)\approx 1/\ln Q^2$, which determines the QCD contributions. The contribution to σ_I, which vanishes only like $(Q^2)^{-1/2}$ and is comparable in magnitude to the QCD term even at $Q^2\approx 100$ GeV^2, does not bother us, since we do not distinguish the current jet from the target jet in our final result. In this case the $\cos\varphi$ term drops out in the two-jet contribution.

The variables x_1, x_2 and x_3 are transformed into jet variables. For example, we can take thrust T and sphericity S. They are related to x_1, x_2 and x_3 by [12]

$$T=\max(x_1,x_2,x_3),\quad S=64\pi^{-2}T^{-2}(1-x_1)(1-x_2)(1-x_3). \tag{9}$$

For $x_1>x_2>x_3$ we have

$$dx_1\,dx_2=\pi^2T\,dT\,dS/64(1-T)(1-\pi^2S/16(1-T))^{1/2}. \tag{10}$$

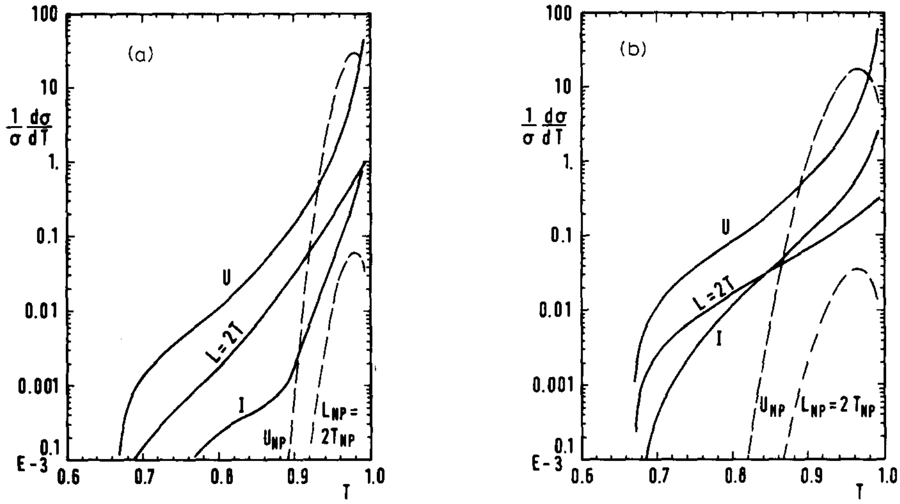

Fig. 2. Partial cross sections u: $d\sigma_u/dT$, L: $d\sigma_L/dT$, I: $d\sigma_I/dT$ and T: $d\sigma_T/dT = (1/2)\, d\sigma_L/dT$ for jet production in electron–proton scattering. The dashed curves are the nonperturbative cross sections. For (a) $W = 100$ GeV, $Q = 25$ GeV, $\Delta T = 0.028$, (b) $W = 50$ GeV, $Q = 25$ GeV, $\Delta T = 0.048$.

In order to calculate $d^2\sigma/dT\,dS$ we sum over the six regions of phase space $x_1 > x_2 > x_3$ etc. consisting of the six permutations of 1,2,3. Then the double differential distributions for the various photon polarization states are integrated over S. For different values of W^2 and Q^2 the resulting distributions $d\sigma_u/dT$, $d\sigma_L/dT = 2d\sigma_T/dT$ and $d\sigma_I/dT$ are shown in fig. 2 and fig. 3. In figs. 2a, b we have $W^2 > Q^2$ ($Q^2 = 625$ GeV^2 and $W^2 = 10^4$ GeV^2 and $W^2 = 2500$ GeV^2). In this case $d\sigma_L/dT = 2d\sigma_T/dT$ and $d\sigma_I/dT$ have a rather steep fall-off with decreasing T. For both choices of W it should be possible to determine the cos 2φ term $d\sigma_T/d\sigma_u$ which is around 15% for T = 0.8 which is outside the nonperturbative region. $d\sigma_I/d\sigma_u$ is roughly of the same magnitude in the case W = 50 GeV,

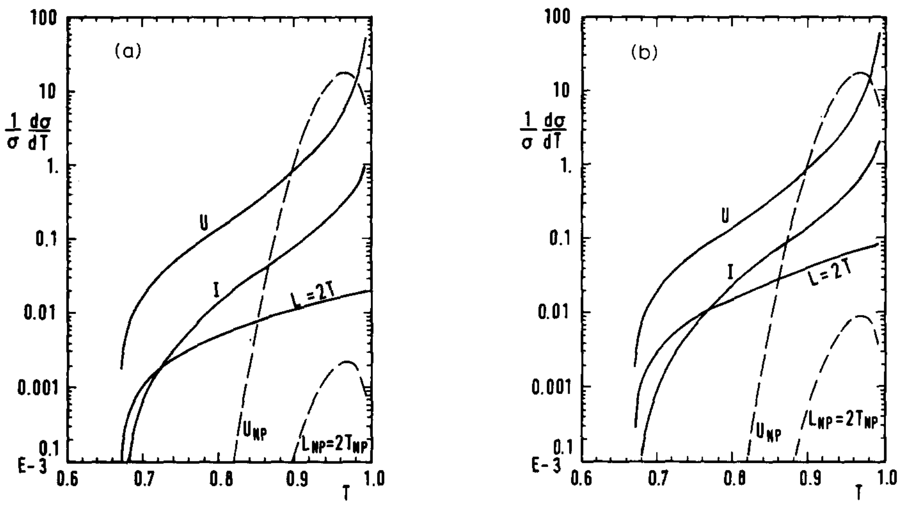

Fig. 3. Same as fig. 2 for other W and Q. (a) W = 50 GeV, Q = 100 GeV, ΔT = 0.048, (b) W = 50 GeV, Q = 50 GeV, ΔT = 0.048.

but smaller for $W = 100$ GeV. The results in figs. 3a, b are for $W = 50$ GeV and two Q values: $Q = 100$ GeV and $Q = 50$ GeV. Here $d\sigma_L = 2d\sigma_T$, as a function of T, is somewhat flatter and smaller than $d\sigma_I$ in the relevant region near $T = 0.8$. In these two cases the determination of the cos φ term should be easier.

The nonperturbative distributions due to the limited transverse momentum of hadrons from quark and gluon fragmentation are also shown in figs. 2a, b and figs. 3a, b. These distributions are calculated from

$$(\sigma_u^0)^{-1}\, d\sigma_u^0/dT = [2(1-T)/(\Delta T)^2]\, \exp[-(1-T)^2/(\Delta T)^2]\ . \tag{11}$$

The usual assumption for the p_T distribution of the particles in a jet is $\exp(-4p_T^2)$, which is gaussian. This has lead us to assume, similar to De Rujula et al. (see ref. [1]), also a gaussian form for the T distribution. The $(1-T)$ factor is for the fact that massive particles cannot be produced collinearly. The width parameter ΔT has been determined from recent jet production experiments in ν–nucleon scattering at $W = 10$ GeV [6]. For the higher W values the ΔT was scaled down with the factor W^{-1}, which gives us $\Delta T = 0.048$ for $W = 50$ GeV. We see from figs. 2 and 3 that $T \leqslant 0.8$ is a safe region to test the QCD cross sections. Concerning the asymmetry terms $d\sigma_T/dT$ and $d\sigma_I/dT$ one could go even to larger T values, since the nonperturbative contribution is really small for $d\sigma_T/dT$ in this region. This latter distribution was computed from eq. (11) with the factor given in eqs. (8) where p_T was chosen 0.4 GeV. Further tests concerning the transverse-longitudinal cross section $d\sigma_I/dT$ are possible in case one distinguishes the final jets: current jet (quark, antiquark or gluon) and the target jet. Of course, then a nonperturbative contribution is present. Results for such cases will be presented elsewhere.

Finally we mention that all our results have been calculated with Q^2 dependent quark and gluon distribution functions. For the Q^2 dependence we adopted the functional form of Buras and Gaemers [11] with parameters taken from the recent analysis of deep inelastic lepton scattering data by Glück and Reya [12].

In conclusion we state that the polarization dependent cross sections σ_L, σ_T and σ_I for jet production in deep inelastic lepton scattering can serve as more sophisticated tools to test QCD in the space-like region of Q^2.

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