PHOTON–PHOTON COLLISIONS AND TWO-JET PRODUCTION TO ORDER α_s

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We calculate the first order QCD correction to the cross section for $\gamma\gamma \to q\bar{q}$, including virtual gluons and bremsstrahlung of soft and hard collinear gluons. The numerical importance of these one-loop corrections to the total cross section for large transverse momentum jet production in $e^+e^- \to e^+e^-$ + two jets is discussed.

The importance of large transverse momentum jet production in photon–photon collisions using e^+e^- storage rings has been extensively discussed in the literature [1,2]. Of all the different jet cross sections, the two-jet production via quark exchange in $\gamma\gamma \to q\bar{q}$ is predicted to have the largest cross section. At the same time, its magnitude is a direct measure of the sum of quark charges to the fourth power, which is an important quantity for distinguishing among different quark models [3]. In view of the importance of this process, it becomes necessary to know the magnitude of the QCD corrections, which arise when gluon exchange and emission to the lowest order process $\gamma\gamma \to q\bar{q}$ are considered. This paper presents the results of such a calculation.

As described in ref. [1], to obtain the cross section for $e^+e^- \to e^+e^-$ + two jets, one has to fold the cross section for the subprocess $\gamma\gamma \to q\bar{q}$ with the equivalent photon energy spectra, which are approximately given by

$$N(x) = [(\alpha/2\pi)\ln\eta]\,[1 + (1-x)^2]/x\,, \quad (1)$$

where $s = 4E^2$, E is the beam energy, m_e is the electron mass and $\eta = s/4m_e^2$ when the electron is not tagged, or $\eta = \theta_{max}^2/\theta_{min}^2$, when the electron is constrained to lie within an angle θ between θ_{min} and θ_{max} in the e^+e^- c.m. frame. We have to know first the cross section for $\gamma\gamma \to q\bar{q}$. In lowest order this is simply

$$d\sigma^0/d\hat{t} = (2\pi\alpha^2/\hat{s}^2)3e_q^4(\hat{t}/\hat{u} + \hat{u}/\hat{t})\,, \quad (2)$$

where $\hat{s}$, $\hat{t}$, and $\hat{u}$ are the Mandelstam variables for the sub-process, and e_q is the fractional charge of the quark.

Up to first order in α_s, the QCD coupling constant, we have to consider virtual gluon corrections, soft and hard collinear gluon bremsstrahlung, and hard acollinear gluon emission. The virtual gluon corrections are given by a series of Feynman diagrams which are topologically the same as for $e^+e^- \to \gamma\gamma$ [4]. They consist of quark self-energy diagrams, vertex corrections, and box diagrams. The soft and hard collinear gluon bremsstrahlung naturally introduce the parameters ϵ and δ, which define the energy and angular resolution of the jets, in completely the same way as in the Sterman–Weinberg formula [5]. We thus find for the cross section including the sum of the virtual, soft, and hard collinear gluon corrections:

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$$\mathrm{d}\sigma/\mathrm{d}\hat{t} = (2\pi\alpha^2/\hat{s}^2)3e_q^4(\hat{t}/\hat{u} + \hat{u}/\hat{t}) \times \{1 + (4\alpha_s/3\pi)[-\ln\delta(4\ln 2\epsilon + 3) - \tfrac{1}{3}\pi^2 + 3 + [2(\hat{t}^2+\hat{u}^2)]^{-1}[(\hat{s}^2+\hat{t}^2)\ln^2(-\hat{t}/\hat{s}) + \hat{u}(\hat{u}-2\hat{s})\ln(-\hat{t}/\hat{s}) + (\hat{t}\leftrightarrow\hat{u})]]\}. \quad (3)$$

As a check on our calculations, we performed them in two different gauges for the gluon. We also used dimensional regularization for the infrared and quark mass singularities [6], and verified that in the limit of small quark and gluon masses the same result was obtained.

The factor $-\ln\delta\,(4\ln 2\epsilon + 3)$, which appears in our result, is the same as in the Sterman–Weinberg formula, since it is a universal factor associated with soft and hard collinear gluon bremsstrahlung from quarks.

To calculate the $O(\alpha_s)$ corrections for physical quantities like the transverse momentum distributions of inclusively produced jets [‡1] in e^+e^- scattering, one has to add also the contribution of hard acollinear gluon emission. Its cross section can be obtained trivially by crossing from the $e^+e^- \to \gamma\gamma\gamma$ formula [7], with appropriate color and charge factors. For completeness we cite here the matrix element squared [‡2] for massless quarks, averaged over the initial spin and summed over color. Using the notation for the four momenta

$$\gamma(k_1) + \gamma(k_2) \to q(p_1) + \bar{q}(p_2) + g(k_3),$$

we obtain

$$|M|^2 = 256\pi^3\alpha^2 e_q^4 \alpha_s \times [(\kappa_1^2 + \kappa_1'^2)/\kappa_2\kappa_2'\kappa_3\kappa_3' + \text{cyclic permutations}]\,s',$$

where $\kappa_i = k_i\cdot p_1$, $\kappa_i' = k_i\cdot p_2$, $s' = 2p_1\cdot p_2$. The ϵ and δ dependence of formula (3) will cancel against the contribution of $|M|^2$ integrated over hard acollinear configurations. Furthermore the quark mass singularities, given by the decays of the initial photons into collinear quark–antiquark pairs, have to be renormalized [9] by the contributions of the one beam and two large transverse momentum jet production cross sections calculated in refs. [1,2]. All this presumably requires an extensive numerical effort. However, we can easily evaluate the importance of the loop corrections arising from those terms of eq. (3), which are independent from ϵ and δ.

We find that the integrated cross section, in leading order for jets with trigger momentum larger than $p_\perp^{\min} = \beta$ is

$$\sigma^0(p_\perp > \beta) = \tfrac{32}{3}\pi\alpha^2 R_{\gamma\gamma}[(\alpha/2\pi)\ln\eta]^2\beta^{-2} \times \{\rho - \tfrac{14}{3} + (\beta^2/2s)[\rho^3 + (\tfrac{51}{2} - \pi^2)\rho - 2.075] + O(\beta^4/s^2)\}, \quad (4)$$

where

$$\rho = \ln(s/\beta^2), \quad R_{\gamma\gamma} = 3\sum_{q=u,d,s,c} e_q^4 = \tfrac{34}{27}. \quad (5)$$

This result was obtained using the method described in appendix A of ref. [1], but extended to include the terms of $O(s^{-1}\ln^n s/\beta^2)$. In comparing this result with eq. (4.2) of ref. [1], where the terms $1 + (1-x)^2$ in the photon spectrum $N(x)$ were approximated by 2, it should be noted that the constant term is different and that it is necessary to calculate also the terms of order $1/s$ times logarithms of s/β^2 to obtain a sensible result. Indeed, for $\sqrt{s} = 30$ GeV and $p_\perp^{\min} = 4$ GeV, the terms $\rho - \frac{14}{3}$ alone would give a negative cross section. We find that jets with $p_\perp > 4$ GeV contribute 0.2 units of R to the e^+e^- total cross section at $\sqrt{s} = 30$ GeV from this lowest order process.

Integrating the α_s terms of eq. (3), disregarding the ϵ, δ terms, which should be combined with the hard acollinear gluon corrections, we find an extra contribution to $\sigma(p_\perp > p_\perp^{\min}$ given by

$$\sigma'(p_\perp > \beta) = \tfrac{32}{3}\pi\alpha^2 R_{\gamma\gamma}[(\alpha/2\pi)\ln\eta]^2\beta^{-2} \times (\alpha_s/\pi)\{-0.461\rho + 0.213 + (\beta^2/s)[0.0333\rho^5 - 0.0833\rho^4 + 0.973\rho^3 - 0.942\rho^2 - 4.759\rho + 17.4] + O(\beta^4/s^2)\}. \quad (6)$$

‡1 An obvious analogue of the angular distribution of the thrust axis is the transverse momentum distribution of the transverse jets. This requires the measurement of the transverse momenta of the final particles and a subsequent determination, e.g., of the transverse thrust axis $\boldsymbol{n}$ which is chosen to maximize the sum $\Sigma_i(|\boldsymbol{p}_{i\perp}\cdot\boldsymbol{n}|)$ over the corresponding semicircle. The transverse momentum of the inclusively produced jet is given by the vectorial sum $\Sigma_i \boldsymbol{p}_{i\perp}$ over the same semicircle.

‡2 For $|M|^2$ we use the normalization of Bjorken and Drell [8].

The different numerical coefficients, but the last one (17.4), are the results of long calculations involving integrals over dilogarithms and higher transcendental functions. They can be expressed in terms of various powers of π and Riemann's zeta-function $\zeta(3)$. We shall discuss their derivation elsewhere. With a systematic expansion to find all the terms, singular as $\beta \to 0$, the constant term 17.4 cannot be obtained. Therefore it was determined numerically. By performing the different integrals numerically we verified that for β values $2\beta/\sqrt{s} < 0.33$ eq. (6) as well as eq. (4) are good approximations within 3%. Note the appearance of terms up to order ρ^5, which arise from the $\ln^2$ terms in eq. (3). With $\alpha_s \approx 0.3$, $p_\perp^{min} = 4$ GeV and $\sqrt{s} = 30$ GeV we find that

$$\sigma'/\sigma^0 \approx -0.11\,. \tag{7}$$

Summarizing, the QCD corrections arising from virtual gluon exchange lower the cross section by approximately $\approx 11\%$. Moreover, we pointed out that in order to obtain sensible results, it is necessary to calculate with the exact expression for the equivalent photon spectrum. We remind that the result (7) does not give the complete $O(\alpha_s)$ correction, since the contribution of the hard acollinear gluon bremsstrahlung, with suitable renormalized quark mass singularities, still has to be evaluated.

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