COMMENT ON TOP QUARK MASS PREDICTIONS

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We discuss the running masses of the known quarks. When these are used in the Glashow determinant condition they produce the t quark mass accessible at PETRA and PEP.

The top quark, welcome by most unified models, has not been found yet. At present a search for it is under way at PETRA which will cover the c.m. energy range up to $2m_t \leqslant 37$ GeV. The previous search [1] sets a lower bound on the $^3S_1$ ground state of toponium: $M_{t\bar{t}}(1\ ^3S_1) > 31.46$ GeV. Many theoretical models predict t-quark masses. However, it might be a delicate matter to decide whether a model is ruled out in the case of the negative answer or it is confirmed by some specific positive result. The point is that the t-quark mass is usually related to masses of other quarks which are known to some accuracy and, as a result, the prediction is uncertain to some extent even if the underlying theory is perfect.

It is our purpose here to summarize the current knowledge of the quark masses and apply it to some particular predictions to see whether the t-quark mass comes out light enough to be observed at PETRA. Thus we do not advocate any new way to evaluate the mass but concentrate on numerical analysis alone.

One of the most interesting predictions is the determinant condition due to Glashow [2]

$$\begin{vmatrix} m_e & m_d & m_u \\ m_\mu & m_s & m_c \\ m_\tau & m_b & m_t \end{vmatrix} = 0 \tag{1}$$

which might have no firm foundation but, clearly, is a generalization of the well-known SU(5) mass pattern [3] which requires for the proportionality of the first two columns. Our claim is that eq. (1) produces the t-quark mass

$$2m_t = (29 \pm 3)\ \mathrm{GeV}$$

which is low enough to be seen at PETRA and PEP.

As is well known [4], one needs to be rather careful in defining a quark mass in QCD since there is no free quark. By "mass" one usually understands the ratio B/A where A and B are introduced through the inverse quark Green function:

$$S^{-1}(p) = A(p^2)\cdot \not{p} - B(p^2).$$

Because of asymptotic freedom one can rely, for large euclidean p^2, on ordinary perturbation theory to find A and B.

The definition of the mass is neither gauge nor renormalization invariant. However, for p^2 large enough the gauge dependence vanishes, at least to leading log order, and one can find the normalization point dependence in a standard way [5]:

$$m(\mu_1)/m(\mu_2) = [\alpha_s(\mu_1)/\alpha_s(\mu_2)]^{4/b}, \tag{2}$$

where α_s is the running coupling constant and b is the coefficient in the Gell-Mann–Low function: $b = 11 - 2n_f/3$. Note that b depends on μ itself since n_f counts the number of quark flavors with quark mass $m_q \ll \mu$. However, for all practical purposes one can approximate $4/b$ by

$$4/b \simeq 1/2$$

and we use this approximation throughout the paper.

Thus, things are rather simple for $\mu \to \infty$. Predictions of unified models refer just to this point since the unification scale is large. For this reason, say, the determinant condition (1) is renormalization- and gauge-invariant. On the other hand, all the measurements which can be interpreted as a manifestation of a quark mass refer to some finite quark virtuality. Therefore, the problem arises as to how continue the "running" mass from finite μ to $\mu \to \infty$. We are going to discuss this problem quark by quark.

$\underline{m_u + m_d}$. One constraint on this mass combination comes from the well-known PCAC-condition

$$(m_d + m_u)\langle 0|\bar{u}u + \bar{d}d|0\rangle = -f_\pi^2 m_\pi^2, \tag{3}$$

where $f_\pi = 130$ MeV is the pion decay constant. The condition is apparently renormalization and gauge invariant and is valid up to $\lesssim 10\%$ corrections. Unfortunately eq. (3) is not sufficient to fix $m_u + m_d$ by itself.

The second constraint is due to the analysis of the QCD sum rules for e^+e^- annihilation [6]

$$\alpha_s(\langle\bar{u}u + d\bar{d}\rangle)^2 = 4.8 \times 10^{-4}\ \mathrm{GeV}^6. \tag{4}$$

As far as we approximate $4/b$ by $1/2$ this condition is again invariant. Combining (3) and (4) we get

$$(m_u + m_d)_\mu \simeq \alpha_s^{4/b}(\mu) \times 15\ \mathrm{MeV}. \tag{5}$$

The uncertainty in the mass is mostly due to the uncertainty in the numerical value in eq. (4). At the worst, it can be a factor of 2 higher [6]. Therefore, we get finally

$$(m_u + m_d)_\mu = \alpha_s^{4/b}(\mu) \times (10\text{–}15)\ \mathrm{MeV}. \tag{6}$$

If this is compared with the original SU(6) estimate by Leutwyler [7] $(m_u + m_d) = 2m_\pi^2 f_\pi/(3\sqrt{2}\, m_\rho \cdot f_\rho) \approx (10\text{–}11)$ MeV then $\alpha_s(\mu)$ has to refer to a low normalization point $\hat{\mu}$ [8] : $\alpha_s(\hat{\mu}) \sim 1$.

$\underline{m_u, m_d}$. The share of each quark in the sum $m_u + m_d$ is fixed by the relation

$$(m_d - m_u)/(m_d + m_u) \approx 0.3, \tag{7}$$

which follows both from the Gell-Mann–Oakes–Renner [9] type analysis and QCD sum rules [10]. Taking it literally we get

$$(m_d)_\mu = \alpha_s^{4/b}(\mu) \times (6.5\text{–}10)\ \mathrm{MeV},$$
$$(m_u)_\mu = \alpha_s^{4/b}(\mu) \times (3.5\text{–}5)\ \mathrm{MeV}. \tag{8}$$

However, relation (7) brings some additional uncertainty (the up-quark share could be slightly larger).

m_s. For the ratio of the masses of the strange and u, $\bar{d}$-quarks one usually takes [9]

$$(m_s + m_d)/(m_u + m_d) = m_{K^0}^2/m_{\pi^0}^2 \approx 14, \tag{9}$$

which relies on the assumption that m_K^2 is small on the scale of characteristic masses of strong interactions. This additional assumption can be violated by, say, 30%. As the result

$$(m_s)_\mu = \alpha_s^{4/b}(\mu) \times (100\text{–}250)\ \mathrm{MeV}. \tag{10}$$

$\underline{m_c}$. So far we discussed light quarks whose mass is lower than the intrinsic mass scale of QCD, $m_q < \Lambda$. Now we proceed to discuss heavy quarks, $m_q \gg \Lambda$, where Λ enters the definition of the running coupling constant in QCD. Therefore there is no need to start with $\mu \gg m_q$ since $\mu \sim m_q$ or even $\mu \ll m_q$ can also be relevant to the short distance physics. Moreover, we must fix the gauge and we choose it to be the Landau gauge hereafter (the gauge dependence will drop out only for $\mu \gg m_q$).

QCD sum rules for charm production in e^+e^- collisions are very sensitive to m_c normalized at the euclidean point $p^2 = -m_c^2$. As a result, it is known to a good accuracy:

$$m_c(m_c) = (1.26 \pm 0.01)\ \mathrm{GeV}. \tag{11}$$

If the normalization point is changed by order of m_c^2 one can evaluate the evolution of the mass by means of first order perturbation theory (in the Landau gauge)

$$m_c(\mu) = m_c^* \left[1 - \frac{\alpha_s(m_c)}{\pi} \frac{m_c^{*2} + \mu^2}{\mu^2} \ln \frac{m_c^{*2} + \mu^2}{m_c^{*2}}\right], \tag{12}$$

where the new mass parameter, m_c^*, is introduced. This parameter denotes the quark mass on the "would be mass shell", extrapolated from the "experimental" value quoted above by means of perturbation theory: for $\alpha_s(m_c) = 0.2$, as determined from the total J/ψ hadronic width, $m_c^* = 1.37$ GeV.

If one wants to choose the normalization point much higher than the mass, $\mu^2 \gg m_c^2$, then one has to abandon the simple first order expression and sum up all the terms of order $(\alpha_s \ln \mu)^n$ which gives the standard asymptotic relation (2). The intermediate region is most difficult. As is emphasized by Shifman and Vainshtein [11], solving the renormalization group equations with account of the mass term is nothing else but to try an interpolation formula which, on the other hand, reproduces exactly the first order calculation for $\mu \lesssim m_c$ and, on the other hand, gives the correct asymptotics for $\mu \gg m_c$. The interpolation procedure is not unique but, as is shown by the same authors, there is no real problem since the matching of relatively low and high μ is very smooth.

For a heavy quark Q (Q = c, b, t, ...) we use the interpolation formula

$$m_Q(\mu) = m_Q^* \left[1 + \frac{b}{4}\frac{\alpha_s(m_Q)}{\pi}\frac{m_Q^{*2}+\mu^2}{\mu^2} \times \ln\frac{m_Q^{*2}+\mu^2}{m_Q^{*2}}\right]^{-4/b} \qquad (13)$$

which reproduces the first order eq. (12) as well as the asymptotic behaviour (see eq. (2)):

$$m_c(\mu) = 1.37\ \mathrm{GeV}\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{4/b} \times (1 \pm 0.03); \quad \mu \gg m_c. \qquad (14)$$

Here we also indicate the error arising from the interpolation procedure, the smoothness of which is illustrated in fig. 1. It is seen that at the matching point ($x \approx 2$) the interpolation coincides with the two limiting curves within 3%.

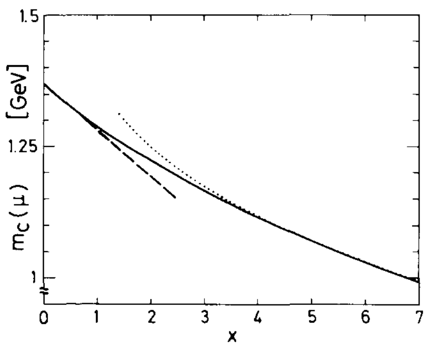

Fig. 1. Running mass of the c-quark as a function of x, $x = [(m_c^{*2}+\mu^2)/\mu^2]\ \ln[(m_c^{*2}+\mu^2)/m_c^{*2}]$. The dashed line is the first order perturbative result, eq. (12), the dotted one refers to the asymptotic behaviour, eq. (14), and the solid line represents the interpolation, eq. (13).

m_b. Here the quark mass is known from the QCD sum rules for quark virtuality of order 1 GeV^2 [12]

$$m_b(m_b^2 - p^2 \approx 1\ \mathrm{GeV}^2) = 4.79 \pm 0.03\ \mathrm{GeV}, \qquad (15)$$

which practically coincides with m_b^*. Note that the mass is fixed to even better precision than for the charmed quark. Using the interpolation eq. (13) we get asymptotically

$$m_b(\mu) = 4.8\ \mathrm{GeV}\left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{4/b} \times (1 \pm 0.01); \quad \mu \gg m_b. \qquad (16)$$

The accuracy in this case is even better since the extrapolation is smoother due to the fall in the running coupling constant. The running b-quark mass is given in fig. 2.

m_t. The effective t-quark mass is given by the same eq. (13), but m_t^* is to be found yet.

Let us emphasize that it is just m_t^* that is most relevant to experimental data. Namely, the mass of the lowest $t\bar{t}$ bound state is close to $2m_t^*$:

$$M_{t\bar{t}}(1\,^3S_1) = 2m_t^* - (0.3\text{–}0.5)\ \mathrm{GeV}.$$

Now we are in a position to find what various models give for the top quark mass. The Glashow determinant condition (1) reads

$$m_t = m_c \frac{m_\tau}{m_\mu}\left[\left(1 - \frac{m_e m_b}{m_\tau m_d}\right) + \frac{m_u}{m_d}\left(\frac{m_\mu m_b}{m_\tau m_c} - \frac{m_s}{m_c}\right)\right] \Big/ \left(1 - \frac{m_e m_s}{m_\mu m_d}\right).$$

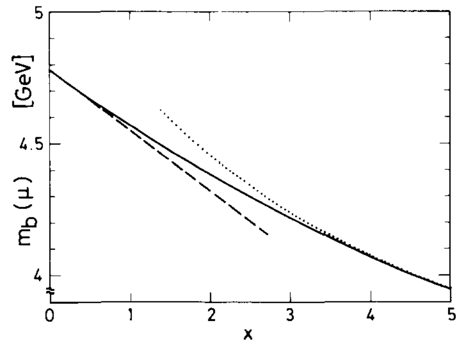

Fig. 2. Running mass of the b-quark as a function of x, $x = [(m_b^{*2}+\mu^2)/\mu^2]\ \ln[(m_b^{*2}+\mu^2)/m_b^{*2}]$. The dashed line is the first order perturbative result, eq. (12) with m_c substituted by m_b, the dotted one refers to the asymptotic behaviour, eq. (16), and the solid line represents the interpolation, eq. (13).

As was already mentioned, the relation refers to masses normalized at $\mu \to \infty$. We extrapolate it down to m_t by means of the equations worked out above. Practically, for all, but t-quark, one can rely on asymptotic expressions, since m_t is much larger than the other masses. To find m_t we use the interpolation formula. In this way we come to the prediction

$$m_t^* = 14.5 \pm 1.5 \text{ GeV}$$

where 4% uncertainty comes from interpolating m_c and the rest is due to the rather poor knowledge of m_d. This estimate relies on the value $\alpha_s(m_c) = 0.2$. Would α_s be larger, the prediction from m_t would go down. Taken at face value, it is ruled out by existing data [1]. Anyhow it will be ultimately proven or disproven very soon.

Another example is the O(10) prediction worked out in ref. [13]

$$\frac{(m_t - m_c + m_u)^3}{m_t m_c m_u} = \frac{(m_\tau - m_\mu + m_e)^3}{m_\tau m_\mu m_e} .$$

Substituting the numbers we find

$$m_t^* \lesssim 12 \text{ GeV},$$

which is lower than the current experimental bound.

As a final example let us mention the simplest relation [14]

$$m_t = m_c \, m_\tau / m_\mu , \tag{17}$$

which produces

$$m_t^* = 18 \pm 1 \text{ GeV}. \tag{18}$$

However, the similar relation $m_b = m_s \, m_\tau / m_\mu$ derived within the same simplified framework does not hold. The final result of ref. [14] is the determinant condition (1) for masses squared. Numerically it reduces essentially to the same prediction (18).

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