VORTEX FREE ENERGY AND STRING TENSION AT STRONG AND INTERMEDIATE COUPLING

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We present results of high temperature expansions up to order g^{-24} for the vortex free energy respectively string tension in pure lattice gauge theories with gauge groups SU(2) and Z_2 in 3 and 4 dimensions. For SU(2) in 4 dimensions the result is compared with Monte Carlo calculations of Creutz and is in good agreement. An intermediate coupling region is seen, where the string tension smoothly interpolates between strong coupling and weak coupling behaviour.

Gauge field theories have been formulated on a lattice [1,2] in order to study the strong coupling region which is inaccessible to ordinary perturbation theory. In the last years many results have been obtained. It has been proven that lattice gauge theories confine static quarks for strong coupling [3]. On the other hand one would like to know if for gauge group SU(2) or SU(3) a continuum limit exists with persisting confinement property. Due to asymptotic freedom [4] the continuum limit of SU(2) or SU(3) lattice gauge theories is supposed to be related to the weak coupling behaviour [5,6]. It is expected to be a limit, where the lattice spacing a and at the same time the bare coupling g go to zero in a way determined by renormalization group equations. The problem of quark confinement in lattice gauge theories is to show that the confining phase extends down to zero coupling.

The connection between strong and weak coupling regions was studied by several authors [7–11]. Of special interest is the intermediate coupling region where a transition from strong coupling to weak coupling behaviour of e.g. the string tension or renormalization group β-function should take place. Monte Carlo calculations for SU(2) in 4 dimensions [8] indicate a rather sharp changeover. In Mack's theory of quark confinement [11], which is based on the assumption that confinement of static quarks is due to condensation of vortices [12–14], the transition signals the

appearance of a nonempty range of distances $a \leqslant d \leqslant d_c(\beta)$ where perturbation theory applies. For values of β above it, vortices need a certain thickness $d_c(\beta) > a$ in order to condense. The center Z_2, respectively Z_3 of the gauge group plays an important role.

In this letter we discuss the transition between strong and weak coupling with the help of high temperature expansions. We consider euclidean lattice gauge theories with gauge groups SU(2) and Z_2 [15] in 3 and 4 dimensions. The gauge field $U(b) \in$ SU(2) respectively $\sigma(b) = \pm 1$ is attached to links b of a hypercubical lattice with lattice spacing a. The action is

$$L(U) = \frac{\beta}{2} \sum_p \text{tr } U(\dot{p}) , \quad \text{for SU(2) ,} \quad (1a)$$

$$L(\sigma) = \beta \sum_p \sigma(\dot{p}) , \quad \text{for } Z_2 , \quad (1b)$$

where $U(\dot{p})$, respectively $\sigma(\dot{p})$ is the product of the group elements on the boundary of an elementary plaquette p. The parameter β is related to the coupling constant g by

$$\beta = 4/g^2 . \quad (2)$$

The sum in eqs. (1) is over all unoriented plaquettes p of the lattice. Expectation values are formed with the path integral measure

$$d\mu = Z^{-1} \exp(L(U)) \prod_b dU(b) , \quad (3a)$$

respectively

$$d\mu = Z^{-1} \exp(L(\sigma)) \prod_b d\sigma(b) , \qquad (3b)$$

where Z is the partition function and $dU(b)$, $d\sigma(b)$ are the Haar measures on the gauge groups.

High temperature i.e. small β expansions are obtained by the well-known method of cluster expansion [16,3]. It is convenient to expand in terms of the coefficients c_j of the series

$$\exp \tfrac{1}{2}\beta \, \mathrm{tr} \, U = N\left[1 + \sum_{j \neq 0} (2j + 1) \, c_j(\beta) \, \chi_j(U)\right]$$

$$\equiv N(1 + h(U)) , \qquad (4a)$$

$$\exp \beta\sigma = N'(1 + k\sigma) \equiv N'[1 + h'(\sigma)] , \qquad (4b)$$

where $\chi_j , j = 1/2, 1, 3/2, ...$ are the irreducible characters of SU(2) and N, N' are normalization factors. Explicitly

$$c_j(\beta) = I_{2j+1}(\beta)/I_1(\beta) , \qquad (5a)$$

$$k = \tanh \beta , \qquad (5b)$$

where I_n are the modified Bessel functions. For $\ln Z$ one obtains expansions in terms of clusters. A cluster $C = (X_1^{n_1}, X_2^{n_2}, ..., X_i^{n_i})$ consists of a connected set of polymers X_1 with multiplicities n_1. Polymers are closed surfaces of plaquettes, which are allowed to possess branching lines. The cluster expansion is

$$\ln Z = \sum_C b(C) \prod_i \frac{[A(X_i)]^{n_i}}{n_i!} , \qquad (6)$$

$$A(X_i) = \int \prod_b dU(b) \prod_{p \in X_i} h(U(\dot{p})) , \quad \text{for SU(2)} ,$$

where $b(C)$ are certain combinatorial coefficients and $A(X_i)$ are the so-called activities of polymers. Details will be published elsewhere [18].

An interesting quantity to study is the free energy of vortices per unit length, respectively area [14,17]. It is defined in the following way. Consider a finite lattice L of sidelengths al_i in $d = 3$ or 4 dimensions. With periodic boundary conditions one gets a partition function Z_+. A quantum of magnetic vortex flux perpendicular to the $x_1 - x_2$ plane can be introduced in L by applying a singular gauge transformation [13,14,17] leading to twisted boundary conditions. The corresponding partition function is denoted Z_-. The free energy of a vortex per unit length, respectively area

is defined by

$$f(l_1, l_2) = \lim_{l_3, l_4 \to \infty} \left(\prod_{i=3}^d l_i^{-1}\right) (\ln Z_+ - \ln Z_-) . \qquad (7)$$

It is a suitable quantity for the characterization of different phases. In the confining phase f should obey an area law [14]

$$f(l_1, l_2) \sim \exp(-\alpha a^2 l_1 l_2) . \qquad (8)$$

In the high temperature expansion of f only a special kind of clusters survive. They contain an odd number of polymers which include a surface cutting L like the $x_1 - x_2$ plane. The expansion leads to an area law as expected:

$$f(l_1, l_2) = 2\exp(-\alpha a^2 l_1 l_2) . \qquad (9)$$

We proved that the coefficient α is in the high temperature expansion equal to the string tension [18], i.e. the corresponding coefficient in the area law for the Wilson loop expectation value [1,3]. It gives the slope of the linear potential between static quarks. A direct expansion for the string tension can be set up. We calculated the series up to order β^{12}. The results are, with $u = c_{1/2}, v = c_1, w = c_{3/2}$: for SU(2) in $d = 3$ dimensions

$$a^2\alpha = -\ln u - 2u^4 + 4u^6 - 6u^4 v - 10u^8$$
$$+ 4u^{10} + 24u^8 v - 36u^6 v^2 - 6v^5 - \tfrac{602}{3}u^{12}$$
$$+ 204u^{10}v - 36u^8 v^2 - 24u^6 v^3 - 24u^4 v^4$$
$$+ 18v^6 - 8u^9 w - 12v^5 w u^{-1} , \qquad (10)$$

for SU(2) in $d = 4$ dimensions

$$a^2\alpha = -\ln u - 4u^4 + 8u^6 - 12u^4 v - 56u^8$$
$$+ 120u^{10} - 168u^8 v - 72u^6 v^2 - 12v^5 - \tfrac{4732}{3}u^{12}$$
$$+ 888u^{10}v - 396u^8 v^2 - 48u^6 v^3 - 48u^4 v^4$$
$$+ 36v^6 - 48u^9 w - 24v^5 w u^{-1} , \qquad (11)$$

for Z_2 in $d = 3$ dimensions

$$a^2\alpha = -\ln k - 2k^4 - 2k^6 - 10k^8 - 16k^{10} - \tfrac{242}{3}k^{12} , \qquad (12)$$

for Z_2 in $d = 4$ dimensions

$$a^2\alpha = -\ln k - 4k^4 - 4k^6 - 56k^8 - 144k^{10} - \tfrac{3616}{3}k^{12} . \qquad (13)$$

The string tension for SU(2) in $d = 3$ dimensions has

been calculated previously [9] up to order β^{14}. Our results agree with theirs.

The case of SU(2) in $d = 4$ dimensions is most interesting. It can be compared with the Monte Carlo simulations of Creutz [8]. In fig. 1 we have plotted $a^2\alpha$ according to eq. (11) and the points given in ref. [8]. The high temperature series is in very good agreement with the Monte Carlo data in the intermediate coupling region between $\beta = 1.6$ and $\beta = 2.25$. At $\beta = 2.25$ the curve matches very nicely the weak coupling fit of Creutz. Above $\beta = 2.3$ the string tension seems to follow the weak coupling asymptotic freedom behaviour. This cannot be expected to be reproduced by the high temperature series (11), which in fact goes through zero near $\beta = 2.4$.
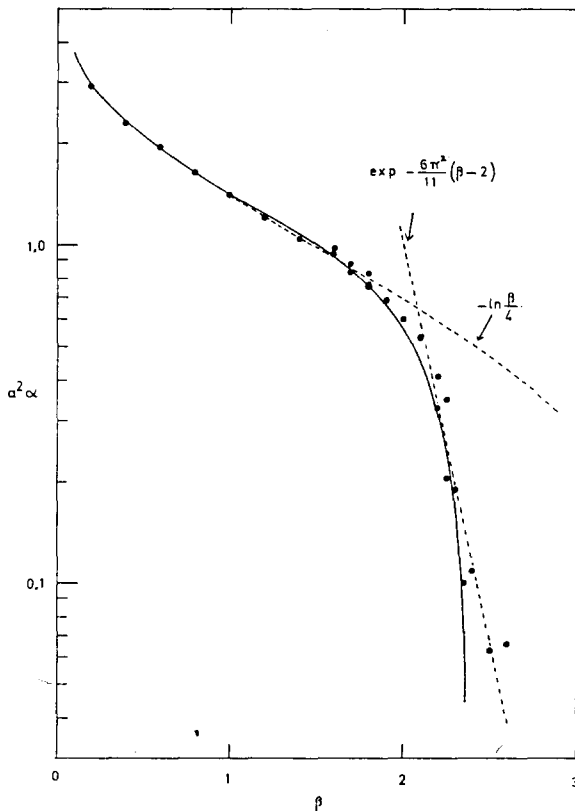
It should be noted that the cluster expansion series (10–13) for α are not power series in β. Instead by using the expansion parameters $0 \leqslant c_j < 1$ one is doing a partial resummation of the usual high temperature series in β. This appears to improve the convergence properties.

In conclusion we see that high temperature cluster expansions yield accurate values of the string tension at strong and intermediate coupling and their prediction joins smoothly to the expected behaviour at weak coupling.

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Note added. After completion of this work we received a paper by N. Kimura, Critical properties of Z(2) lattice gauge systems from strong-coupling expansions (Hokkaido University HOU-HP-80-01), where the high temperature series for the string tension of Z_2 lattice gauge theory has been calculated up to order k^{14} for $d = 3, 4$ and 5. Our results agree with theirs.

Fig. 1. The string tension α times the lattice spacing squared as a function of $\beta = 4/g^2$ for SU(2) lattice gauge theory in $d = 4$ dimensions. The solid line represents the result of cluster expansions up to 12th order [formula (11)]. The dots are results of a Monte Carlo calculation of Creutz [8]. The dashed lines are the lowest order high temperature curve and the weak coupling fit of ref. [8].

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