THE ON-SHELL QCD QUARK FORM FACTOR AND ITS DETERMINATION FROM TWO-PARTICLE CORRELATIONS IN e^+e^- ANNIHILATION

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Opposite side correlations in hadron jets produced in e^+e^- annihilation provide a possibility to measure directly the on-shell QCD quark form factor. Comparing recent PLUTO data with the leading log prediction yields 0.54 ± 0.1 GeV for the QCD parameter Λ.

At presently available e^+e^- energies ($\sqrt{q^2} \leqslant 35$ GeV) one finds essentially two classes of events within perturbative QCD: well separated 3-jet (and less probably 4- etc. jet) events, and acollinear 2-jet events. The former are due to very early large angle gluon emissions and occur at short distances, such that the asymptotic freedom prediction for the running quark-gluon coupling α_s can be used (N_f = number of flavors)

$$\alpha_s(q^2) = 12\pi/b \ln(q^2/\Lambda^2)\,, \quad b = 33 - 2N_f\,. \tag{1}$$

The acollinear 2-jet events reflect the deflection of the initially back-to-back quark-antiquark pair due to multiple small angle gluon emission: the highly off-shell quark (antiquark) radiates soft and collinear gluons until its invariant mass has reached $\sqrt{q_0^2} \geqslant \Lambda$.

The quark evolution from high invariant masses $\approx\sqrt{q^2}$ down to $\sqrt{q_0^2}$ has its clear manifestation in the well-known scaling violation of the fragmentation functions, $D_f^h(x, q^2)$, which is caused by the conversion of invariant mass into transverse momenta of radiated gluons, and therefore leads to a deviation from back-to-back direction by an accolinearity angle $\theta \approx 2p_\perp/\sqrt{q^2}$, where $p_\perp$ is the total radiated transverse momentum.

It has been pointed out by Dokshitzer, D'yakonov and Troyan [1] that the deflection of the quark–antiquark pair from the original back-to-back direction can be described by an effective quark form factor which can be directly measured in a suitably energy-weighted angular correlation in e^+e^- annihilation into hadrons.

In this note we shall discuss the QCD prediction for the on-shell electromagnetic form factor, where special emphasis is put on the correct normalization over the whole kinematical range, and shall compare with recent data [2].

Let us start with the zeroth order result in QCD for the 2-particle inclusive cross-section ($e^+e^- \to h_1 + h_2 + X$) for particles h_1 and h_2 belonging to opposite jets

$$\frac{d\sigma_0^{h_1h_2}}{dx_1 dx_2 d\cos\theta} = \frac{3}{2}\sigma_{\mu\mu} \times \sum_f Q_f^2 [D_{0f}^{h_1}(x_1) D_{0\bar{f}}^{h_2}(x_2) + D_{0\bar{f}}^{h_1}(x_1) D_{0f}^{h_2}(x_2)] \times \delta(\cos\theta - 1) \tag{2}$$

where $x_i = 2E_i/\sqrt{q^2}$ are the fractional energies carried by particle h_1 and h_2, $\pi - \theta$ is the angle between them ($\theta = 0$ corresponds to back-to-back production), and $\sigma_{\mu\mu} = 4\pi\alpha^2/3q^2$. The physical meaning of the bare fragmentation functions implies the sum rule

$$\sum_h \int_0^1 dx\, x D_{0f}^h(x) = 1\,. \tag{3}$$

Following DDT [1], we define an energy-weighted angular correlation ("acollinearity distribution")

$$\frac{dw}{d\cos\theta} = \frac{1}{\sigma_{tot}} \sum_{h_1h_2} \int_0^1 dx_1 x_1 \int_0^1 dx_2 x_2 \frac{d\sigma^{h_1h_2}}{dx_1 dx_2 d\cos\theta} \tag{4}$$

with normalization

$$\int_{-1}^{+1} \mathrm{d}\cos\theta \, \frac{\mathrm{d}w}{\mathrm{d}\cos\theta} = 1 \,. \tag{5}$$

From eq. (2) one obtains the zeroth order QCD result

$$\frac{\mathrm{d}w_0}{\mathrm{d}\cos\theta} = \delta(\cos\theta - 1) \,. \tag{6}$$

Eqs. (2) and (6) agree with the naive parton model, if one neglects the transverse momentum spread, i.e. assumes a transverse momentum distribution $f(\boldsymbol{p}_\perp) = \delta^{(2)}(\boldsymbol{p}_\perp)$. In a more realistic version of the parton model transverse momentum smearing is taken into account by a distribution with Gaussian momentum cut off

$$f_{\text{parton}}(\boldsymbol{p}_\perp) = \frac{1}{4\langle p_\perp\rangle^2} \, \mathrm{e}^{-\pi/4 \, p_\perp^2/\langle p_\perp\rangle^2} \,, \tag{7}$$

with a constant $\langle p_\perp\rangle \approx 0.3$ GeV. The x_1, x_2 dependence of the inclusive cross-section is still given by eq. (2), but the sharp angular correlation (6) gets smeared into

$$(\mathrm{d}w/\mathrm{d}\cos\theta)_{\text{parton}} = \tfrac{1}{2}A \, \mathrm{e}^{-A \, \sin^2(\theta/2)}, \quad A = \pi/\langle\delta\rangle^2 \,, \tag{8}$$

where the mean jet opening angle $\langle\delta\rangle$ is defined by $\langle\delta\rangle = \langle n\rangle \langle p_\perp\rangle/\sqrt{q^2}$. Since no evidence has been found for a deviation from the 2-jet picture in $\mathrm{e^+e^-}$ annihilation for c.m. energies below 10 GeV, the parton prediction (8) should describe adequately the data at $\sqrt{q^2} = 7.7$ and 9.4 GeV (see fig. 1).

At large acollinearity angles the angular correlation should reliably be given by the $\mathrm{O}(\alpha_s)$ contribution, i.e. by hard single gluon emission $\mathrm{e^+e^-} \to \mathrm{q}\,\bar{\mathrm{q}}\,\mathrm{g}$, which can be exactly calculated in QCD [3]. For small angles, however, the $\mathrm{O}(\alpha_s)$ contribution as well as all higher order contributions are infrared divergent due to both soft and collinear singularities. In order to obtain a physical sensible result, one therefore has to sum an infinite set of Feynman diagrams. Although the complete solution of this problem is not known, it has been shown by DDT [1] that a reliable summation can be done if the total radiated transverse momentum satisfies $\Lambda^2 \ll p_\perp^2 \ll q^2$. The latter region is characterized by large values of $\ln^2(q^2/p_\perp^2)$ and requires summation of all contributions of the type $(\alpha_s \ln^2(q^2/p_\perp^2))^n$; Employing the socalled "planar" gauge, the dominant

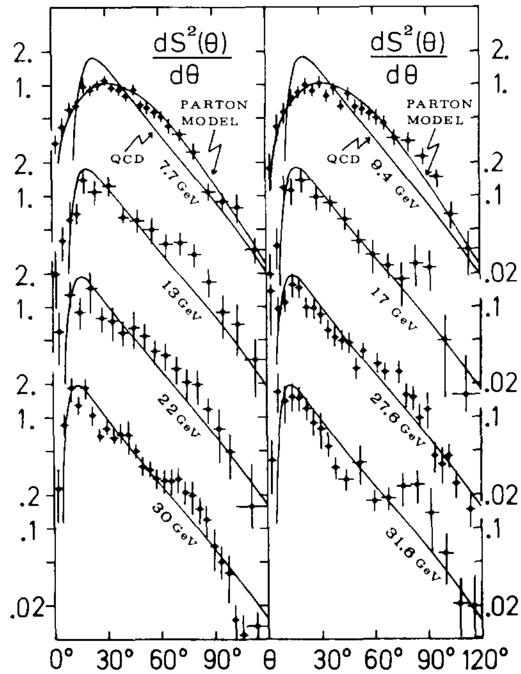

Fig. 1. The acollinearity distribution $\mathrm{d}w/\mathrm{d}\theta$ [2]. The QCD prediction is shown for $\Lambda = 0.5$ GeV. The parton model result (8) is shown with $A = 6.3$.

Feynman diagrams have a ladder like structure (no interference diagrams), and can be summed to all orders. As a result one obtains the "DDT formula"

$$\frac{\mathrm{d}\sigma^{\mathrm{h}_1\mathrm{h}_2}}{\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}\cos\theta} = \frac{3}{2}\sigma_{\mu\mu} \sum_{\mathrm{f}} Q_{\mathrm{f}}^2 \frac{\partial}{\partial\cos\theta}$$

$$\times \{[D_{\mathrm{f}}^{\mathrm{h}_1}(x_1,p_\perp^2)D_{\mathrm{f}}^{\mathrm{h}_2}(x_2,p_\perp^2) + D_{\mathrm{f}}^{\mathrm{h}_2}(x_1,p_\perp^2)D_{\mathrm{f}}^{\mathrm{h}_2}(x_2,p_\perp^2)]\,T^2(q^2,p_\perp^2)\} \,. \tag{9}$$

Comparison of this expression with the zeroth order result (2) shows clearly two effects reflecting the quark evolution according to QCD: the bare fragmentation functions have been replaced by the q^2 dependent scale breaking fragmentation functions, $D_{\mathrm{of}}^{\mathrm{h}}(x) \to D_{\mathrm{f}}^{\mathrm{h}}(x, q^2)$, and the fragmentation functions have been multiplied by the square of a QCD form factor $T(q^2, p_\perp^2)$. Since the q^2 dependent fragmentation functions satisfy still the sum rule (3), one obtains immediately from eq. (9) the angular correlation for real gluon emission ($p_\perp, \theta \neq 0$)

$$(\mathrm{d}w/\mathrm{d}\cos\theta)_{\mathrm{real}} = \partial T^2(q^2, p_\perp^2)/\partial\cos\theta\,. \qquad (10)$$

The uncalculable fragmentation functions have dropped out completely, and one is left with a direct measurement of the QCD form factor T. Eqs.(9), (10) have been derived in the region $\Lambda^2 \ll p_\perp^2 \ll q^2$. If one *defines* the form factor T in such a way that eq. (10) holds over the whole kinematical range $0 \leqslant p_\perp^2 \leqslant q^2$, the normalization (5) tells us immediately that $w(p_\perp^2) = T^2(q^2, p_\perp^2)$ should be interpreted as the probability that the total radiated transverse momentum is less than $p_\perp$ with $0 \leqslant w(p_\perp^2) \leqslant w(q^2) = 1$ [‡1].

In the region $\Lambda^2 \ll p_\perp^2 \ll q^2$ DDT obtained ($C_F = \frac{4}{3}$):

$$T(q^2, p_\perp^2) \approx \int_1^2 \mathrm{d}z \exp\left[-\frac{\alpha_s(p_\perp^2)}{2\pi} C_F \ln^2(p_\perp^2/q^2)\cdot\frac{1}{z^2}\right]$$

$$= 1 - \frac{\alpha_s(p_\perp^2)}{4\pi} C_F \ln^2\left(\frac{p_\perp^2}{q^2}\right) + \frac{7}{6}$$

$$+ \frac{7}{6}\frac{1}{2!}\left[\frac{\alpha_s(p_\perp^2)}{4\pi} C_F \ln^2\left(\frac{p_\perp^2}{q^2}\right)\right]^2 \mp \dots\,. \qquad (11)$$

In view of the well-known exponentiation of the leading log contributions in QED, which has also been found in QCD [4], the non-exponential form (11) is somewhat surprising. One has to remember, however, that (11) has been derived after several approximations from a more complete expression replacing eq. (9) [‡2]. Moreover, it is seen from the perturbation expansion given in (11) that the numerical deviation from the simple exponential

$$S(q^2, p_\perp^2) \equiv \exp[-\{\alpha_s(p_\perp^2)/4\pi\} C_F \ln^2(p_\perp^2/q^2)] \qquad (12)$$

is irrelevant for all physical applications [‡3], and it is therefore tempting to conjecture [5,6] that the *analytically* correct angular correlation is given by eq. (10) with T being replaced by the exponential form factor S, eq. (12). It is interesting to note, that the form factor S is identical to the QCD electromagnetic form factor for an on-shell massless quark ($p^2 = 0$), if $p_\perp$ is replaced by a small regularization mass μ for the gluons: $F_{\text{on-shell}}(q^2) = S(q^2, \mu^2)$.

The appearance of the on-shell form factor in eq. (10) is not accidental but rather is required by the infrared finiteness of the integrated acollinearity distribution. In an integration over $\mathrm{d}w/\mathrm{d}\cos\theta$ one has to add to eq. (10) the contributions from virtual gluon exchanges, which are infrared divergent in finite order in α_s. But the sum of all virtual gluon corrections is nothing else than the total cross section for the exclusive channel $e^+e^- \to q\,\bar{q}$, which by definition is given by the square of the on-shell quark electromagnetic form factor. Thus we have

$$(\mathrm{d}w/\mathrm{d}\cos\theta)_{\mathrm{virtual}} = \delta(\cos\theta - 1) F^2_{\text{on-shell}}(q^2). \qquad (13)$$

Adding the real and virtual contributions and integrating we obtain [‡4]

$$\int_{-1}^{+1} \mathrm{d}\cos\theta \left[\left(\frac{\mathrm{d}w}{\mathrm{d}\cos\theta}\right)_{\mathrm{real}} + \left(\frac{\mathrm{d}w}{\mathrm{d}\cos\theta}\right)_{\mathrm{virtual}}\right]$$

$$= F^2_{\text{on-shell}}(q^2) + S^2(q^2, q^2) - S^2(q^2, \mu^2) = 1 \qquad (14)$$

(since $S(q^2, q^2) = 1$), and one observes that the infrared contributions from virtual exchanges have been exactly cancelled by the divergent contributions from real emissions. This proofs that the simple exponential (12) is the correct QCD expression to be used in the acollinearity distribution (10) in the leading log region

For a study of the θ dependence of S one needs the correct relation between the total transverse momentum $p_\perp$ and the acollinearity angle θ. DDT used $p_\perp^2 = q^2\tan^2(\theta/2)$, which restricts the kinematically allowed region to $0 \leqslant \theta \leqslant \pi/2$ and leads to a vanishing acollinearity distribution at 90°. Since the experimentally accessible region is $0 \leqslant \theta \leqslant \pi$, this relation cannot be correct, and we shall use instead

$$p_\perp^2 = q^2\sin^2(\theta/2)\,. \qquad (15)$$

Support for the correctness of this relation comes from two sources. The first is the naive parton model, where relation (15) is found to hold (this is the reason for the $\sin^2(\theta/2)$ dependence in (8)). The second source is the exact $O(\alpha_s)$ result for S [3,7]

[‡1] For the zeroth order result (6) one obtains $w_0 = 1$ for $p_\perp > 0$ and $w_0 = 0$ for $p_\perp = 0$, while the parton model (8) gives $w_{\text{parton}} = (1 - \exp(-A\,p_\perp^2/q^2))$ with $p_\perp^2 = q^2\sin^2(\theta/2)$.

[‡2] See the first paper cited in [1].

[‡3] E.G. at $\sqrt{q^2} = 30$ GeV the difference between S and T is less than 1% for $\theta > 20°$ and less than 10% for $10° < \theta < 20°$.

[‡4] Note, that in the integration the explicit form of the leading log result (12) is required only near $p_\perp = 0$, where use of the LLA is justified, if the real gluons have a fictitious mass μ, yielding $S(q^2, p_\perp^2 + \mu^2)$ in eq. (10).

$$S(q^2, p_\perp^2) = 1 - C_F \frac{\alpha_s}{4\pi}[\ln^2(\sin^2(\theta/2)) + 3\ln(\sin^2(\theta/2)) + 10.2] + O(\alpha_s^2) , \qquad (16)$$

where we have dropped terms which vanish in the limit of small acollinearity. A comparison of eq. (16) with eq. (12) leads immediately to the relation (15).

It remains to discuss the appearance of the running coupling $\alpha_s(p_\perp^2)$ in the exponential form factor (12). Since the acollinearity distribution is characterized by two large invariants ($p_\perp^2$ and q^2), the determination of the correct argument requires a calculation beyond the leading logarithm approximation (LLA), i.e. a summation of single logs. Even though the final answer to this question is not yet known, there are strong indications from the work of DDT and others [8], that $\alpha_s(p_\perp^2)$ is the correct choice.

Let us discuss the kinematic range, in which the LLA may be considered to be reasonable. Since the asymptotic freedom prediction (1) for $\alpha_s(p_\perp^2)$ will be used, we obtain the limitation $\theta \gg 2\Lambda/\sqrt{q^2}$. However, the realistic restriction at small acollinearity angles comes from consideration of confinement effects, which we estimate using the parton model formula (8). We are then led to the condition $\theta \gg \langle\delta\rangle = \langle n\rangle\langle p_\perp\rangle/\sqrt{q^2} \approx 2(\langle p_\perp\rangle/\langle x\rangle)/\sqrt{q^2}$, i.e. $\theta \gg 18°, 7°$ at $\sqrt{q^2} = 13, 31.6$ GeV, respectively, if we use $\langle p_\perp\rangle = 0.3$ GeV, $\langle x\rangle = 0.15$.

Concerning large acollineatiry angles one has to remember that single logs have been neglected in the LLA. In order to ensure the dominance of the double log terms, one therefore requires $\ln^2(q^2/p_\perp^2) \gg \ln(q^2/p_\perp^2)$, which yields the constraint $\sin(\theta/2) \ll 1/\sqrt{e}$, i.e. $\theta \ll 75°$. While this angle is large enough to provide a useful comparison with experimental data, we think that this limit is not serious for the following reasons: 1) in the exact $O(\alpha_s)$ result, eq. (16), the single log term is very large, already at 20°, and opposite in sign compared to the leading log term, and seems therefore to invalidate the LLA. If one includes, however, the finite term, it is seen that the non-leading terms cancel almost completely [‡5], and that the double log term constitutes a good approximation. If this cancellation occurs also in higher orders, the leading log result would be much better than comparison between leading and non-leading terms would indicate; ii) the QCD form factor (12) is correctly normalized over the whole kinematical range $0 \leqslant \theta \leqslant \pi$ ($S(q^2, q^2) = 1$). Therefore it is not unlikely that it constitutes a good approximation also at large acollinearity angles. Already at 40° S^2 has reached a value of about 0.75, and not much room is left for a variation between 40° and 180°.

[‡5] Actually, the sum of the non-leading terms in eq. (16) has a zero at $\theta = 21°$.

The data at the lowest energies, $\sqrt{q^2} = 7.7$ and 9.4 GeV, do not agree with the QCD prediction (fig. 1), since at these energies the jet fragmentation is governed by hadronization and not by perturbation theory. Instead the data are well described by the parton formula (8). At the higher energies, however, the data agree with QCD as well as could possibly be expected. In particular we observe that the maximum of $dw/d\theta$ is correctly predicted by QCD. Since agreement is found over nearly the whole angular interval (i.e. $10° < \theta < 120°$), this confirms our conjecture about the validity of the LLA. It is worthwhile to mention that the good overall agreement at large angles depends critically on the relation (15) and the choice $\alpha_s(p_\perp^2)$. If we would have used $\alpha_s(q^2)$ instead of $\alpha_s(p_\perp^2)$ in the form factor S, the result would have drastically changed. E.g. at 31.6 GeV the maximum of $dw/d\theta$ would occur at $\theta = 3°$ (for $\Lambda = 0.5$ GeV), which is certainly excluded by the data. To shift the maximum to the experimentally observed position, requires an unreasonably large value of Λ, $\Lambda = 2.5$ GeV.

Actually the data displayed in fig. 1 show some fluctuations around the smooth theoretical curve, preventing us from a precise determination of the scale parameter Λ from a fit to $dw/d\theta$.

The most optimal observable for a quantitative QCD test is the square of the form factor $S(\theta)$, which can be obtained from the experimental data on $dw/d\theta$ by a straightforward integration from zero angle up to θ. Since the quantity $w(\theta) = S^2(\theta)$ has the simple physical interpretation as the total probability that the acollinearity angle is less than θ, it should be a smooth, monotonously increasing function of θ, in which possible statistical fluctuations are washed out. If we remember that the measurement of $dw/d\theta$ requires an energy-weighting (eq. (4)), it can be expected that $S^2(\theta)$ provides a suitably inclusive observable for which a save QCD prediction can be made.

It is seen (fig. 2) that theory and experiment are within experimental errors well compatible with each

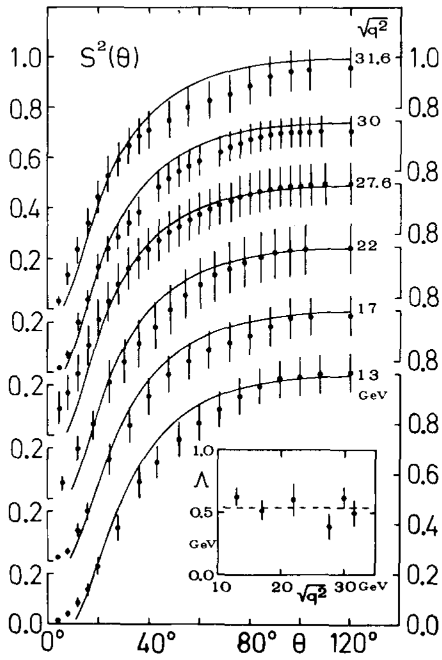

Fig. 2. The integrated acollinearity distribution and the QCD prediction (12) for $\Lambda = 0.5$ GeV. Also shown is the QCD scale parameter Λ as obtained from best fits to the data points using cut offs at $\theta = 18°$ at 13 GeV and $\theta = 14°$ at 17, 22, 27.6, 30 and 31.6 GeV.

other for acollinearity angles outside the confinement region. The good agreement allows us to determine at each energy the only free parameter, the QCD scale Λ, from a best χ^2-fit to the integrated data. Using a cut off at small angles to exclude hadronization effects, we obtain the Λ values shown in fig. 2. The fact that the Λ values obtained at different energies are well compatible with a common value, $\Lambda = 0.54 \pm 0.1$ GeV, is a direct test of the asymptotic freedom behavior of the QCD quark-gluon coupling (1). Note that similar values for Λ have been found in other reactions [9].

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