A NULL RESULT IN MASSLESS QCD: BEAM–EVENT ASYMMETRY IN $e^+e^- \to q\bar{q}g$ WITH LONGITUDINALLY POLARIZED BEAMS

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The imaginary part of the amplitude $e^+e^- \to q\bar{q}g$ is calculated in the one-loop approximation. It is found to be directly proportional to the Born term contribution. This leads to the vanishing of the beam–event asymmetry in $e^+e^- \to q\bar{q}g$ with longitudinally polarized beams contrary to the result in $e^+e^- \to 1^{--} \to ggg$.

It is well known that the spin-averaged cross section $e^+e^- \to q\bar{q}g$ involves 4 structure functions H_1–H_4 when using unpolarized or transversely polarized beams [1,2]. These can be measured by an angular correlation analysis between the hadron plane and the beam direction [1,2]. In the case of longitudinally polarized beams there is an additional contribution from a 5th structure function H_5 which can be isolated by determining a beam–event asymmetry to be defined later. Theoretical interest in a measurement of H_5 derives from the fact that H_5 obtains contributions only from the imaginary part of the amplitude $e^+e^- \to q\bar{q}g$, involving, to lowest order, the imaginary part of 1-loop contributions, or, in a different language, final state interaction effects between the final state quanta. Thus, H_5 will be contributed to in $O(\alpha_s^2)$, whereas H_1–H_4 are of $O(\alpha_s)$. Since the non-abelian character of QCD, viz. the gluon self-coupling, manifests itself only at the $O(\alpha_s^2)$ level, measurement of H_5 and comparison to the QCD prediction is, in principle, an important test of the non-abelian character of QCD.

In the corresponding case $e^+e^- \to 1^{--} \to ggg$ the final state interactions involve triple and quartic gluon couplings. These have been calculated by de Rújula et al. [3]. Although nonzero, the final state interaction is so small that the detection of the beam–event asymmetry which it produces is virtually impossible [3].

In the case of $e^+e^- \to q\bar{q}g$ we find that H_5 and therewith the final state interaction effect is identically zero. It is the purpose of this note to expose this surprising result.

We start by expressing the differential cross section for $e^+e^- \to q(p_1)\bar{q}(p_2)g(p_3)$ as a product of the lepton tensor $L_{\mu\nu}$ and the hadron tensor $H_{\mu\nu}$,

$$d\sigma = L^{\mu\nu}H_{\mu\nu}\,dP\,, \tag{1}$$

where dP denotes the phase space element. It is convenient to split up (1) into contributions from the symmetric and antisymmetric lepton and hadron tensor combinations

$$d\sigma = (L^{\{\mu\nu\}}H_{\{\mu\nu\}} + L^{[\mu\nu]}H_{[\mu\nu]})\,dP\,. \tag{2}$$

In the pure electromagnetic case the antisymmetric lepton tensor can be seen to obtain contributions only from longitudinally polarized beams. One has

$$L_{[\mu\nu]} = (2\mathrm{i}m/q^2)\epsilon_{\mu\nu\rho\sigma}(s^{(+)} + s^{(-)})^\rho q^\sigma \,, \tag{3}$$

where $q = p^{(+)} + p^{(-)}$ is the total four-momentum and $s^{(+)}$ and $s^{(-)}$ are covariant polarization vectors of the positron and electron beam, respectively. In the center-of-mass system it has the simple form

$$L_{[ij]} = \mathrm{i}\epsilon_{ijk}v_k(\xi_\parallel^{(+)} + \xi_\parallel^{(-)}) \,, \tag{4}$$

with $\xi_\parallel^{(\pm)}$ denoting the degree of longitudinal polarization of the positron and electron beams and $\boldsymbol{v}$ is a unit vector in the electron beam direction.

We would like to mention that an antisymmetric $L_{[\mu\nu]}$ is also obtained for unpolarized beams when γ–Z^0 interference effects are present. In this case one has

$$L_{[ij]} = \mathrm{i}\epsilon_{ijk}v_k \,, \tag{5}$$

where the effective coupling now is

$$2Q_\mathrm{f} a v_\mathrm{f}\, \mathrm{Re}\, \beta + 2va(v_\mathrm{f}^2 + a_\mathrm{f}^2)|\beta|^2 \,, \tag{6}$$

instead of Q_f^2 when multiplying with the antisymmetric hadron tensor. Q_f is the electric charge of the quark, and the neutral current couplings in (6) are defined as in ref. [4].

Expanding the hadron tensor in a gauge invariant basis one readily identifies 4 symmetric and 1 antisymmetric contribution:

$$H_{\mu\nu} = (q^2 g_{\mu\nu} - q_\mu q_\nu)H_1 + \hat{p}_{1\mu}\hat{p}_{1\nu}H_2 + \hat{p}_{2\mu}\hat{p}_{2\nu}H_3 + (\hat{p}_{1\mu}\hat{p}_{2\nu} + \hat{p}_{1\nu}\hat{p}_{2\mu})H_4 + (\hat{p}_{1\mu}\hat{p}_{2\nu} - \hat{p}_{1\nu}\hat{p}_{2\mu})H_5 \,, \tag{7}$$

where $\hat{p}_\mu = q^2 p_\mu - p\cdot q q_\mu$.

Since one has $H_{\mu\nu} = H_{\nu\mu}^*$ from the hermiticity of the electromagnetic current, one concludes that H_1–H_4 are real and H_5 is imaginary. Thus, H_5 is the only hadronic structure function that contributes to $L_{[\mu\nu]}H^{[\mu\nu]}$.

The contribution of the symmetric part of the lepton tensor which contains only even powers of the longitudinal polarization (including of course the zeroth power) can be eliminated by defining suitable measurements. One defines the asymmetry

$$A = \frac{(\mathrm{d}\sigma/\mathrm{d}\cos\eta)(\cos\eta = |\cos\eta|) - (\mathrm{d}\sigma/\mathrm{d}\cos\eta)(\cos\eta = -|\cos\eta|)}{(\mathrm{d}\sigma/\mathrm{d}\cos\eta)(\cos\eta = |\cos\eta|) + (\mathrm{d}\sigma/\mathrm{d}\cos\eta)(\cos\eta = -|\cos\eta|)} \,, \tag{8}$$

where η is the angle between the beam axis and the normal of the (oriented) event plane [3]. A is directly proportional to H_5. If polarization reversal is feasible one can define asymmetry cross sections

$$\mathrm{d}\sigma_A = \mathrm{d}\sigma(\xi^{(\pm)} = |\xi^{(\pm)}|) - \mathrm{d}\sigma(\xi^{(\pm)} = -|\xi^{(\pm)}|) \,, \tag{9}$$

which again are proportional to H_5.

Eq. (8) corresponds to measuring a correlation $\langle s\,[\boldsymbol{p}_1 \times \boldsymbol{p}_2]\rangle$ where s is the longitudinal spin vector and $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are two of the three hadronic momenta. Since the corresponding operator $s[\boldsymbol{p}_1 \times \boldsymbol{p}_2]$ is odd under time reversal it is well known that nonvanishing contributions arise only in the presence of final state interaction.

Returning to the antisymmetric part of eq. (2) one calculates

$$2(\hat{p}_{1\mu}\hat{p}_{2\nu} - \hat{p}_{1\nu}\hat{p}_{2\mu})H_5 = \sum_{\lambda_1\lambda_2\lambda_3}\{[\bar{u}(\lambda_1)T_{\mu\rho}v(\lambda_2)\epsilon^\rho(\lambda_3)]\,[\bar{u}(\lambda_1)T_{\nu\sigma}v(\lambda_2)\epsilon^\sigma(\lambda_3)]^+ - (\mu \leftrightarrow \nu)\} \,, \tag{10}$$

where λ_1, λ_2 and λ_3 are the helicities of the quark, antiquark and gluon and $T_{\mu\rho}$ denotes the amplitude for $e^+e^- \to q\bar{q}g$. Performing the spin sums in (10) one obtains

$$2(\hat{p}_{1\mu}\hat{p}_{2\nu}-\hat{p}_{1\nu}\hat{p}_{2\mu})H_5 = \mathrm{Tr}\,(\not{p}_1 T_{\mu\rho}\not{p}_2 \bar{T}_{\nu\rho} - \not{p}_1 T_{\nu\rho}\not{p}_2 \bar{T}_{\mu\rho}) \tag{11a}$$

$$= \mathrm{Tr}\,(\not{p}_1 T_{\mu\rho}\not{p}_2 \bar{T}_{\nu\rho} - \not{p}_1 \tilde{\tilde{\bar{T}}}_{\mu\rho}\not{p}_2 \tilde{\bar{T}}_{\nu\rho}) \,, \tag{11b}$$

where $\bar{T}_{\mu\rho} = \gamma_0 T^{+}_{\mu\rho}\gamma_0$ and the ($\sim$) operation denotes reversal of the order of γ-matrices. The more convenient form (11b) was obtained by exploiting the fact that the trace of a product of γ-matrices is invariant under cyclical *and* anticyclical permutations of the γ-matrices.

Expanding $T_{\mu\rho}$ in a standard set of (non-gauge invariant) covariants one has [‡1] ($P = p_1 - p_2$)

$$\begin{aligned} T_{\mu\rho} &= (B_1 q_\mu + B_2 P_\mu + B_3 p_{3\mu})\gamma_\rho + \gamma_\mu(B_4 q_\rho + B_5 P_\rho) + B_6 \gamma_\mu\gamma_\rho \not{q} + B_7 g_{\mu\rho}\not{q} \\ &+ (C_1 q_\mu q_\rho + C_2 q_\mu P_\rho + C_3 P_\mu q_\rho + C_4 P_\mu P_\rho + C_5 p_{3\mu} q_\rho + C_6 p_{3\mu} P_\rho)\not{q} \,. \end{aligned} \tag{12}$$

Note that the expansion of $T_{\mu\rho}$ contains only odd powers of γ-matrices, according to the masslessness of the fermions in our approach. Since $\bar{T}_{\mu\rho} = \tilde{\bar{T}}_{\mu\rho}$ for the Born term, it is clear that the two terms in (11b) cancel and the Born term has no contribution to H_5.

There are some further observations we want to make about the expansion (12). We have ordered the invariants according to their dimension: there are seven B_i with dimension $[q^2]^{-1}$ and six C_i with lower dimension $[q^2]^{-2}$. Note that the Born term contributes only to the invariants B_i since the covariance structure of the Born term is determined from the Feynman diagrams as $\gamma_\rho(\not{p}_1 + \not{p}_3)\gamma_\mu$ and $\gamma_\mu(\not{p}_2 + \not{p}_3)\gamma_\rho$ which populate only the covariants of dimension $[q^2]^{1/2}$.

From gauge invariance we have $q_\mu T_{\mu\rho} = p_{3\rho} T_{\mu\rho} = 0$ which gives 7 constraints on the 13 amplitudes in (12). There are thus 6 independent amplitudes which agrees with the number of independent amplitudes using helicity counting [6]. As stated before, the Born term only populates the amplitudes B_i. Setting the $C_i = 0$ one can solve the homogeneous equation $M^B_{ij} B_j = 0$ (M^B_{ij} is the 7×7 constraint matrix which can be seen to have rank 6) since $\det M^B_{ij} = 0$. One finds a unique solution which by necessity has the structure of the Born term. Thus, the gauge constraints are so restrictive in the case of $e^+e^- \to q\bar{q}g$ as to fix the Born term structure already uniquely.

The calculation of the imaginary parts of the 1-loop contributions fig. 1 have been done using dimensional regularization [7] [‡2]. The result is surprisingly that all $\mathrm{Im}\, C_i \equiv 0$. This is true also for the gauge invariant subset of graphs corresponding to an QED type abelian theory with no gluon self-coupling.

Since the imaginary parts of the 1-loop contributions populate only the higher dimensional invariants B_i the

[‡1] There are only 12 *independent* covariants in (12). An identity linking the 13 covariants can be derived as in ref. [5].

[‡2] The calculations were done in the Feynman gauge. In this gauge the result $\mathrm{Im}\, C_i \equiv 0$ holds true for each of the 11 graphs of fig. 1 separately. For the self-energy and vertex corrections this is not so difficult to understand from the fact that there is not enough structure in the diagrams to populate the covariants of higher dimensions. For the box diagrams the vanishing of the $\mathrm{Im}\, C_i$ results from the cancellation of a large number of contributing terms. A systematic understanding of these cancellations is still lacking.

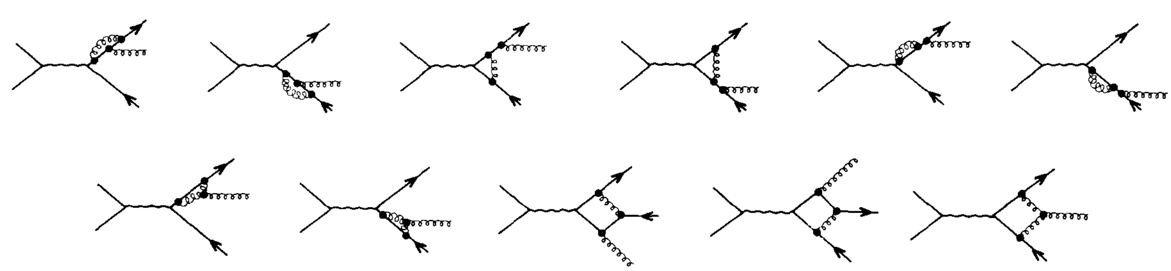

Fig. 1. One-loop contributions to $e^+e^- \to q\bar{q}g$.

same reasoning as above implies that these *must* have the same structure as the Born term.

Thus one has

$$\mathrm{Im}\, T_{\mu\rho}^{1\text{-loop}} = a\alpha_s T_{\mu\rho}^{\mathrm{Born}} , \tag{13}$$

where $\alpha_s a$ denotes the factor of proportionality of the two contributions. Hence the imaginary parts of the 1-loop contributions are directly proportional to the Born term. Then one immediately concludes from eq. (11) that H_5 will be zero, i.e. there is no observable final state interaction effect in this order. The same holds true, of course, also for the QED type abelian (massless!) theory mentioned above.

This can also be stated differently. By explicit calculation one finds from folding the Born term with an amplitude having the general structure of eq. (12) that the r.h.s. of (11) depends only on Im C_1, Im C_2, Im C_3, Im C_4 and Im C_5. Since these are absent in the 1-loop imaginary terms, one reaches the same conclusion.

We would like to end this note with some concluding remarks. The vanishing of the final state interaction effect in $e^+e^- \to q\bar{q}g$ appears to result from the fact that the gauge conditions are very restrictive in the massless quark case. For massive quarks the number of amplitudes in (12) is doubled and the gauge structure no longer so restrictive In fact, one finds a nonvanishing H_5 in this case [8]. Similarly, the gauge structure for $e^+e^- \to 1^{--} \to ggg$ is quite different and therefore a nonvanishing H_5 comes as no surprise [3]. It would be interesting to find out whether the present result holds also in higher orders. Higher orders could change the present conclusion by generating either more structure in the real or in the imaginary parts of $e^+e^- \to q\bar{q}g$. It would also be interesting to find out whether the absence of a final state interaction effect holds true also in the scattering case at space-like q^2.

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