TEST OF QED IN e⁺e⁻ ANNIHILATION AT ENERGIES BETWEEN 12 AND 31.6 GeV

TASSO Collaboration

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We have measured the reactions $e^+e^- \rightarrow e^+e^-$, $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \gamma\gamma$ at c.m. energies between 12 and 31.6 GeV. Excellent agreement with the predictions of QED has been found, resulting in cut off parameters $\Lambda_+ > 112$ GeV and $\Lambda_- > 139$ GeV for the first process and $\Lambda_+ > 34$ GeV and $\Lambda_- > 42$ GeV (95% c.l.) for the last one. A limit on the Weinberg angle of $\sin^2\theta_W < 0.55$ (95% c.l.) has been obtained.

High energy e^+e^- annihilation into lepton pairs tests the validity of quantum electrodynamics (QED) down to very small distances and probes the pointlike structure of leptons. Furthermore, at momentum transfers as large as 1000 GeV², accessible at PETRA, modifications of the pure electromagnetic processes due to the interference of the photon with the weak neutral gauge boson Z⁰ are expected to become observable.

We report on measurements of the reactions $e^+e^ \rightarrow e^+e^-$, $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \gamma\gamma$. The experiment was performed with the TASSO detector at the $e^+e^$ storage ring PETRA. Data were taken at c m. energies W of 12, 13, 17, 27.4, 27.7, 30.0 and 31.6 GeV and between 29.9 and 31.46 GeV in steps of 20 MeV.

The detector and the reconstruction procedure for charged particles have been described previously [1,2]. For the analysis of charged tracks in the central detector a momentum resolution of $\sigma_p/p = 0.02 p$ (p in GeV/c) and angular resolutions of $\sigma_{\phi} = 4$ mrad in azimuth (perpendicular to the beam) and $\sigma_{\theta} = 7$ mrad in polar angle were achieved. In part of the experiment muons were positively identified by a muon detection system [2] which covers a solid angle of 45% of 4π and has a muon detection efficiency greater than 99%. For the analysis of charged pair events a pair trigger was used, which demanded 2 or 3 charged tracks to be coplanar within 27°, as measured in the plane perpendicular to the beam. For each track the minimum momentum transverse to the beam was 0.20 GeV/c at W = 12 GeV and 0.32 GeV/c at $W \ge 13$ GeV. The pair trigger covered 2π in azimuth and a polar angle θ between 35° and 145°. This trigger was run in parallel with a multitrack trigger. The reaction $e^+e^- \rightarrow \gamma\gamma$ was studied with lead-scintillator shower counters described previously [2]. They cover a solid angle of 52% of 4π and were triggered for this reaction by requiring more than 2 GeV electromagnetic energy deposited in each of two shower counter situated opposite in azimuth.

The luminosity was measured by small angle Bhabha scattering around $\theta \approx 3^{\circ}$ with a system of 8 identical scintillation and shower counter telescopes. The symmetry of the set-up with respect to the interaction

point provides a counting rate almost independent of the position of the beam in the interaction region. Each arm consists of an acceptance scintillation counter, a somewhat larger coincidence scintillation counter and a lead glass shower counter. The Bhabha event rate was counted by demanding a coincidence between an acceptance counter in one arm and a coincidence counter in the opposite arm with a minimum deposited energy in both shower counters. The geometrical acceptance of the system is 1.4 msterad. Radiative corrections up to order α^3 were taken into account [3]. The main sources of systematic uncertainties were due to deflections in the magnetic fields of the central and compensating coils (especially at low beam energies) and backscattering of particles from the shower counters. The systematic error was estimated to be $\pm 4\%$.

Event selection for charged pairs. The analysis procedure for Bhabha and μ pair events was almost identical. The selection of pair events required:

(i) two oppositely charged tracks $^{\pm 1}$ with an acollinearity angle in space of less than 10° ;

(ii) the momentum of each track to be larger than 1.0(2.5) GeV/c for $W \le 17 (\ge 27.4) \text{ GeV}$;

(iii) each track to lie within $\Delta z = 10$ cm and the vertex of the two tracks to lie within $\Delta z = 6$ cm of the nominal interaction point, where z is the coordinate along the beam direction;

(iv) the difference between the time of flight of two particles, as measured with the inner time of flight counters, to be less than 3 ns and the velocity of each particle to be within $0.5 < \beta < 1.5$.

These cuts effectively removed all background from cosmic rays, beam gas scattering, hadronic processes and 2γ processes, e.g. $e^+e^- \rightarrow e^+e^-e^+e^-$ and $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$. The residual contamination from these sources was less than 0.5%. The pair event sam-

^{‡1} Due to the finite resolution in curvature measurement 0.1% (0.6%) of the tracks at $W \le 17$ GeV ($W \ge 27.4$ GeV) had the wrong charge assignment. The inclusion of events with the same charges had no effect on the shape of angular distributions.

ple selected in this way consisted of 94% Bhabha scattering events, 5% μ pairs and a less than 0.8% contamination by $e^+e^- \rightarrow \tau^+\tau^-$ where both τ 's had decayed into 1 charged particle + neutrals.

 $e^+e^- \rightarrow e^+e^-$. The Bhabha event sample was obtained by subtracting from the pair events the contamination of μ pairs and τ pairs. The subtraction was done with the help of the direct measurements of μ pair production (see below) and of τ pair production [2] in this experiment.

The determination of the cross section required corrections, typically of the order of 20%, due to energy losses and showering of the electrons in a total of 0.13 radiation lengths in the beam pipe and other material in front of the track chambers. The corrections and the influence of the cuts described above on the acceptance were determined by Monte Carlo methods. Bhabha events were generated [4] including the effects of hard and soft photons and internal radiative corrections [3]. The events were tracked through the detector simulating electromagnetic interactions with the shower program EGS [5] and then analyzed in the same way as the data. The Monte Carlo calculations were checked with about 10% of the experimental data where secondary tracks from showere were not excluded.

To compare directly to the first order QED cross section the data were corrected for acceptance, efficiencies of the detector and track reconstruction programs and radiative effects. The radiative corrections included loops over lepton pairs, vector meson resonances and the hadronic vacuum polarization [3]. The systematic uncertainty in the determination of the large angle Bhabha cross section was estimated to be less than $\pm 4\%$.

The measured acollinearity angle distribution is shown in fig. 1. The solid curve represents the result of a Monte Carlo simulation as described above. The distributions are in good agreement. Note that the region of small acollinearity angles ($\zeta < 2^{\circ}$) is dominated by the resolution, whereas the large angles are accounted for by radiative effects. The corrected differential cross sections, $d\sigma/d\Omega$, weighted by $s = W^2$, are plotted in fig. 2 as a function of $\cos \theta$. The scattering angle θ was measured between the positive particles of the initial and final state. The QED curve gives a good fit to the data at all energies. This shows that the



Fig. 1. The measured acollinearity angle distribution for Bhabha scattering events at W = 30.7 GeV compared with a Monte Carlo simulation (curve) including radiative corrections up to order α^3 .

observed ratio between small angle Bhabha scattering (for which the mass squared of the virtual photon is small $|q^2| \leq 1 \text{ GeV}^2$) and large angle scattering ($|q^2|$ = 450 GeV² at cos $\theta = 0$, W = 30 GeV) is in agreement with QED. A comparison of the cross sections for wide angle Bhabha scattering integrated over $|\cos \theta| < 0.8$ with the QED cross sections is given in table 1.

The corrected differential cross sections were compared to the modified QED cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left[\frac{10 + 4x + 2x^2}{(1 - x)^2} |F_S|^2 - \frac{2(1 + x)^2}{(1 - x)} \operatorname{Re}(F_S |F_T^*) + (1 + x^2) |F_T|^2 \right],$$

where $x = \cos \theta$. Any deviation from pure QED is expressed by modifying the spacelike and timelike parts of the cross section through form factors F_S and F_T . If time reversal invariance holds both form factors can



Fig. 2. The differential cross sections $s d\sigma/d\Omega$ for Bhabha scattering at different energies. The curve is the QED prediction. The insert shows the corrections to the measured Bhabha cross section due to electromagnetic effects up to order α^3 (full curve, labelled em) and the electroweak interference of the Weinberg–Salam model with $\sin^2\theta_W = 0.23$ (dash-dotted curve, labeled ewi).

be taken to be real. It is customary to parametrize them as follows [6]:

$$F_{\rm S}(q^2) = 1 \mp \frac{q^2}{q^2 - \Lambda_{\rm S^{\pm}}^2}, \quad F_{\rm T}(s) = 1 \mp \frac{s}{s - \Lambda_{\rm T}^2},$$

with $q^2 = -\frac{1}{2}s(1-x)$.

Table 1

Integrated luminosities, event rates and ratios of the measured cross section to the QED cross section for Bhabha scattering, μ pair and γ pair production. The errors given are statistical only.

W	∫£dt	$e^+e^- \rightarrow e^+e^-$		$e^+e^- \rightarrow \mu^+\mu^-$		$e^+e^- \rightarrow \gamma\gamma$	
(GeV)	(nb ⁻¹)	no. of events	o ^{meas} /o ^{QED}	no. of events	$\sigma^{\text{meas}}/\sigma^{\text{QED}}$	no. of events	$\sigma^{\text{meas}}/\sigma^{\text{QED}}$
12	96	655	1.00 ± 0.04	15	0.97 ± 0.25	16	0.99 ± 0.27
13	31	144	0.96 ± 0.09		-	_	-
17	39	118	1.00 ± 0.10	-	-	-	-
27.5	330	360	1.04 ± 0.06	_	-		-
30.7	2438	2580	0.99 ± 0.02	70	1.08 ± 0.13	48 a)	0.94 ± 0.18

a) Only part of the data sample was used.

The signs of the cut-off parameters Λ express different metrics of QED modified by a propagator. In practice one fits $\lambda = 1/\Lambda^2$ leaving the sign free. Limits on Λ_+ and Λ_- are then derived from the corresponding positive and negative bounds on λ . The data were fitted to the modified QED cross section by treating the λ parameters as free and allowing the normalization to vary within the estimated systematic uncertainty. Since we are able to determine the charge of the scattered leptons, separate information on the spacelike and timelike form factors can be extracted from the data at backward scattering angles where the spacelike and the timelike amplitudes become comparable. The results of a combined fit to the data at all energies (assuming both $\Lambda_{S} \neq \Lambda_{T}$ and $\Lambda_{S} = \Lambda_{T}$) are summarized in table 2. Similar results have been obtained by other PETRA experiments [7].

At W = 30 GeV the interference of the photon with the weak current, for example through the proposed Z^0 boson, should produce observable deviations from the pure WED cross section. In the Weinberg–Salam model the maximum expected deviation is about -2%at scattering angles between 80° and 130° for $\sin^2\theta_W$ = 0.23 (see insert at fig. 2). We fitted the data including the standard Weinberg–Salam model according to the following formula [8]:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left(\left(\frac{3+x^2}{1-x} \right)^2 - \frac{D(s)}{2(1-x)} \left\{ 2(1+x)^3 (g_V^2 + g_A^2) \right\} \right)$$

+
$$[8 - (1 - x)^3] (g_V^2 - g_A^2)$$
,

Reaction	Fitted parameters	Lower limits (95% c.l.)			
	(GeV ⁻²)	Λ ₊ (GeV)	Λ_ (GeV)		
$e^+e^- \rightarrow e^+e^-$	$1/\Lambda_{\rm S}^2 = (1.7 \pm 4.1) \times 10^{-5}$	108	138		
	$1/\Lambda_T^2 = (7.9 \pm 12.6) \times 10^{-5}$	59	88		
	$1/\Lambda^{\frac{1}{2}} = (1.4 \pm 4.0) \times 10^{-5}$ ($\Lambda_{\rm S} = \Lambda_{\rm T}$)	112	139		
$e^+e^- \rightarrow \mu^+\mu^-$	$1/\Lambda_{\rm T}^2 = (4.2 \pm 6.9) \times 10^{-5}$	80	118		
$e^+e^- \rightarrow \tau^+\tau^-$ [2]	$1/\Lambda_{\rm T}^2 = (2.0 \pm 10.2) \times 10^{-5}$	73	82		
$e^+e^- \rightarrow \gamma\gamma$	-	34	42		
	Reaction $e^+e^- \rightarrow e^+e^-$ $e^+e^- \rightarrow \tau^+\tau^-$ [2] $e^+e^- \rightarrow \gamma\gamma$	Reaction Fitted parameters (GeV ⁻²) $e^+e^- \rightarrow e^+e^ 1/\Lambda_S^2 = (1.7 \pm 4.1) \times 10^{-5}$ $1/\Lambda_T^2 = (7.9 \pm 12.6) \times 10^{-5}$ $1/\Lambda_T^2 = (1.4 \pm 4.0) \times 10^{-5}$ $(\Lambda_S = \Lambda_T)$ $e^+e^- \rightarrow \tau^+\tau^ 1/\Lambda_T^2 = (4.2 \pm 6.9) \times 10^{-5}$ $e^+e^- \rightarrow \tau^+\tau^ 1/\Lambda_T^2 = (2.0 \pm 10.2) \times 10^{-5}$ $e^+e^- \rightarrow \gamma\gamma$	Reaction Fitted parameters (GeV ⁻²) Lower fifting Λ_+ (GeV) $e^+e^- \rightarrow e^+e^ 1/\Lambda_Z^2 = (1.7 \pm 4.1) \times 10^{-5}$ 108 $1/\Lambda_T^2 = (7.9 \pm 12.6) \times 10^{-5}$ 59 $1/\Lambda_T^2 = (1.4 \pm 4.0) \times 10^{-5}$ 112 $(\Lambda_S = \Lambda_T)$ $(\Lambda_S = \Lambda_T)$ $e^+e^- \rightarrow \mu^+\mu^ 1/\Lambda_T^2 = (4.2 \pm 6.9) \times 10^{-5}$ 80 $e^+e^- \rightarrow \tau^+\tau^ 1/\Lambda_T^2 = (2.0 \pm 10.2) \times 10^{-5}$ 73 $e^+e^- \rightarrow \gamma\gamma$ $-$ 34	ReactionFifted parameters (GeV^2)Lower limits (93% C.l.) Λ_{+} (GeV) $e^+e^- \rightarrow e^+e^ 1/\Lambda_S^2 = (1.7 \pm 4.1) \times 10^{-5}$ $1/\Lambda_T^2 = (7.9 \pm 12.6) \times 10^{-5}$ $1/\Lambda_T^2 = (1.4 \pm 4.0) \times 10^{-5}$ $1/\Lambda_S^2 = \Lambda_T$)108 112 139 $(\Lambda_S = \Lambda_T)$ $e^+e^- \rightarrow \mu^+\mu^-$ $e^+e^- \rightarrow \tau^+\tau^- [2]$ $1/\Lambda_T^2 = (2.0 \pm 10.2) \times 10^{-5}$ $1/\Lambda_T^2 = (2.0 \pm 10.2) \times 10^{-5}$ 80 118 82 24	

Table 2 Results for QED cut-off parateters.

where

$$D(s) = \frac{1}{2\pi\alpha} \frac{s}{s - M_z^2}, \quad M_z = 74.6 \text{ GeV/sin } 2\theta_W,$$

 $g_{\rm V} = (4\sin^2\theta_{\rm W} - 1)/(2\sin 2\theta_{\rm W}),$

 $g_{\rm A} = -1/(2\sin 2\theta_{\rm W}).$

We find an upper limit for the Weinberg angle of $\sin^2 \theta_W < 0.55$ at 95% confidence level.

 $e^+e^- \rightarrow \mu^+\mu^-$. The reaction e^+e^- was studied at W = 12 GeV and near W = 30.7 GeV. The events were selected by applying in addition to the criteria (i)-(iv) the following cuts:

(v) the momentum of each track had to be greater than one third of the beam momentum;

(vi) at least one track had to be positively identified as a muon in the muon detection system.

The background from τ pairs was estimated to be 5% (2%) at W = 30.7 GeV (12 GeV) and was subtracted from the data. The background due to cosmic rays and the two photon process $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ was found to be negligible. The data were corrected for detector and analysis program efficiencies, acceptance and radiative effects [3].

The measured differential μ pair cross section for W = 30.7 GeV is shown in fig. 3 together with the QED expectation. The angular distribution is consis-



Fig. 3. (a) The differential cross section for μ pair production at W = 30.7 GeV. The curve is the QED prediction. (b) The energy distribution of photons from γ pair production at W = 30.7 GeV. The curve shows the expected resolution. (c) The differential cross section for γ pair production at W = 30.7 GeV. The curve is the QED prediction.

tent with that for a spin 1/2 particle. The total cross

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sections are in good agreement with the QED prediction as shown in table 1.

In order to quantify any deviation from QED the data were compared to the following modified QED cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4s} \left(1 + \cos^2\theta\right) |F_{\mathrm{T}}|^2 \,,$$

with

$$F_{\rm T}(s) = 1 \pm \frac{s}{s - \Lambda_{\pm}^2} \; .$$

The result of the fit in terms of the cut-off parameter is given in table 2. Similar limits have been given by other PETRA experiments [7].

The value obtained for Λ_{-} can be directly converted into limits on the Weinberg angle:

$$0.25 - \frac{37.3 \text{ GeV}}{\Lambda_{-}} < \sin^2 \theta_{\text{W}} < 0.25 + \frac{37.3 \text{ GeV}}{\Lambda_{-}}.$$

Our result is $\sin^2 \theta_W < 0.57$ with 95% confidence. The observed forward-backward asymmetry $(\sigma_F - \sigma_B)/(\sigma_F + \sigma_B)$, where $\sigma_F (\sigma_B)$ is the cross section integrated over $0 \le \cos \theta \le 0.8$ ($-0.8 \le \cos \theta \le 0$), was measured to be -0.01 ± 0.12 . Within the Weinberg-Salam model an asymmetry of -0.06 would be expected. The effect of the weak interaction on QED cut-off parameters has been discussed in more detail in the literature [9].

 $e^+e^- \rightarrow \gamma\gamma$. The reaction e^+e^- was studied at W = 12 GeV and around W = 30.7 GeV. Events were selected by requiring two shower clusters in the shower counters with more than 2 GeV electromagnetic energy deposited in each. The two showers had to be collinear within 25° and no charged track should have been detected in the tracking devices. Background due to cosmic ray showers was reduced by demanding that no more than 10 out of the 152 shower counters registered more than 100 MeV energy each. The residual background was removed by a visual scan of all event candidates. In fig. 3b the energy distribution of the photons is shown for the data at W = 30.7 GeV. The final event sample was obtained by demanding both photon energies to be larger than 6 GeV (2 GeV) at W = 30.7 GeV (12 GeV). The effect of these cuts as well as the corrections due to the geometrical acceptance were calculated by using the above described $e^+e^- \rightarrow e^+e^-$

event sample and analyzing these events in a similar way. The data were corrected for acceptance, detector efficiencies and radiative effects [3]. The systematic uncertainties in the cross section determination were estimated to be $\pm 12\%$. The differential γ pair cross section for W = 30.7 GeV is shown in fig. 3c as function of $|\cos \theta|$, where θ is defined as the angle between the difference vector of the two photon momenta and the beam direction. A good agreement with QED is obtained. The number of events and a comparison of the cross section integrated over the acceptance with the QED cross section are given in table 1.

To search for a deviation from the QED prediction the data were compared to the following cross section [10]:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{s} \frac{1 + \cos^2\theta}{\sin^2\theta} \left(1 \mp \frac{s^2 \sin^2\theta}{2\Lambda_{\mathrm{e}^{\pm}}^4}\right).$$

The first term describes the electron exchange diagram. The term proportional to Λ_e accounts for the contribution of a hypothetical excited electron. The resulting lower limits for the cut-off parameters are given in table 2. Similar limits were given by the JADE Collaboration [7].

Conclusions. In a previous publication [2] we presented results from τ pair production, some of which are also given in table 2. By studying the reactions $e^+e^- \rightarrow e^+e^-$, $e^+e^- \rightarrow \mu^+\mu^-$, $e^+e^- \rightarrow \tau^+\tau^-$ and $e^+e^ \rightarrow \gamma\gamma$ we have shown that QED is valid up to momentum transfers of $\approx 1000 \text{ GeV}^2$, or equivalently down to distances of the order 10^{-16} cm. Within these limits electrons, muons and taus behave as pointlike particles. A search for electro-weak interference effects within the framework of the Weinberg–Salam model led to a limit of $\sin^2\theta_W < 0.55$ (95% c.1.).

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