COMPARISON OF e^+e^- ANNIHILATION WITH QCD AND DETERMINATION OF THE STRONG COUPLING CONSTANT

TASSO Collaboration

R. BRANDELIK, W. BRAUNSCHWEIG, K. GATHER, V. KADANSKY, K. LÜBELSMEYER, P. MÄTTIG, H.-U. MARTYN, G. PEISE, J. RIMKUS, H.G. SANDER, D. SCHMITZ, A. SCHULTZ von DRATZIG, D. TRINES and W. WALLRAFF
I. Physikalisches Institut der RWTH Aachen, Germany [7]

H. BOERNER, H.M. FISCHER, H. HARTMANN, E. HILGER, W. HILLEN, L. KOEPKE, G. KNOP, P. LEU, B. LÖHR [5], R. WEDEMEYER, N. WERMES and M. WOLLSTADT
Physikalisches Institut der Universität Bonn, Germany [7]

H. BURKHARDT, D.G. CASSEL [1], D. HEYLAND, H. HULTSCHIG, P. JOOS, W. KOCH, P. KOEHLER [2], U. KÖTZ, H. KOWALSKI, A. LADAGE, D. LÜKE, H.L. LYNCH, G. MIKENBERG [3], D. NOTZ, J. PYRLIK, R. RIETHMÜLLER, M. SCHLIWA [6], P. SÖDING, B.H. WIIK and G. WOLF
Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

R. FOHRMANN, M. HOLDER, G. POELZ, O. RÖMER, R. RÜSCH and P. SCHMÜSER
II. Institut für Experimentalphysik der Universität Hamburg, Germany [7]

D.M. BINNIE, P.J. DORNAN, N.A. DOWNIE, D.A. GARBUTT, W.G. JONES, S.L. LLOYD, D. PANDOULAS, J. SEDGBEER, S. YARKER and C. YOUNGMAN
Department of Physics, Imperial College London, UK [8]

R.J. BARLOW, I. BROCK, R.J. CASHMORE, R. DEVENISH, P. GROSSMANN, J. ILLINGWORTH, M. OGG, B. ROE [4], G.L. SALMON and T. WYATT
Department of Nuclear Physics, Oxford University, UK [8]

K.W. BELL, B. FOSTER, J.C. HART, J. PROUDFOOT, D.R. QUARRIE, D.H. SAXON and P.L. WOODWORTH
Rutherford Laboratory, Chilton, UK [8]

Y. EISENBERG, U. KARSHON, D. REVEL, E. RONAT and A. SHAPIRA
Weizmann Institute, Rehovot, Israel [9]

J. FREEMAN, P. LECOMTE, T. MEYER, SAU LAN WU and G. ZOBERNIG
Department of Physics, University of Wisconsin, Madison, WI USA [10]

Received 19 May 1980

We have analyzed 1113 events of the reaction $e^+e^- \to$ hadrons at CM energies of 12 and 30 GeV in order to make a detailed comparison with QCD. Perturbative effects can be well separated from effects depending on the quark and gluon fragmentation parameters to yield a reliable measurement of the coupling constant α_s. At 30 GeV, the result is $\alpha_s = 0.17 \pm 0.02$ (statistical) ± 0.03 (systematic). QCD model predictions, using the fragmentation parameters determined along with α_s, agree with both gross properties of the final states and with detailed features of the three-jet states.

Direct evidence for gluon bremsstrahlung was first obtained at PETRA from the analysis of e^+e^- annihilation into hadrons at center-of-mass energies W near 30 GeV [1–5]. We have shown [1,2] that events are produced with a planar structure excluding a pure $q\bar{q}$ fragmentation mechanism, that the emitted particles cluster around 2 or 3 axes with an average transverse momentum of about 300 MeV/c, and that a few percent of the events show a clear three-jet topology [6]. Since then more data have been taken in this energy range permitting a more detailed comparison with quantum chromodynamics (QCD).

The TASSO detector and the event selection criteria have been described previously [7,8] [‡1]. Charged particles were accepted over 87% of the total 4π solid angle; for two thirds of the data also the neutral energy was measured over 52% of 4π [9]. We obtained 194 events of single-photon annihilation into hadrons at $W = 12$ GeV, and 919 events in the range $27.4 \leqslant W \leqslant 31.6$ GeV (hereafter denoted as $W = 30$ GeV).

A quantitative comparison of the data with perturbative QCD depends on the value of the coupling constant α_s as well as on the nonperturbative fragmentation of quarks and gluons into hadrons. To describe the fragmentation we employed the general framework of Field and Feynman [10] but treated the parameters describing the fragmentation process as free parameters. We selected a kinematic region where hard gluon bremsstrahlung dominates and determine from the fraction of such events the value of α_s at $W = 30$ GeV almost independent of the fragmentation parameters. With a single set of fragmentation parameters a good description of our data at different energies is then obtained, both in the perturbative and the nonperturbative regions. This demonstrates the consistency of the analysis.

Two specific QCD models were used, one by Hoyer et al. [11] and a later extension by Ali et al. [12], leading to similar results. The results we quote are from the second model which includes second order terms in α_s. The description of the fragmentation process involves the following three parameters:

(i) σ_q. The distribution of the transverse momentum k_T of the quarks in the jet cascade is assumed to be $\sim\exp(-k_T^2/2\sigma_q^2)$.

(ii) $P/(P+V)$. Here P/V is the ratio of primordial pseudoscalar mesons to vector mesons produced in the fragmentation process.

(iii) a_F. The primordial quark fragmentation function $f^h(z)$ of a quark into a hadron h in the Field–Feynman model is taken to be the same for u, d, and s quarks,

$$f^h(z) = 1 - a_F + 3a_F(1-z)^2 ,$$

where

$$z = (E + p_\parallel)_h/(E + p_\parallel)_q .$$

Our results are insensitive to assumptions for the primordial fragmentation functions of the c and b quarks; for simplicity they have been taken to be 1. In the model of Ali et al. [12], a Q^2-dependent form of these fragmentation functions is used.

In principle this model [12] involves a fourth parameter σ_g. As the gluon is assumed to split into a quark–antiquark pair, the distribution of the transverse momentum q_T of the quarks with respect to the gluon is

[1] On leave from Cornell University, Ithaca, NY, USA.
[2] On leave from FNAL, Batavia, IL, USA.
[3] On leave from Weizmann Institute, Rehovot, Israel.
[4] On leave from the University of Michigan, Ann Arbor, MI, USA.
[5] Now at SLAC, Stanford, CA, USA.
[6] Now at mbp Bremen, Germany.
[7] Supported by the Deutsches Bundesministerium für Forschung und Technologie.
[8] Supported by the UK Science Research Council.
[9] Supported by the Minerva Gesellschaft für die Forschung mbH, Munich.
[10] Supported in part by the US Department of Energy contract WY-76-C-02-0881.
[‡1] In addition to the criteria described in refs. [7,8], events at $W = 30$ GeV for which the sum of the observed momenta in one hemisphere with respect to the beam direction was less than 750 MeV/c were removed.

taken to be $\exp(-q_T^2/2\sigma_g^2)$. However, the result of the present analysis is insensitive to the value of this parameter, the reason being that, in the gluon jets, the transverse momentum spread due to σ_g and that due to σ_q from the hadronization of quarks are hardly distinguishable. Changing σ_g from 0 to 0.5 GeV/c makes no difference in the results presented here.

The QCD models also involve a thrust cut-off T_0 separating the perturbative from the nonperturbative region [11,12]. The T_0 value used in the models is 0.95 at W = 30 GeV. We verified that increasing T_0 to 0.975 did not change our results on α_s.

In our analysis we used the sphericity tensor [13]

$$M_{\alpha\beta} = \sum_j p_{j\alpha} p_{j\beta} ,$$

$$\alpha, \beta = x, y, z , \quad j = 1, \ldots, N \text{ particles} ,$$

with eigenvectors $\hat{n}_1, \hat{n}_2, \hat{n}_3$ and corresponding normalized eigenvalues $Q_k = \Sigma(p_j \cdot \hat{n}_k)^2 / \Sigma p_j^2$, which satisfy $Q_1 + Q_2 + Q_3 = 1$ and which are ordered such that $0 \leqslant Q_1 \leqslant Q_2 \leqslant Q_3$. In terms of these Q_k, the sphericity S, the aplanarity A and the variable Y are given by

$$S = \tfrac{3}{2}(Q_1 + Q_2) , \quad A = \tfrac{3}{2} Q_1 , \quad Y = \tfrac{1}{2}\sqrt{3}(Q_2 - Q_1) .$$

The plane determined by $\hat{n}_2$ and $\hat{n}_3$ is called the event plane; $\hat{n}_3$ gives the sphericity axis.

For the determination of α_s we used the data at W = 30 GeV. To reduce background of hard photon bremsstrahlung we removed events where the angle θ_n between the normal to the event plane $\hat{n}_1$ and the beam direction is larger than 80°. This cut reduced the sample to 777 events. The distribution of these events in the triangular plot with axes S and A is shown in fig. 1.

Two-jet events have small sphericity S and aplanarity A, while three-jet candidates have large sphericity and small aplanarity. For the determination of α_s we used the region $S > 0.25$ which, in the absence of a new flavor threshold [8,14], is mainly sensitive to effects of hard gluon bremsstrahlung. This region contains 128 events. These are also shown projected onto the S and A axes as histograms [‡2]. We divided the two-dimensional S versus A distribution into 5 bins and fitted the two parameters α_s and σ_q for a wide range of values of $P/(P + V)$ and a_F. The distribution from the QCD model in this fit included acceptance and radiation effects and was normalized to the *total* sample

[‡2] The 80° cut on θ_n does not remove all possible contributions from hard photon emission with large angles to the beam direction. The charged particles from such events will be emitted into two noncollinear jets. Of the 128 e events only 6 have this topology. In our comparison with the QCD Monte Carlo models we have taken both the collinear and the hard noncollinear photon bremsstrahlung into account [15].

Table 1
Fitted values [a)] of α_s and σ_q for different input values of a_F and $P/(P + V)$.

a_F		$P(P+V)$				
		0.1	0.3	0.5	0.7	0.9
0.1	α_s	0.17 ± 0.03	0.17 ± 0.03	0.17 ± 0.03	0.16 ± 0.03	0.16 ± 0.03
	σ_q	0.44 ± 0.11	0.46 ± 0.10	0.47 ± 0.10	0.48 ± 0.09	0.48 ± 0.08
0.3	α_s	0.17 ± 0.04	0.16 ± 0.04	0.16 ± 0.04	0.15 ± 0.04	0.15 ± 0.04
	σ_q	0.42 ± 0.12	0.44 ± 0.11	0.46 ± 0.10	0.47 ± 0.09	0.48 ± 0.08
0.5	α_s	0.17 ± 0.04	0.16 ± 0.04	0.16 ± 0.04	0.15 ± 0.04	0.14 ± 0.04
	σ_q	0.35 ± 0.12	0.38 ± 0.12	0.41 ± 0.12	0.43 ± 0.10	0.44 ± 0.09
0.7	α_s	0.17 ± 0.03	0.17 ± 0.04	0.16 ± 0.04	0.15 ± 0.05	0.14 ± 0.05
	σ_q	0.28 ± 0.09	0.30 ± 0.10	0.33 ± 0.10	0.36 ± 0.10	0.39 ± 0.09
0.9	α_s	0.17 ± 0.03	0.17 ± 0.04	0.16 ± 0.04	0.15 ± 0.04	0.14 ± 0.05
	σ_q	0.21 ± 0.08	0.23 ± 0.08	0.26 ± 0.08	0.30 ± 0.09	0.33 ± 0.08

a) All fits have acceptable confidence levels, with χ^2 between 1.2 and 5.4 for 3 degrees of freedom.

of 777 events. The results presented in table 1 show that α_s is fixed by the data nearly independently of the values of $P/(P + V)$ and a_F. Their systematic variation produces an uncertainty of α_s of only ±0.017. The average result from table 1 is $\alpha_s = 0.16 \pm 0.04$. The error contains the effects of the correlation between α_s and σ_q. We emphasize that this result is independent of input assumptions for the values of the Field–Feynman fragmentation parameters.

The accuracy of the determination of α_s can be improved at the price of slightly more reliance on the fragmentation model. This was done by experimentally determining the fragmentation parameters from the data including the region $S < 0.25$ where two-jet events dominate. We note that for each event the average squared transverse momentum $\langle p_T^2 \rangle_{out} (= \langle (p \cdot \hat{n}_1)^2 \rangle)$ perpendicular to the event plane is most sensitive to σ_q, the single particle fractional momentum distribution dN/dx (where x = momentum of particle/beam energy) is most sensitive to a_F, and the distribution of charged multiplicity n_{ch} is most sensitive to $P/(P + V)$. These distributions are well described by the parameters

$$a_F = 0.57 \pm 0.20 , \quad \sigma_q = 0.32 \pm 0.04 \text{ GeV}/c ,$$
$$P/(P + V) = 0.56 \pm 0.15 .$$

This result is insensitive to the precise value of α_s, but the values are correlated. We have not optimized the determination of these parameters by making an effort to minimize the correlations. We used these parameter values to obtain a mofe accurate value for α_s from a fit to the S–A distribution of the high sphericity data ($S > 0.25$). The result is $\alpha_s = 0.17 \pm 0.02$. The agreement with the α_s value found before shows the consistency of the procedure. The QCD predictions with these parameter values are shown as curves in the histograms of fig. 1 [‡3].

Within the context of the QCD models used, we estimated the systematic error in our determination of α_s by repeating the analysis using a variety of different distributions and investigating the effects of different kinematical cuts on the data. The final result is α_s

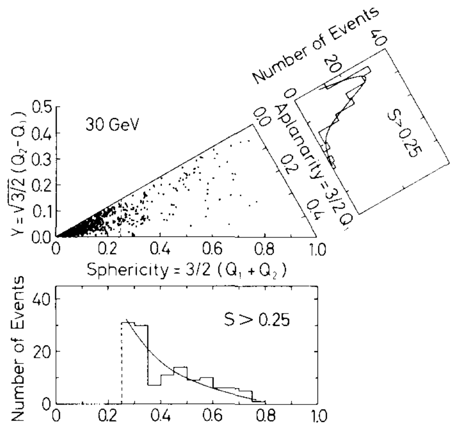

Fig. 1. Distribution of the observed events as a function of sphericity S and aplanarity A, for the data at W = 30 GeV. The events with $S > 0.25$ are projected onto the S and A axes as histograms. The curves show the result of fitting the QCD model (including acceptance and radiative effects) to the observed two-dimensional triangular plot distribution. They are normalized to the total event sample (777 events for all values of S).

= 0.17 ± 0.02 (statistical) ± 0.03 (systematic) [‡4]. Previous results on α_s from PETRA experiments have been given in refs. [5,12,16].

In the remainder of this paper we compare our data in various different ways with the QCD model using the above determined parameters. Fig. 2 shows this comparison for the sphericity S and aplanarity A, as well as for the inclusive x distribution of the charged particles both at W = 12 GeV (using $\alpha_s = 0.21$ for the running coupling constant) and at 30 GeV. Since the 12 GeV data were not used in determining any of the parameters, the good agreement shows that we have indeed succeeded in separating the perturbative effects which are strong at 30 GeV and hardly observable at 12 GeV, from the nonperturbative effects which are generally assumed to be independent of energy.

So far we have considered gross features of the

‡3 In this analysis only the charged particles were used. In order to check the validity of this procedure, the QCD model predictions were also compared with the subsample (310 events) of the data that included the measurement of the neutral energies. Good agreement was found.

‡4 It has been suggested that the contributions of higher order terms in $\alpha_s(Q^2)$ may be effectively absorbed in the variable Q^2 substituting $Q^2 \to P^2$ where P^2 is the invariant mass square of the virtual quark which radiates the hard gluon. In the kinematic region used for the α_s fit the average value of P^2 was estimated from the QCD Monte Carlo model to be $\langle P^2 \rangle \approx 140$ GeV^2 at $W^2 = Q^2 = 900$ GeV^2.

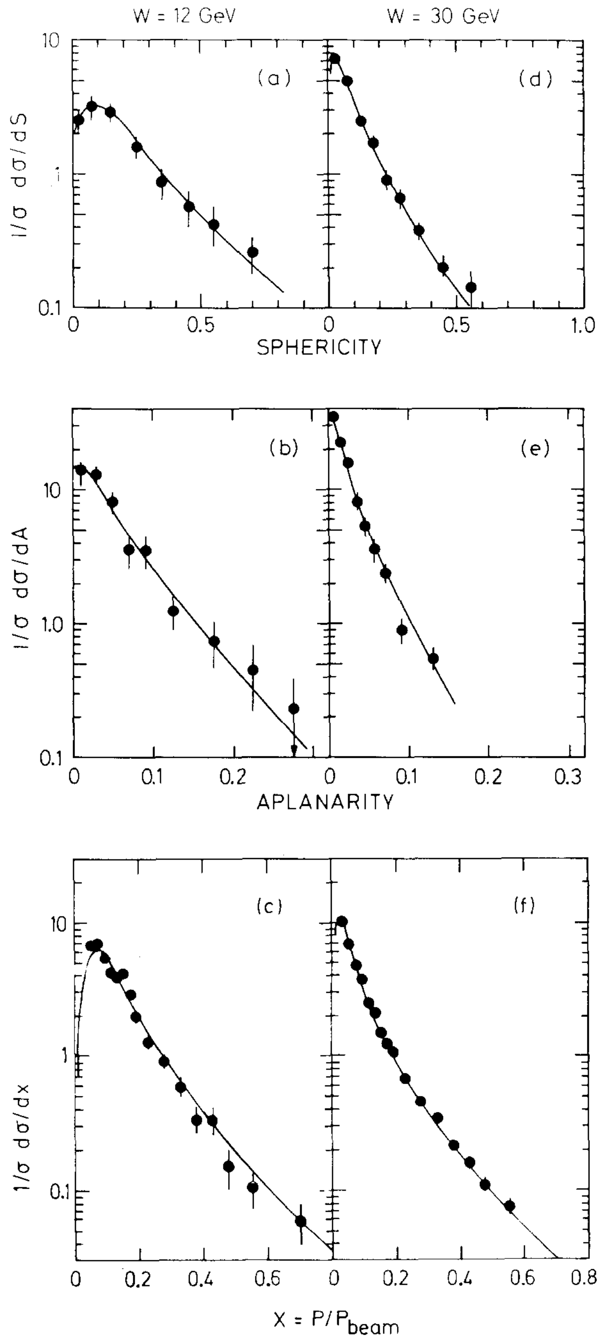

Fig. 2. Comparison of the data with the QCD model (curves) at $W = 12$ GeV for (a) sphericity S, (b) aplanarity A and (c) the single particle inclusive x distribution for charged particles; and at $W = 30$ GeV for (d) S, (e) A, and (f) x. The experimental as well as the theoretical distributions are corrected for acceptance and radiative effects as described in refs. [15, 18].

event distributions. We now extend the test of the QCD model considering more specific features of the *planar* high sphericity events at $W = 30$ GeV. There were 77 events with $S > 0.25, A < 0.08$. We analyzed these for three-jet structure using the method of generalized sphericity [17].

The procedure is as follows. Let q_j be the projections of the momenta p_j of the observed particles on the event plane. Consider a partition of these N vectors q_j into three nonoverlapping subsets. For each subset k, we diagonalize the 2×2 tensor $\Sigma_i\, q_{j\alpha} q_{j\beta}$ where $\alpha, \beta = 1, 2$ correspond to the components in the event plane, and thereby find the sphericities S_k $(k = 1, 2, 3)$ and the sphericity axes $\hat{m}_k$. Note that for a jet-like subset the sphericity is small. Therefore we determine the best admissible partition by minimizing the quantity $S_1 + S_2 + S_3$. If the event under consideration is indeed a three-jet event, we have then associated each observed particle to one of the three jets, and we have reconstructed the three-jet axes $\hat{m}_k$.

In fig. 3a we show the distribution of the squared transverse momenta p_T^2 of the charged hadrons for the 3×77 observed jets, where the p_T of each hadron is calculated with respect to the associated jet axis. It is compared with the corresponding distribution for events at 12 GeV analyzed as two-jets and, therefore, without any cuts in S or A. The p_T^2 behaviour is found to be the same in the two cases. It also compares well with the QCD model (curve) at $W = 30$ GeV.

To discuss the p_T behaviour of the planar events further, it is useful to define the quantity "trijettiness" [17]

$$J_3 = \frac{1}{N-3} \sum_{j=1}^{N} \frac{q_{Tj}^2}{\frac{1}{2}\sigma_q^2},$$

where we took $\sigma_q = 0.33$ GeV/c and where q_{Tj} is the transverse momentum in the event plane with respect to the associated jet axis, i.e. $q_{Tj} = [q_j \times \hat{m}_k|$. For a three-jet event, J_3 is expected to be near 1 while for a uniform disk-like distribution J_3 is substantially larger than 1.

Fig. 3b shows the J_3 distribution. It peaks at lower J_3 values than the distribution expected for disk-like events [‡5] (dashed curve). The peaking at low J_3 values

‡5 The planarity cut ($S > 0.25, A < 0.08$) was applied to a phase space distribution using as inputs the observed average charged multiplicity and the ratio of neutral to charged energies of the hadrons.

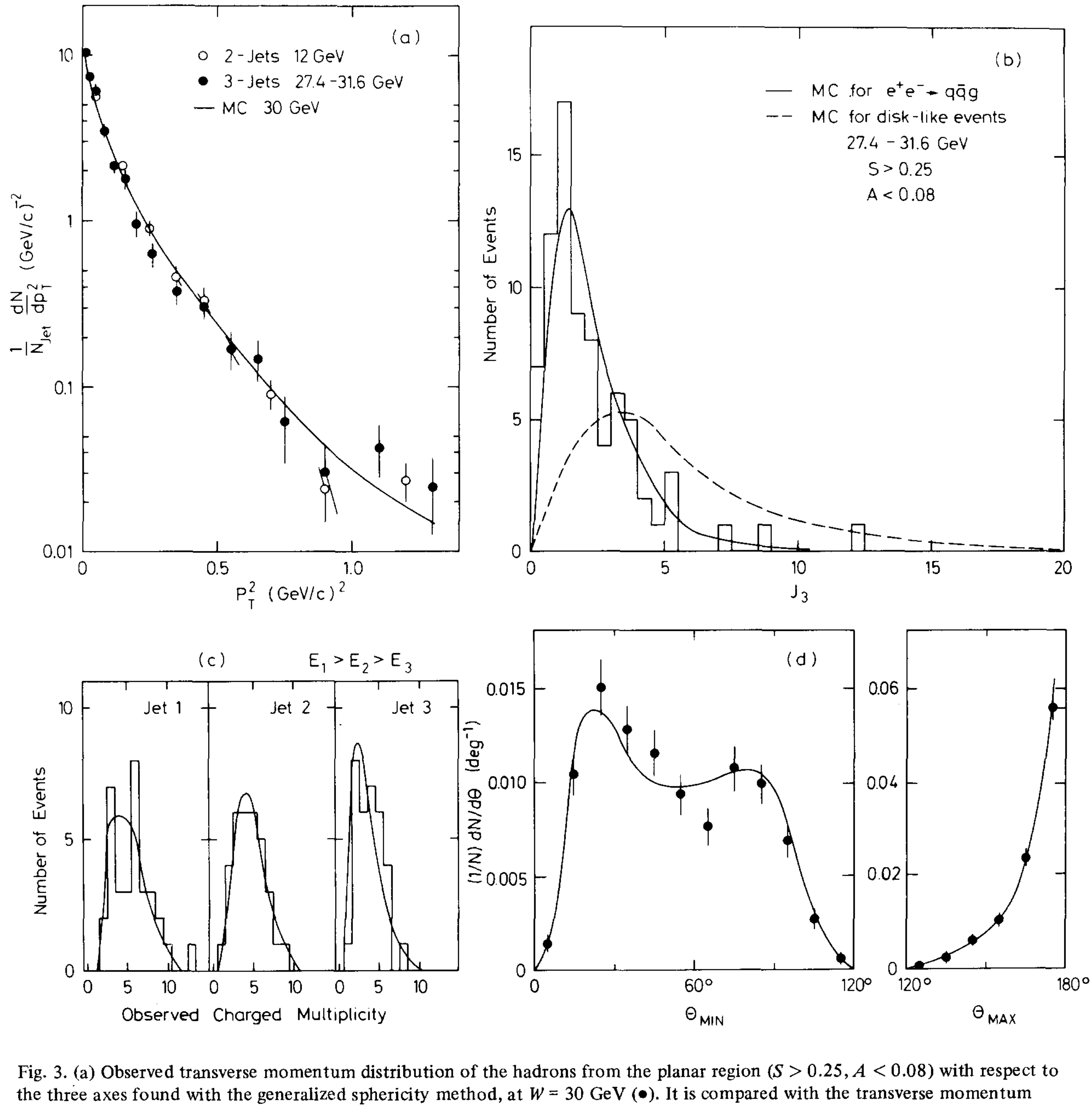

Fig. 3. (a) Observed transverse momentum distribution of the hadrons from the planar region ($S > 0.25, A < 0.08$) with respect to the three axes found with the generalized sphericity method, at W = 30 GeV (•). It is compared with the transverse momentum distribution relative to the sphericity axis for *all* events (no S or A cut) at W = 12 GeV, analyzed as two-jet events (○). It is also compared with the result from the QCD model at 30 GeV (curve). (b) Comparison of the trijettiness distribution for the planar event sample at W = 30 GeV, with the distribution for disk-like events (dashed curve) and the QCD model (solid curve). The curves are normalized to the number of the observed events. (c) Observed charged multiplicity distributions for the jets of highest (E_1), medium (E_2), and lowest (E_3) energy as reconstructed by the generalized sphericity method, for the events in the planar region at W = 30 GeV. The curves are calculated from the QCD model, normalized to the number of observed events. (d) Distribution of the smallest and the largest angle between any of the three jets, when all events (no S or A cuts) at W = 30 GeV are analyzed with the generalized sphericity method. The curves show the results from the QCD model.

is a consequence of the strong collimation of the particles around three-jet axes. The observed J_3 distribution agrees well with the QCD prediction (solid curve).

As a further test we show in fig. 3c a comparison between the observed (i.e. uncorrected) and the calculated charged multiplicity distribution ‡6 of the three jets as reconstructed by our method, ordered by their energies ($E_1 > E_2 > E_3$). The energies E_i are the reconstructed parton energies calculated from the angles between the three jet axes, assuming the parton masses to be zero. An additional requirement, $\theta_n < 53°$, was imposed to ensure almost complete acceptance for all particles of the three jets in the drift chamber. Again good agreement with the QCD model is found.

Finally, fig. 3d shows the distribution of the smallest (θ_{min}) and the largest angle (θ_{max}) between any of the three jets. These angles are between 0° and 120° for θ_{min}, and between 120° and 180° for θ_{max}. The distributions are plotted for all events with $\theta_n < 80°$, but without sphericity or aplanarity cuts. Thus, the distribution of θ_{max} shows the transition from two-jet events (θ_{max} near 180°) to large angle gluon bremsstrahlung events. The minimum angle θ_{min} is small for two-jet events, larger for three-jet events. Both distributions are consistent with the expectation from the QCD model.

In summary, we have made a detailed comparison between QCD models and our data on $e^+e^- \to$ hadrons at $W = 12$ and 30 GeV. We have measured a value $\alpha_s = 0.17 \pm 0.02$ (statistical) $\pm$ 0.03 (systematic error) for the coupling constant at $W = 30$ GeV and showed that the uncertainties due to quark and gluon fragmentation are unimportant in our analysis. QCD models were found to give a consistent description of the data.

We wish to acknowledge the tremendous efforts of the PETRA machine group which made this investigation possible, as well as the invaluable help provided by the technical support groups, in particular the Hallendienst and the DESY computer center. We gratefully acknowledge the help of Dr. G. Söhngen. We are also indebted to the engineers and technicians of the collaborating institutions. We have enjoyed numerous discussions with Drs. T. Walsh and P. Zerwas. The Wisconsin group wishes to thank the Physics Department and especially the High Energy Group and the Graduate School Research Committee of the University for support. Those of us from abroad wish to thank the DESY Directorate for the hospitality extended to us while working at DESY. One of us (P.K.) would like to thank the Alexander von Humboldt Foundation for support through a Humboldt Award.

‡6 Charged particles with transverse momenta below 100 MeV/c relative to the beam direction are not included in the analysis.

References

[1] B.H. Wiik, Proc. Intern. Neutrino Conf. (Bergen, Norway, 18–22 June 1979) p. 113;
P. Söding, Proc. EPS Intern. Conf. on High energy physics (Geneva, Switzerland, 27 June–4 July 1979) p. 271.
[2] TASSO Collab., R. Brandelik et al., Phys. Lett. 86B (1979) 243.
[3] MARK J Collab., D.P. Barber et al., Phys. Rev. Lett. 43 (1979) 830.
[4] PLUTO Collab., Ch. Berger et al., Phys. Lett. 86B (1979) 418.
[5] JADE Collab., W. Bartel et al., DESY Report 79/80 (Dec. 1979).
[6] J. Ellis, M.K. Gaillard and G.G. Ross, Nucl. Phys. B111 (1976) 253;
T.A. DeGrand, Y.J. Ng and S.-H. Tye, Phys. Rev. D16 (1977) 3251.
[7] TASSO Collab., R. Brandelik et al., Phys. Lett. 83B (1979) 261.
[8] TASSO Collab., R. Brandelik et al., Z. Phys. C4 (1980) 87.
[9] TASSO Collab., R. Brandelik et al., DESY Report 80/12 (1980).
[10] R.D. Field and R.P. Feynman, Nucl. Phys. B 136 (1978) 1.
[11] P. Hoyer, P. Osland, H.E. Sander, T.F. Walsh and P.M. Zerwas, Nucl. Phys. B 161 (1979) 349.
[12] A. Ali, E. Pietarinen, G. Kramer and J. Willrodt, DESY Report 79/86 (1979), to be published.
[13] J.D. Bjorken and S.J. Brodsky, Phys. Rev. D1 (1970) 1416.
[14] TASSO Collab., R. Brandelik et al., Phys. Lett. 88B (1979) 199.
[15] F.A. Berends and R. Kleiss, to be published. We are grateful to the authors for the permission to use their $q\bar{q}\gamma$ event generator.
[16] MARK J Collab., D.P. Barber et al., Phys. Lett. 89B (1979) 139.
[17] S.L. Wu and G. Zobernig, Z. Physik C2 (1979) 107.
[18] TASSO Collab., R. Brandelik et al., Phys. Lett. 89B (1980) 418.