TEST OF SOME CURRENT IDEAS IN QUARK CONFINEMENT PHYSICS BY MONTE CARLO COMPUTATIONS FOR FINITE LATTICES

G. MACK
II. Institut für Theoretische Physik der Universität Hamburg, Fed. Rep. Germany

and

E. PIETARINEN
Deutsches Elektronen-Synchrotron DESY, Hamburg, Fed. Rep. Germany

Received 9 June 1980

We present some new results of Monte Carlo computations for pure SU(2) Yang–Mills theory on a finite lattice. They support consistency of asymptotic freedom with quark confinement, validity of a block cell picture, and ideas based on a vortex condensation picture of quark confinement.

In this letter we present some new results of Monte Carlo computations for pure SU(2) Yang–Mills theory on a finite lattice in four dimensions. They will be compared with expectations that are based on current ideas in quark confinement physics.

We consider a system on a finite hypercubic lattice of size $d \times d \times d \times d$ with lattice spacing $a = 1$. The random variables $U(b) \in \mathrm{SU}(2)$ are attached to links b of the lattice, and the action is [1]

$$L(U) = \tfrac{1}{2}\beta \sum_p \mathrm{Tr}\,[U(\dot{p}) - 1]\,. \qquad (1)$$

Summation is over all unoriented plaquettes p of the lattice. $U(\dot{p}) = U(b_4) \ldots U(b_1)$ is the parallel transporter around a plaquette p with boundary consisting of links $b_1 - b_4$. The expectation value of an observable $A(U)$ is given by

$$\langle A \rangle = Z^{-1} \int \prod_b \mathrm{d}U(b)\, A(U) \exp L(U). \qquad (2)$$

It will in general depend on the choice of the boundary conditions. They are described below. $\mathrm{d}U$ is the normalized Haar measure on SU(2). The partition function Z is defined by the requirement that $\langle 1 \rangle = 1$.

Following 't Hooft we consider the quantity [2]

$$\mu(\beta, d) = -\tfrac{1}{2} \ln [Z(\text{block, t.b.c.})/Z(\text{block, p.b.c.})]\,. \qquad (3)$$

It involves the ratio of partition functions for the block of size $d \times d \times d \times d$ with periodic boundary conditions (p.b.c.), and with twisted periodic boundary conditions (t.b.c.). The latter are obtained from the former by a singular gauge transformation which acts on the restriction of the configuration U to the boundary of the block. It agrees with an ordinary gauge transformation locally, but changes the parallel transporter $U(C)$ into $-U(C)$ for every closed path C on the boundary of the block which winds once around the intersection of the x^3, x^4 plane with the block. Such a singular gauge transformation changes the number of vortex souls [3] that wind through the block from even to odd and from odd to even.

Since $\mu = 0$ at $\beta = 0$, it can be recovered from its derivative

$$\partial\mu/\partial\beta = \tfrac{3}{2}(d/a)^4 \left(\langle \mathrm{Tr}\, U(p) \rangle_{\mathrm{pbc}} - \langle \mathrm{Tr}\, U(p) \rangle_{\mathrm{tbc}}\right). \qquad (4)$$

This quantity was computed for blocks of size up to $5 \times 5 \times 5 \times 5$. We used a Monte Carlo procedure very similar to that described by Creutz [4,5]. Results are shown in figs. 1 and 2. Error bars indicate the expected statistical error. For each of the two terms in (4) it is given by the mean square deviation of the averages

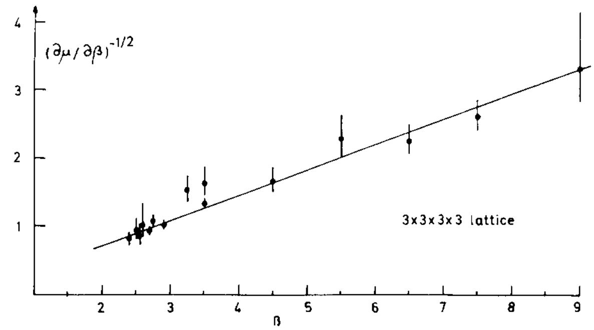

Fig. 1. Comparison of data for large values of β with perturbation theory. The straight line represents a fit by $\partial\mu/\partial\beta = 7.28 \times \beta^{-2}$.

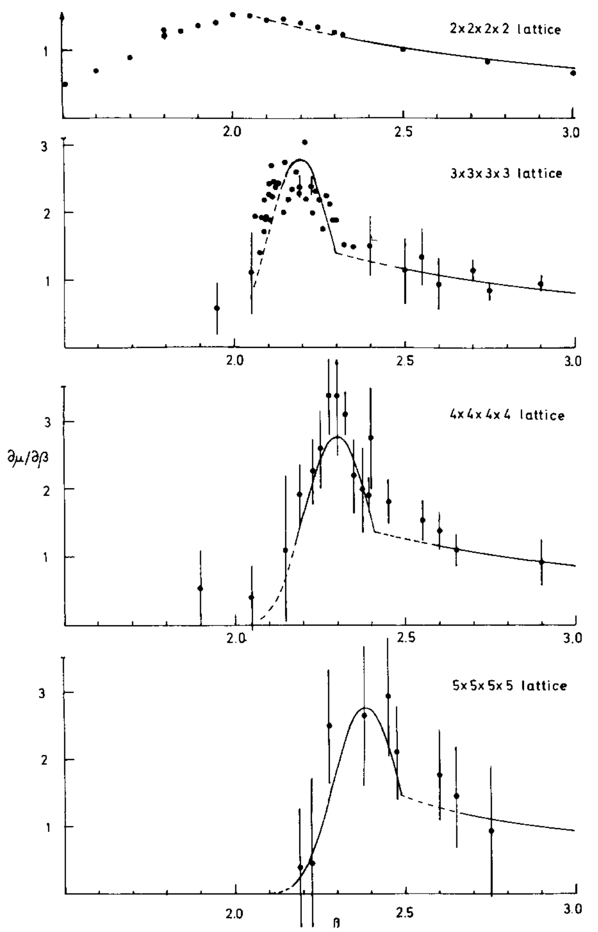

Fig. 2. Data for $\partial\mu/\partial\beta$ at intermediate values of β. The curves on the right represent the prediction of perturbation theory with expansion coefficients determined in fig. 1, viz. $\partial\mu/\partial\beta = 7.28[\beta - (11/6\pi^2)\ln d^2/9a^2]^{-2}$. The curve in the middle is the prediction of eqs. (5)–(7), with $\alpha_0 = 0.6$ and $\beta_1 = 2.06$. Lowering α_0 to 0.54 would raise it by 10%. For further discussion see text.

in $M = 5-10$ subsamples divided by $\sqrt{M}$. A similar quantity in Z(N)-theories was computed before by Groeneveld et al. [6].

For the purpose of the following discussion we introduce a length scale $d_c = d_c(\beta)$ which is fixed by the string tension α as follows:

$$\alpha = \alpha_0/d_c^2, \tag{5}$$

whenever this produces a value $d_c \geqslant a$. Otherwise we put $d_c = a$. α_0 is a constant whose value will be discussed below. In the fits we set $\alpha_0 = 0.6$. We compare the data with the following theoretical predictions [7].

For β such that d is substantially larger than $d_c(\beta)$ we shall compare with the derivative of the formula

$$\mu(\beta, d) = [d/d_c(\beta)]^2 \exp(-\alpha d^2). \tag{6}$$

We use the following expression for the string tension:

$$\alpha = \alpha_0 \exp[-\tfrac{6}{11}\pi^2(\beta - \beta_1)]. \tag{7}$$

It is supposed to be valid outside the high temperature region. According to Creutz, this expression fits Monte Carlo data for the string tension for $\beta > 2.17$. In our fits we use a value $\beta_1 = 2.06$ which is slightly smaller than Creutz's value 2.09.

For d smaller than $d_c(\beta)$ we compare with the lowest nontrivial orders of perturbation theory for $\partial\mu/\partial\beta$

$$\partial\mu/\partial\beta = a_1 x^{-2} + \dots, \tag{8a}$$

where x is the perturbation theoretic renormalization group invariant,

$$x = \beta - (11/6\pi^2)\ln(d^2/a^2) + x_0 + \dots. \tag{8b}$$

This is supposed to hold for sufficiently large d/a. The coefficient a_1 has not yet been computed analytically, but it can be determined from the data at large values of β. From the plot in fig. 1 one obtains $a_1 \approx 7.3$. For values of α_0, β_1 as given above, the condition $d \leqslant d_c(\beta)$ is fulfilled for $\beta \geqslant 2.47$ if $d = 3a$, for $\beta \geqslant 2.58$ if $d = 4a$, and for $\beta \geqslant 2.66$ if $d = 5a$, etc.

Eq. (8a) is obtained from the perturbation expansion of μ. The latter contains no term proportional β because there exist configurations U on the lattice which are locally a pure gauge [i.e. $U(\dot{p}) \equiv 1$] and which satisfy the boundary conditions. This is true both for periodic *and* for twisted periodic boundary conditions. (The same would not be true in an abelian theory.)

From our data and their comparison with the theoretical predictions (5)–(8) we draw the following conclusions.

(1) *Consistency of asymptotic freedom and finite string tension* α. The shape of the curve $\partial\mu/\partial\beta$ versus β is approximately constant [‡1] for $d/a \geqslant 3$ while the position shifts to larger values of β for increasing d. The rate of the shift is consistent with the prediction of asymptotic freedom in perturbation theory, i.e. the result depends on β and d only through the low order perturbation theoretic renormalization group invariant x. (Note that the fit eq. (6) has this property.) It is significant that this is true at some values of β where low order perturbation theory itself does not fit the data for $\partial\mu/\partial\beta$. This is in agreement with the effective Z(2) theory of quark confinement of ref. [7]. Assuming that this kind of behavior persists for larger values of d/a as well, one may conclude that $\mu(\beta,d)$ will have a finite continuum limit when this limit is taken in agreement with the hypothesis of asymptotic freedom – viz. $a \to 0, \beta \to \infty$ so that x remains fixed. The limiting value incorporates the factor $\exp(-\alpha d^2)$ which is characteristic of vortex condensation and confinement of (static) quarks.

(2) *Validity of a block cell picture*. High temperature expansions would predict that [8]

$$\mu(\beta,d) = [d/a]^2 \exp(-\alpha d^2) \tag{9}$$

for large d. This expression agrees with eq. (6) in the high temperature phase where $d_c = a$. But elsewhere it differs by a factor $[d_c(\beta)/a]^2$ which depends exponentially on β and is therefore very important. The modification (6) of eq. (9) is based on a block cell picture. One relates the SU(2) theory on a lattice with lattice spacing a to another lattice gauge theory on a lattice with a larger lattice spacing d_c. This latter theory is supposed to be amenable to high temperature expansions, so that eq. (9) can be used for it. Both Wilson's renormalization group approach [9] and the effective Z(2) theory [7] of quark confinement use such a block cell picture. Our data support it because they are in agreement with eq. (6), whereas extrapolation of eq. (9) would strongly disagree with them at intermediate values of β.

‡1 The same is not true for $d/a = 2$. We attribute this to the too small size of the block. There is some indication that small-size effects have not yet gone away completely for $d/a = 3$ either.

(3) *Range of validity of low order perturbation theory*. Low order perturbation theory fits the data for all values of β which are such that $d \leqslant d_c(\beta)$.

(4) *Vortex condensation*. Condensation of vortices [2,3,7] of thickness less than d is expected to lead to a suppression factor $\exp(-\alpha d^2)$ in $\mu(\beta,d)$. Such a factor is incorporated in the fit (6) and its presence is supported by the data.

(5) *Comparison with predictions of the effective Z(2) theory of quark confinement [7]*. In this theory, eqs. (6) and (7) can be derived, including an approximate value 2.0 for the parameter β_1. The quantity $d_c(\beta)$ is identified as the minimal thickness of vortices that can condense at inverse temperature β; when $d < d_c(\beta)$ none such can fit through the block and low order perturbation theory is then supposed to be valid. The parameter α_0 is given by the string tension in Wegner's Z(2) lattice gauge theory [10] model just above its transition temperature and it is supposed to be computable by high temperature expansions. This gives a value around 0.6. All these results are in agreement with our data. There is some uncertainty about the precise theoretical value of α_0. High temperature cluster expansion for the Z(2) theory to 14th order [11] gives 0.62, whereas extrapolation of the $2k$ = 8-, 10-, 12-, 14-th order result by a linear function of $1/k$ would give 0.54. Given the string tension, α_0 fixes the overall normalization of expression (6). Lowering α_0 from 0.6 to 0.54 would result in raising the corresponding curves in fig. 2 by 10%.

It is also of some interest to investigate whether the coupling parameter $\beta_{\text{eff}}(d)$ of the effective Z(2) theory can be approximated by μ. If so, $d_c(\beta)$ should be the solution of

$$\mu(\beta, d_c(\beta)) = \beta_c. \tag{10}$$

In the effective Z(2) theory, interaction of vortices is taken into account in a kind of mean field approximation, but the further approximation $\beta_{\text{eff}} = \mu$ mutilates it (among other things). Internal consistency requires that one allows for a shift of β_c away from the inverse transition temperature 0.44 of Wegner's Z(2) model to compensate for this. The appropriate value of β_c can be determined theoretically if the two leading terms $\mu = a_0 - a_1/x + ...$ in the perturbation expansion of μ are known. Our data indicate that eq. (10) works, with a value of β_c that is higher than 0.44 by a factor 2. From our data we can also extract $a_0 \approx 3.8$.

It is easy to see why the approximation $\beta_{eff} = \mu$ requires a value of β_c larger than 0.44. If eq. (6) were valid right down to $d = d_c$, then eq. (10) would hold with a value of $\beta_c = \exp(-\alpha_0) = 0.54$–$0.58$. But this would require negative values of $\partial\mu/\partial\beta$ for $d \leqslant d_c/\sqrt{\alpha_0} \approx 1.3 d_c$, whereas μ is expected to grow monotonically with β (and the data confirm this). Substituting positive for negative values of $\partial\mu/\partial\beta$ will increase the area $\beta_c = \mu(d_c)$ underneath the curve $\partial\mu/\partial\beta$.

The authors wish to thank G. Münster for discussions.

References

[1] K. Wilson, Phys. Rev. D10 (1974) 2445.
[2] G. 't Hooft, Nucl. Phys. B153 (1979) 141;
G. Mack and V.B. Petkova, Ann. Phys. (NY) 125 (1980) 117.
[3] G. Mack, Properties of lattice gauge theory models at low temperatures, DESY 80/C3, and to be published in: Recent developments in gauge field theories, eds. G. 't Hooft et al. (Plenum, New York).
[4] M. Creutz, Phys. Rev. Lett. 43 (1979) 553.
[5] M. Creutz, Solving quantized SU(2) gauge theory, preprint BNL-26847, Brookhaven (September 1979).
[6] J. Groeneveld, J. Jurkiewicz and C.P. Korthals Altes, Phys. Lett. 92B (1980) 312.
[7] G. Mack, Predictions of a theory of quark confinement, DESY 80/21.
[8] G. Münster, High temperature expansions for the free energy of vortices resp. the string tenstion in lattice gauge theories, DESY 80/44.
[9] K. Wilson, Cargèse lectures 1979, to be published in: Recent developments in gauge field theories, eds. G. 't Hooft et al. (Plenum, New York).
[10] F. Wegner, J. Math. Phys. 12 (1971) 2259.
[11] N. Kimura, Critical properties of Z(2) lattice gauge systems from strong coupling expansions, preprint Sapporo (1980).