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HOW THICK ARE CHROMO-ELECTRIC FLUX TUBES?

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We analyse the space dependence of the expectation value of the chromo-electric energy density in the presence of a static quark-antiquark pair by means of the strong coupling expansion on a lattice and by the relativistic string model. Both methods indicate that the transversal width of the field energy distribution increases without bound, when the quark-antiquark separation goes to infinity.

1. Introduction

It is a widespread belief that the confinement of quarks is associated with the formation of color electric flux tubes. Thus, the chromo-electric field energy density (above the vacuum)

$$\mathscr{E}(x) \propto \langle q\bar{q}|\,\mathrm{Tr}\,\boldsymbol{E}(x)^2|q\bar{q}\rangle - \langle q\bar{q}|q\bar{q}\rangle\langle 0|\,\mathrm{Tr}\,\boldsymbol{E}(x)^2|0\rangle \tag{1}$$

in the ground state $|q\bar{q}\rangle$ of the gluon field in the presence of an infinitely heavy quark-antiquark pair is expected to be supported essentially on a tube-like region as shown in fig. 1. Outside this region, $\mathscr{E}(x)$ falls off exponentially with a characteristic length inversely proportional to the glueball mass. A convenient measure for the width of the flux tube is

$$\sigma^2 = \frac{\int d^2x_\perp x_\perp^2 \mathscr{E}(x)}{\int d^2x_\perp \mathscr{E}(x)}\,, \qquad x_\perp = (x_1, x_2)\,, \qquad x_3 = \tfrac{1}{2}L \tag{2}$$

(the quarks are supposed to be located at $x = 0$ and $x = (0, 0, L)$ respectively).

σ^2 is a renormalization group invariant, because $\mathscr{E}(x)$ is only multiplicatively renormalized. In other words,

$$\sigma^2 = L^2 f(\Lambda L)\,, \tag{3}$$

where Λ is a physical mass parameter such as the square root of the string tension. When $L \ll 1/\Lambda$, the flux tube of fig. 1 degenerates to a ball with the quark-antiquark

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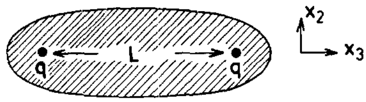

Fig. 1. Support of $\mathscr{E}(x)$.

pair at its center. The chromo-electric field inside the ball is then essentially equal to an electrostatic dipole field so that

$$\sigma^2 = \tfrac{1}{4}L^2\left\{1 + \mathrm{O}\left(\frac{1}{\ln \Lambda L}\right)\right\}, \qquad \left(L \ll \frac{1}{\Lambda}\right). \tag{4}$$

On the other hand, if $L \gg 1/\Lambda$, perturbation theory is not applicable. From the flux tube picture we rather expect σ^2 to grow much slower than L^2 as $L \to \infty$. It is even possible (and in fact true in strong coupling lattice gauge theories) that σ^2 approaches a constant $\sigma_\infty^2 < \infty$ in this limit.

In this article we address the question of whether or not $\lim_{L\to\infty} \sigma^2 = \sigma_\infty^2$ is finite (in the continuum). If indeed σ_∞^2 were finite, we would expect that $\alpha \cdot \sigma_\infty^2$ (where α is the string tension) extrapolates smoothly from the strong to the weak coupling domain on the lattice. This, we find, is not the case: the 12th order strong coupling expansion of σ_∞^2 presented in sect. 2 rather indicates that $(\alpha \cdot \sigma_\infty^2)^{-1}$ vanishes at a relatively large value of the bare coupling constant g^2. σ^2 can also be estimated from the string model approximation to the Wilson loop correlation functions (sect. 3). Although the string model assumes an infinitely thin bare string connecting the quark with the antiquark, σ^2 comes out to be non-zero, because the quantum mechanical wave function of the string has a non-vanishing width. In sect. 4 we summarize and comment on our results.

2. Strong coupling expansion of σ_∞^2

In this section we consider the standard pure SU(2) gauge theory on a four-dimensional euclidean lattice [1]. Our notation is as in ref. [2]. Thus, the link variables are denoted by $U(b)$ and expectation values of gauge-invariant combinations $\mathcal{O}$ of the bond variables are given by

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod_b \mathrm{d}U(b)\,\mathcal{O} \exp \mathscr{L}, \tag{5}$$

$$\mathscr{L} = \tfrac{1}{2}\beta \sum_{\mathrm{p}} \mathrm{Tr}\, U(\dot{\mathrm{p}}), \qquad g^2 = \frac{4}{\beta} = \text{bare coupling constant}. \tag{6}$$

The sum in eq. (6) extends over all unoriented plaquettes p on the lattice and $U(\dot{\mathrm{p}})$ is the product of the link variables associated with the boundary of p.

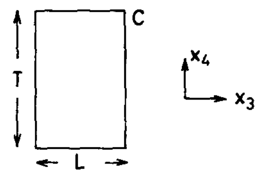

Fig. 2. A Wilson loop $\mathscr{C}$ located in the (x_3, x_4) plane at $x_1 = x_2 = 0$.

Suppose $\mathscr{C}$ is a rectangular Wilson loop as shown in fig. 2. The euclidean equivalent of eq. (1) then reads*

$$\mathscr{E}(x) = \tfrac{1}{2} \lim_{T\to\infty} \frac{\langle \mathrm{Tr}\, U(\mathscr{C})\, \mathrm{Tr}\, U(\dot{\mathrm{p}}_x)\rangle - \langle \mathrm{Tr}\, U(\mathscr{C})\rangle\langle \mathrm{Tr}\, U(\dot{\mathrm{p}}_x)\rangle}{\langle \mathrm{Tr}\, U(\mathscr{C})\rangle} \tag{7}$$

Here, $U(\mathscr{C})$ denotes the ordered product of the link variables along $\mathscr{C}$ and p_x is a plaquette at x parallel to the (x_3, x_4) plane. The leading contribution to $\mathscr{E}(x)$ in the standard strong coupling expansion comes from the minimal connected surface of plaquettes bounded by $\mathscr{C}$ and passing through p_x. $\mathscr{E}(x)$ therefore decreases exponentially, when x goes to infinity in a direction perpendicular to the plane containing $\mathscr{C}$. We thus expect (but did not prove rigorously) that for sufficiently small β

$$|\mathscr{E}(x)| \leq c_1 \exp\left[-c_2(|x_1| + |x_2|)\right], \tag{8}$$

where c_1 and c_2 are constants independent of L. It follows that σ_∞^2 is finite in the strong coupling domain.

We now proceed to calculate σ_∞^2 up to 12th order in β. First note that $\mathscr{E}(x)$ is a function of x_1 and x_2 alone for $L = \infty$. Taking this into account, we may rewrite eq. (7) as

$$\mathscr{E}_\infty(x_1, x_2) = -\frac{\partial}{\partial\tilde{\beta}}\, \alpha(\beta, \tilde{\beta}, x_1, x_2)\big|_{\tilde{\beta}=\beta}, \tag{9}$$

where $\alpha(\beta, \tilde{\beta}, x_1, x_2)$ is a generalized string tension:

$$\alpha(\beta, \tilde{\beta}, x_1, x_2) = -\lim_{T,L\to\infty} \frac{1}{T \cdot L} \ln \langle \mathrm{Tr}\, U(\mathscr{C})\rangle_{\tilde{\beta}, x_1, x_2}. \tag{10}$$

The expectation value $\langle \ldots \rangle_{\tilde{\beta}, x_1, x_2}$ is defined as $\langle \ldots \rangle$ [eq. (5)], but with all couplings β replaced by $\tilde{\beta}$ along the two-dimensional plane parallel to $\mathscr{C}$ and passing through $(x_1, x_2, 0, 0)$. The point of this manipulation is that the generalized string tension (10) can be calculated in precisely the same manner as the ordinary string tension. In particular, the strong coupling graphs needed are exactly those of ref. [2]. Only the

* For simplicity, we only take the contribution of the component E_3 of the electric field parallel to the flux tube into account.

activities change somewhat to include the dependence on $\tilde{\beta}$ and x_1, x_2. As an expansion parameter, we use the variable*

$$u = \frac{I_2(\beta)}{I_1(\beta)} = \tfrac{1}{4}\beta + \mathrm{O}(\beta^3) \tag{11}$$

instead of β (I_ν denotes a Bessel function, cf. ref. [3] sect. 8.445). The results of our computations are

$$\mathscr{E}_\infty(0,0) = \frac{\mathrm{d}u}{\mathrm{d}\beta}\{u^{-1} - 4u^3 - 24u^5 - 92u^7 - \tfrac{5360}{9}u^9 - \tfrac{3078332}{1215}u^{11} + \cdots\}, \tag{12a}$$

$$\mathscr{E}_\infty(1,0) = \frac{\mathrm{d}u}{\mathrm{d}\beta}\{u^3 + 2u^5 + \tfrac{68}{3}u^7 + \tfrac{21524}{405}u^9 + \tfrac{852436}{1215}u^{11} + \cdots\}, \tag{12b}$$

$$\mathscr{E}_\infty(1.1) = \frac{\mathrm{d}u}{\mathrm{d}\beta}\{2u^7 + 12u^9 + \tfrac{350}{3}u^{11} + \cdots\}, \tag{12c}$$

$$\mathscr{E}_\infty(2,0) = \frac{\mathrm{d}u}{\mathrm{d}\beta}\{u^7 + \tfrac{118}{3}u^{11} + \cdots\}, \tag{12d}$$

$$\mathscr{E}_\infty(2,1) = \frac{\mathrm{d}u}{\mathrm{d}\beta}\{4u^{11} + \cdots\}, \tag{12e}$$

$$\mathscr{E}_\infty(3,0) = \frac{\mathrm{d}u}{\mathrm{d}\beta}\{u^{11} + \cdots\}. \tag{12f}$$

When $|x_1| + |x_2| \geq 4$, $\mathscr{E}_\infty(x_1, x_2)$ vanishes to the order considered.

On a lattice, the defining formula (2) for σ^2 takes the form

$$\sigma_\infty^2 = \frac{\sum_{x_1,x_2=-\infty}^{\infty} (x_1^2 + x_2^2)\mathscr{E}_\infty(x_1, x_2)}{\sum_{x_1,x_2=-\infty}^{\infty} \mathscr{E}_\infty(x_1, x_2)}. \tag{13}$$

Inserting the expansions (12) yields

$$\sigma_\infty^2 = 4\{u^4 + 2u^6 + \tfrac{92}{3}u^8 + \tfrac{37724}{405}u^{10} + \tfrac{1412551}{1215}u^{12} + \cdots\}. \tag{14}$$

We have plotted this polynomial in fig. 3. As expected, σ_∞^2 is rising monotonically from zero at $\beta = 0$ to about 0.5 at $\beta = 2$. For $\beta \geq 2$, σ_∞^2 rises steeply, but in that region the strong coupling expansion is hardly reliable: according to the celebrated Monte Carlo calculations of the string tension by Creutz [4], a rapid crossover from strong to weak coupling behaviour takes place just there.

* This choice is motivated by the expansion of the Boltzmann factor in eq. (5) into characters of SU(2); see ref. [2] for details.

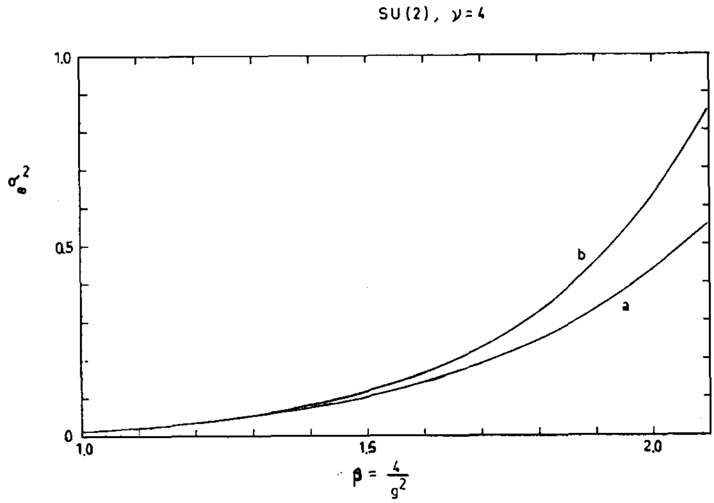

Fig. 3. Plot of σ_∞^2 for the SU(2) guage theory in $\nu=4$ dimensions [eq. (14)]. a and b denote the 10th and 12th order curves respectively.

As explained in sect. 1, $\alpha \cdot \sigma_\infty^2$ should approach a constant for $\beta \gtrsim 2$, if $\sigma_\infty^2 < \infty$ in the continuum limit. Using the strong coupling series for α obtained in ref. [2], we get

$$(\alpha \cdot \sigma_\infty^2)^{-1} = \frac{-1}{4u^4 \ln u}\left\{1 - 2u^2 - 4u^4\left(\frac{20}{3} + \frac{1}{\ln u}\right) + 4u^6\left(\frac{2179}{405} + \frac{2}{\ln u}\right) - u^8\left(\frac{244903}{1215} - \frac{48}{\ln u} - \frac{16}{(\ln u)^2}\right) + \cdots\right\}. \tag{15}$$

The corresponding 6th (a) and 8th (b) order curves are shown in fig. 4. Rather than approaching a constant, $(\alpha \cdot \sigma_\infty^2)^{-1}$ seems to vanish at about*

$$\beta_R \simeq 1.9 . \tag{16}$$

This value is just below the crossover region $\beta \gtrsim 2$ so that we are inclined to conclude that σ_∞^2 diverges at β_R.

To check whether our procedure to determine β_R is significant, we have also calculated $(\alpha \cdot \sigma_\infty^2)^{-1}$ for the $\mathbb{Z}_2$ gauge theory in $\nu=3$ and 4 dimensions. With $x = \mathrm{tgh}\beta$, the results are respectively

$$(\alpha \cdot \sigma_\infty^2)^{-1} = \frac{-1}{2x^4 \ln x}\left\{1 - 3x^2 - x^4\left(11 + \frac{2}{\ln x}\right) + x^6\left(19 + \frac{4}{\ln x}\right) - x^8\left(42 - \frac{18}{\ln x} - \frac{4}{(\ln x)^2}\right) + \cdots\right\}, \quad (\nu=3), \tag{17}$$

* We choose the symbol β_R because the phenomenon observed here is very much the same as the surface roughening of a phase boundary in three dimensions (see below).

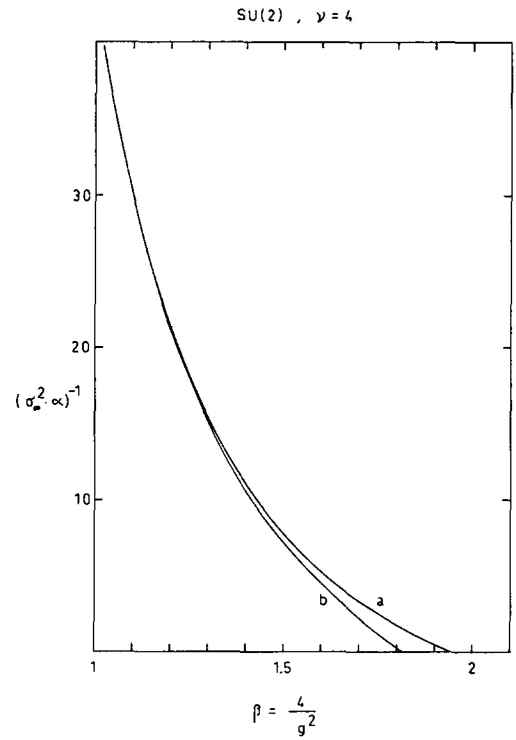

Fig. 4. Plot of $(\alpha \cdot \sigma_\infty^2)^{-1}$ versus β for the SU(2) gauge theory in $\nu = 4$ dimensions [eq. (15)]. a and b denote the 6th and 8th order curves respectively.

$$(\alpha \cdot \sigma_\infty^2)^{-1} = \frac{-1}{4x^4 \ln x}\left\{1 - 3x^2 - x^4\left(21 + \frac{4}{\ln x}\right)\right.$$
$$\left. + x^6\left(15 + \frac{8}{\ln x}\right) - x^8\left(176 - \frac{40}{\ln x} - \frac{16}{(\ln x)^2}\right) + \cdots\right\}, \quad (\nu = 4)\,. \qquad (18)$$

The corresponding curves plotted in figs. 5 and 6 suggest (with subjective errors)

$$\beta_R = 0.51 \pm 0.02\,, \qquad (\nu = 3), \qquad (19)$$

$$\beta_R = 0.44 \pm 0.02\,, \qquad (\nu = 4)\,. \qquad (20)$$

For $\nu = 3$, the width σ_∞^2 measures the thickness of an interface between two regions of opposite magnetization in the dual Ising model at inverse temperature $\beta^* = -\frac{1}{2}\ln x$. The width of the interface has previously been found to diverge at $\beta_R^* \simeq 0.39$ corresponding to $\beta_R \simeq 0.50$ [7]. This value is in good agreement with our result (19). Note also that β_R is well below the critical inverse temperature $\beta_c = 0.76\ldots$, which makes the reliability of the strong coupling expansion plausible. Concerning the four-dimensional case, we remark that β_R is practically equal to the self-dual point

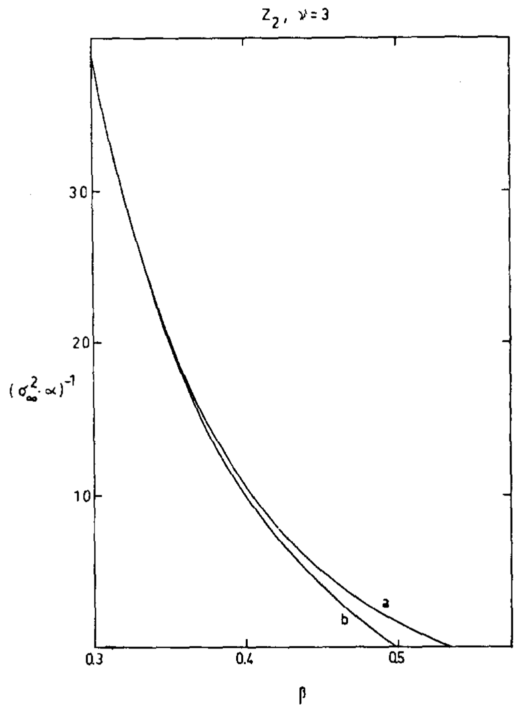

Fig. 5. Same as fig. 4, but for the $\mathbb{Z}_2$ gauge theory in $\nu = 3$ dimensions.

$\beta_c = 0.44 \ldots$. This coincidence is hardly accidental, but we have no explanation for it at present. Altogether, the $\mathbb{Z}_2$ results encourage the conclusion that the roughening transition we found for the SU(2) model is not a mere artifact of our procedure.

3. String model predictions for σ^2

3.1. APPROXIMATE CALCULATION OF $\mathscr{E}(x)$

The string model provides an approximation for correlation functions of Wilson loops, which is expected to be sensible for large, smooth loops [5, 6]. By eq. (7), $\mathscr{E}(x)$ is proportional to

$$G(\mathscr{C}, \tilde{\mathscr{C}}) = \langle \mathrm{Tr}\, U(\mathscr{C})\, \mathrm{Tr}\, U(\tilde{\mathscr{C}})\rangle - \langle \mathrm{Tr}\, U(\mathscr{C})\rangle\langle \mathrm{Tr}\, U(\tilde{\mathscr{C}})\rangle\,, \tag{21}$$

where $\mathscr{C}$ is the Wilson loop shown in fig. 2 and $\tilde{\mathscr{C}}$ is a small loop located at x. In its crudest form, the string model gives

$$G(\mathscr{C}, \tilde{\mathscr{C}}) \propto \exp\left[-M^2 A(\mathscr{C}, \tilde{\mathscr{C}})\right], \tag{22}$$

where M^2 is the string tension and $A(\mathscr{C}, \tilde{\mathscr{C}})$ denotes the area of the smallest connected surface with boundary $\mathscr{C} \cup \tilde{\mathscr{C}}$.

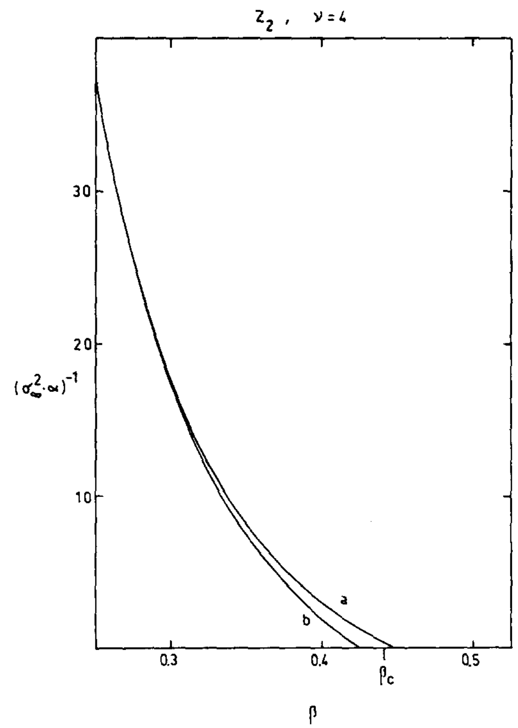

Fig. 6. Same as fig. 4, but for the $\mathbb{Z}_2$ gauge theory in $\nu = 4$ dimensions.

It is impossible to calculate $A(\mathscr{C}, \tilde{\mathscr{C}})$ in general. Since we are only interested in what happens, when $\mathscr{C}$ is much larger than $\tilde{\mathscr{C}}$ and the distance between $\tilde{\mathscr{C}}$ and the x_1, $x_2 = 0$ plane, we may replace $\mathscr{C}$ by a large circle of radius R. For $\tilde{\mathscr{C}}$ we choose a small circle parallel to $\mathscr{C}$ with radius r and center $(h, 0, 0, 0)$, $h \geq 0$ (cf. fig. 7). For such an arrangement of loops $\mathscr{C}$ and $\tilde{\mathscr{C}}$, the minimal surface can be determined explicitly.

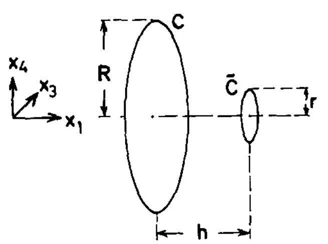

Fig. 7. Configuration of Wilson loops used to calculate σ^2 from the string model. $\mathscr{C}$ and $\tilde{\mathscr{C}}$ are circles in the planes $x_1 = 0$ and $x_1 = h$, respectively.

Namely, it is a surface of revolution about the x_1 axis: denoting the aximuthal angle by φ, we have

$$x(z,\varphi)=(z,0,\rho(z)\cos\varphi,\rho(z)\sin\varphi)\,,\qquad 0\leqslant z\leqslant h\,, \tag{23}$$

$$\rho(z)=\frac{1}{\omega}\,\mathrm{ch}\,\omega(z-z_0)\,. \tag{24}$$

The constants ω and z_0 must be adjusted such that

$$\rho(0)=R=\frac{1}{\omega}\,\mathrm{ch}\,\omega z_0\,, \tag{25a}$$

$$\rho(h)=r=\frac{1}{\omega}\,\mathrm{ch}\,\omega(z_0-h)\,. \tag{25b}$$

The area of the surface (23) is

$$A=\frac{\pi}{\omega}\{h+R_+\sqrt{\omega^2R^2-1}-\mathrm{sign}\,(z_0-h)r_+\sqrt{\omega^2r^2-1}\}\,. \tag{26}$$

For fixed h, r and $R\to\infty$, the transcendental equations (25) can be solved,

$$\omega=\frac{1}{h}\ln\frac{R}{r}+\mathrm{O}\left[\left(\ln\frac{R}{r}\right)^{-2}\right],\qquad z_0\geqslant h\,, \tag{27}$$

so that

$$A=\pi(R^2-r^2)+\pi\frac{h^2}{\ln(R/r)}+\mathrm{O}\left[\left(\ln\frac{R}{r}\right)^{-4}\right]. \tag{28}$$

From this result we first of all see that $\mathscr{E}(x)$ approaches a value independent of h in the limit $R=\infty$, h, r fixed. More precisely,

$$\mathscr{E}(x)\propto\exp\left[-M^2\frac{\pi x_\perp^2}{\ln(R/r)}\right],\qquad (R\gg|x_\perp|,r) \tag{29}$$

so that the width

$$\sigma^2=\frac{\ln(R/r)}{M^2\pi} \tag{30}$$

diverges logarithmically for $R\to\infty$. In other words, the string model suggests that

$$\sigma^2\sim\sigma_0^2\ln(L/\lambda)\qquad \text{for } L\to\infty\,, \tag{31}$$

for some constants σ_0^2 and λ. The acutal value for λ cannot be extracted from the string model, because this would require to take the local limit $r\to 0$, whereas the string approximation assumes $Mr\gg 1$.

3.2. HAMILTONIAN CALCULATION OF THE WIDTH OF THE STRING WAVE FUNCTIONAL

We here present an alternative derivation of the scaling law (31) by a straightforward quantum mechanical argument. We thus consider a string

$$x_k(s) = \left(x_1(s), x_2(s), \frac{L}{\pi}s\right), \qquad 0 \leq s \leq \pi, \tag{32}$$

with fixed ends

$$x_\perp(0) = x_\perp(\pi) = 0, \qquad x_\perp = (x_1, x_2). \tag{33}$$

As a hamiltonian we choose*

$$H = M^2 L + \frac{\pi}{2M^2 L}\int_0^\pi ds (p_\perp^2 + M^4 x_\perp'^2), \qquad x_\perp' = \frac{d}{ds} x_\perp \tag{34}$$

where $p_\perp$ is the canonical momentum

$$p_\perp(s) = -i\frac{\delta}{\delta x_\perp(s)}. \tag{35}$$

Correspondingly, the ground-state wave functional $\psi[x_\perp]$ is gaussian:

$$\psi[x_\perp] = \exp\left[-\tfrac{1}{2}\int_0^\pi ds\, dr x_\perp(s)\cdot x_\perp(r) H(s,r)\right], \tag{36}$$

$$H(s,r) = \frac{2M^2}{\pi}\sum_{n=1}^\infty n \sin(ns)\sin(nr). \tag{37}$$

We now ask for the probability $P(x_\perp)\, d^2x_\perp$ that the string passes through an area $d^2x_\perp$ at $x = (x_1, x_2, \frac{1}{2}L)$ (see fig. 8). This will give us a measure for the width of the wave functional (31). $P(x_\perp)$ is also gaussian, of course, and a little algebra gives

$$P(x_\perp) = \frac{1}{\pi\delta^2}\exp\left(-\frac{x_\perp^2}{\delta^2}\right), \tag{38}$$

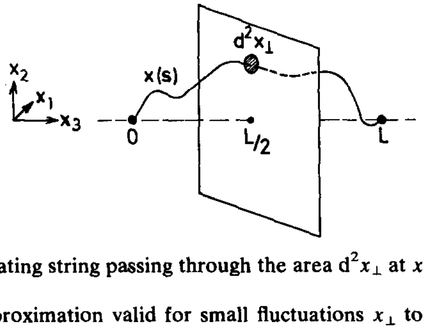

Fig. 8. Fluctuating string passing through the area $d^2x_\perp$ at $x = (x_1, x_2, \frac{1}{2}L)$.

* This is the gaussian approximation valid for small fluctuations $x_\perp$ to the full hamiltonian of the relativistic string model.

$$\delta^2 = \frac{1}{\pi M^2} \sum_{k=0}^{k_{\max}} \frac{1}{k+\frac{1}{2}}, \qquad k_{\max} = L/\lambda \,. \tag{39}$$

Here, we introduced a lower cutoff λ on the wavelengths of the string vibrations. One can interpret λ as an intrinsic thickness of the string or merely as an effective parameter, beyond which the string model is no longer applicable to chromo-electric flux tubes. We now see that as $L \to \infty$, the width δ^2 of the string wave functional diverges logarithmically

$$\delta^2 \sim \frac{1}{\pi M^2} \ln \frac{L}{\lambda} \,. \tag{40}$$

Note that this is an infrared divergence: as $L \to \infty$ more and more vibrations have wavelengths l larger than λ. The sum (34) can then be approximated by

$$\delta^2 \sim \frac{1}{\pi M^2} \int_\lambda^L \frac{dl}{l}, \tag{41}$$

which diverges for large wavelengths l. In particular, the divergence of the width δ^2 is independent of precisely how the cutoff λ is introduced.

4. Conclusions

Our strong coupling calculations suggest that the SU(2) lattice gauge theory in four dimensions undergoes a "roughening" transition at about $\beta_R = 1.9$, where the transversal width of the field energy distribution $\mathscr{E}(x)$ of an infinitely long flux tube diverges. This transition is very similar to the roughening of a phase boundary in the three-dimensional Ising magnet [7, 8], which in fact is dual to the $\mathbb{Z}_2$ gauge theory. Based on this correspondence, Itzykson [9] and Hasenfratz [10] have recently pointed out that transitions of the roughening type might be a common phenomenon in gauge theories, too.

The significance of the roughening transition for the continuum gauge theory is that the width σ^2 of the gluon field energy distribution $\mathscr{E}(x)$ in the presence of a static quark-antiquark pair separated by a distance L diverges as $L \to \infty$. How precisely σ^2 diverges, is difficult to derive from the lattice gauge theory. The string model approximation to the Wilson loop correlation functions, on the other hand, predicts

$$\sigma^2 \sim \sigma_0^2 \ln\left(\frac{L}{\lambda}\right), \qquad (L \to \infty) \tag{31}$$

with some constants σ_0^2 and λ. If we take the string model literally, the divergence (31) is explained to arise from the large, quantum mechanical fluctuations of a thin "bare" flux tube connecting the quark with the antiquark. We finally mention that the string model has its counterpart in the theory of phase boundaries, where it is called the drumhead model [11].

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