GRAND UNIFICATION AT THE SUBCOMPONENT LEVEL

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Received 7 July 1980

The Casalbuoni–Gatto subcomponent model of quarks and leptons is generalized allowing for an arbitrary number of subcomponents. It is shown that there are only a limited number of cases where the subcolour can be embedded in a semi-simple grand unification scheme. The most interesting models lead to an SU(7) ⊗ SU(7) grand unification at the subcomponent level. In one of them there is also a natural place for a hypercolour (technicolour) group $SU(2)_{hc}$.

In a recent paper [1] Casalbuoni and Gatto proposed models for composite quarks and leptons based on the "conventional" idea of confinement. The main assumption is the existence of a confined $SU(3)_{sc}$ "subcolour". The quarks and leptons contain three subcomponents in an antisymmetric state under $SU(3)_{sc}$ in the same way as baryons are antisymmetric three-quark composites in $SU(3)_c$ (c is colour).

As was shown in ref. [1], there are only four different possibilities if the full gauge group is $SU(n) \otimes SU(3)_{sc}$ ($n \geqslant 6$) and if the following assumptions are fulfilled:

(1) The subcomponent fermions are in the fundamental $(n, 3)$ representation of $SU(n) \otimes SU(3)_{sc}$. Their antiparticles are in the complex conjugate representation $(n^*, 3^*)$. This assures the absence of triangular anomalies at the subcomponent level.

(2) The quarks and leptons are three-subcomponent spin-1/2 states totally symmetric under $SU(n) \otimes Sl(2, C)$, where $Sl(2, C)$ is the Lorentz group for undotted and dotted Weyl-spinor indices. (The symmetry is the consequence of antisymmetry under $SU(3)_{sc}$.)

(3) The gauge group $SU(n)$ contains $SU(5) \otimes SU(n-5)_H$, where SU(5) is the grand unification group of Georgi and Glashow [2] and $SU(n-5)_H$ is a "horizontal" gauge group connecting the different standard $(10 \oplus 5^*)$ SU(5) families of quarks and leptons.

(4) The standard SU(5) families span a representation of the horizontal gauge group $SU(n-5)_H$.

The basic processes for proton decay: $u + d \to \bar{u} + e^+$ or $u + u \to \bar{d} + e^+$, are subconstituent-rearrangement reactions. Consequently, the spatial extension of quarks and leptons (the confinement radius for subcolour) has to be of the order of the inverse of the SU(5) grand unification mass [3] (in any way, smaller than 10^{-14} GeV^{-1}) in order to avoid an contradiction with the present lower limit of the proton lifetime τ_p. (For a recent review of experimental limits on τ_p see e.g. ref. [4].) On the other hand, the confinement radius for ordinary colour is about 1 GeV^{-1}. The large difference in the scale parameters of $SU(3)_{sc}$ and $SU(3)_c$ makes it rather difficult (if not impossible) to imagine that subcolour and colour could have the same strength at some common grand-unification scale. The situation changes, however, in models with a larger subcolour group $SU(2k+1)_{sc}$ ($k \geqslant 2$) where quarks and leptons are composed of $(2k+1)$ subcomponents.

The purpose of the present letter is to look for $SU(n) \otimes SU(2k+1)_{sc}$ subcomponent models satisfying the above assumptions (1)–(4) with the only change that we allow for any odd number $(2k+1)$ of subcomponents inside the quarks and leptons. It turns out that, besides a few exceptional cases, there are two infinite series of such models exhausting the full set of possible k values ($k = 1, 2, 3, \ldots$). Among these cases there are, however, only 11 different possibilities with $2k + 1 \leqslant n$, where a semi-simple grand unification in

[1] Supported by Bundesministerium für Forschung und Technologie, Bonn, Fed. Rep. Germany.

$SU(n) \otimes SU(n)$ is possible. We shall see below that only 2 of these 11 models seem to be interesting for a reasonable grand unification scheme, both of them with $n = 7$. The subcolour groups are $SU(5)_{sc}$ and $SU(7)_{sc}$ in the two cases, respectively.

Let us first look for the possible $SU(n) \otimes SU(2k+1)_{sc}$ subcomponent models with $2k + 1$ subcomponents in quarks and leptons satisfying the above assumptions. We shall consider only the cases $n \geqslant 7$ because it can be easily shown that for $n = 6$ there is strictly speaking no solution. The only possibility would be the $n = 6$ model discussed in ref. [1] but it does not really satisfy assumption (4) as the $U(1)_H$ quantum numbers are different for the 10 and 5^* within a SU(5) family.

According to assumption (2) we have to look for the $(2k + 1)$-subcomponent states totally symmetric with respect to $SU(n) \otimes Sl(2, C)$. In the Lorentz group we use undotted and dotted Weyl-spinor indices, therefore $Sl(2, C) \equiv SU(2)_l \otimes SU(2)_r$, where $SU(2)_l$ and $SU(2)_r$ act on undotted and dotted indices, respectively. We shall denote, as usual, the Young tableau with rows of length $\lambda_1, \lambda_2, \ldots, \lambda_s$ by $(\lambda_1, \lambda_2, \ldots, \lambda_s)$. The low lying spin-1/2 composite fermion states can belong, in $SU(2)_l \otimes SU(2)_r$, either to the Young tableau $(a + b + 1, a + b, a, a)$ or to the Young tableau $(a + b + 1, a + b + 1, a + 1, a)$ for $a, b \geqslant 0$. In the two cases the spin-1/2 is provided by the column of length 1 and 3, respectively. The columns of length 4 and 2 are scalars. In the former case the only possibility is to put 2 undotted and 2 dotted indices in the column, whereas in the latter one it is possible to put either two undotted or two dotted indices. The number of different spin configurations is, therefore, in both cases equal to $2b + 1$.

The total symmetry under $SU(n) \otimes Sl(2, C)$ is realized by identical Young tableaus in $Sl(2, C)$ and $SU(n)$. For the standard low mass SU(5) families we need either 10 and 5^* or 10^* and 5 in the reduction $SU(n) \supset SU(5) \otimes SU(n-5)_H$. In the former case the quarks and leptons have to be composed of the subcomponents in $(n, 3)$, whereas in the latter case of the antisubcomponents in $(n^*, 3^*)$. The unwanted right-handed $10 \oplus 5^*$ states with V + A couplings in the Glashow–Weinberg–Salam $SU(2)_v \otimes U(1)_y$ are assumed to have large masses [1]. For the states of the three $(n, 3)$ subcomponents there are four possibilities:

(a) 10 and 5^* in SU(5) with equivalent representations of $SU(n-5)_H$;

(b) 10^* and 5 in SU(5) with equivalent representations of $SU(n-5)_H$;

(c) 10 and 5 in SU(5) with complex conjugate representations of $SU(n-5)_H$;

(d) 10^* and 5^* in SU(5) with complex conjugate representations of $SU(n-5)_H$.

What is left is a straightforward discussion of the $SU(5) \otimes SU(n-5)_H$ reduction of the two sorts of Young tableaux introduced above. In cases (a) and (b) we have to require identical Young tableaux in $SU(n-5)_H$, whereas in cases (c) and (d) the Young tableaux in $SU(n-5)_H$ have to be complex conjugates of the 10 and 5 in SU(5). The condition for the complex conjugate Young tableaux can be best imposed on the sequence of "horizontal and vertical separating lines" illustrated in fig. 1. Namely, in complex conjugate Young tableaux both the horizontal and the vertical separating lines have to be equal in opposite order.

The lengthy but otherwise straightforward investigation of all the possibilities gives the following solutions:

(1) In $SU(7) \otimes SU(5 + 2b)_{sc}$ ($b = 0, 1, 2, \ldots$) the Young tableau $(b + 2, b + 1, 1, 1)$ with $(b + 1)$ different spin structures and (1) = doublet in $SU(2)_H$.

(2) In $SU(7) \otimes SU(3 + 2b)_{sc}$ ($b = 0, 1, 2, \ldots$) the Young tableau $(b + 1, b + 1, 1)$ with $(b + 1)$ different spin structures and (0) = singlet in $SU(2)_H$. $b = 0$ corresponds to the $SU(7) \otimes SU(3)_{sc}$ model in ref. [1].

(3) In $SU(12) \otimes SU(7)_{sc}$ the Young tableau (2, 2, 2, 1) [=56 628-dimensional representation of SU(12)] with one spin structure and $(1, 1, 1)$ = 35-plet in $SU(7)_H$.

(4) In $SU(10) \otimes SU(9)_{sc}$ the Young tableau (3, 3, 2, 1) [=304 920-dimensional representation of SU(10)]

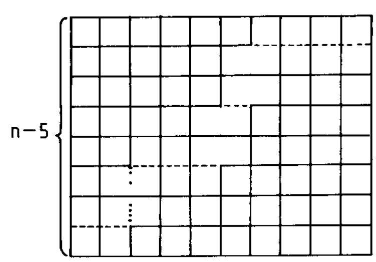

Fig. 1. The complex conjugate Young tableaux of $SU(n-5)_H$ put together in a quadrangle consisting of columns of length $n - 5$. The "horizontal separating lines" are dashed, the "vertical" ones are dotted.

with 2 different spin structures and (3, 2, 1, 1) = 175-plet in $SU(5)_H$.

(5) In $SU(8) \otimes SU(3)_{sc}$ the Young tableau (2, 1) [= 168-dimensional representation of SU(8)] with 2 different spin structures and (1) = triplet in $SU(3)_H$. This is one of the models in ref. [1].

(6) In $SU(10) \otimes SU(9)_{sc}$ the Young tableau (3, 2, 2, 2) [= 110 880-dimensional representation of SU(10)] with one spin structure and (3, 2, 1, 1) = 175-plet in $SU(5)_H$.

(7) In $SU(10) \otimes SU(9)_{sc}$ the Young tableau (3, 3, 2, 1) = [= 304 920-dimensional representation of SU(10)] with 2 different spin structures and (3, 3, 1) = 315-plet in $SU(5)_H$.

(8) In $SU(8) \otimes SU(3)_{sc}$ the Young tableau (1, 1, 1) [= 56-dimensional representation of SU(8)] with 1 spin structure and (1) = triplet in $SU(3)_H$. This is one of the models in ref. [1].

If we require $(2k + 1) \leqslant n$, in order to have a semi-simple grand unification $SU(n) \otimes SU(n)$, then the two infinite sequences in (1) and (2) are reduced to the following cases:

(1a) In $SU(7) \otimes SU(5)_{sc}$ the Young tableau (2, 1, 1, 1) (= 224-dimensional representation in SU(7)] with one spin structure and (1) = doublet in $SU(2)_H$.

(1b) In $SU(7) \otimes SU(7)_{sc}$ the Young tableau (3, 2, 1, 1) [= 2940-dimensional representation of SU(7)] with two different spin structures and (1) = doublet in $SU(2)_H$.

(2a) In $SU(7) \otimes SU(3)_{sc}$ the Young tableau (1, 1, 1) [= 35-dimensional representation of SU(7)] with one spin structure and (0) = singlet in $SU(2)_H$. This is one of the models in ref. [1].

(2b) In $SU(7) \otimes SU(5)_{sc}$ the Young tableau (2, 2, 1) [= 490-dimensional representation of SU(7)] with two different spin structures and (0) = singlet in $SU(2)_H$.

(2c) In $SU(7) \otimes SU(7)_{sc}$ the Young tableau (3, 3, 1) [= 3528-dimensional representation of SU(7)] with 3 different spin structures and (0) = singlet in $SU(2)_H$.

These are the 11 possibilities mentioned in the introduction.

Among these 11 cases, 3 [(5), (8) and (2a)] have $SU(3)_{sc}$ as subcolour group [1] which seems to be too small for a unification with $SU(3)_c$. The rather exotic cases (3), (4), (6) and (7) have a very large number of SU(5) families which ruin the asymptotic freedom at high energies; therefore we do not consider them any more here. What is left is either $SU(7) \otimes SU(5)_{sc}$ [cases (1a) and (2b)] or $SU(7) \otimes SU(7)_{sc}$ [cases (1b) and (2c)] both leading to an $SU(7) \otimes SU(7)$ grand unification.

Semi-simple grand unification in $SU(7) \otimes SU(7)$ means that above some point the coupling constants of the two SU(7) factors become equal. This has to be above the energy of the substructure of quarks and leptons when the subcomponents are the relevant degrees of freedom in the renormalization group equations. The subcomponents are in the fundamental representation (7, 7) of $SU(7) \otimes SU(7)$ and for equal coupling constants the lagrangian has a discrete symmetry under the exchange of the two SU(7) factors. Below the grand unification point this symmetry is spontaneously broken in such a way that the subcolour coupling constant becomes large at much higher energies (say, near $M_X = 10^{14}$ GeV) than the QCD coupling constant α_s. Below M_X the effective fermion degrees of freedom entering the renormalization group equations are the standard SU(5) families of quarks and leptons composed of (5 or 7) subcomponents. The other possible composite states (e.g. higher SU(5) representations etc.) are assumed to lie near M_X.

We begin the qualitative discussion of the renormalization group equations for the coupling constants [5] in the simpler case of $SU(7)_{sc}$ subcolour. In the usual notation $\alpha_j \equiv g_j^2/4\pi$ we have up to one-loop order $[t \equiv \ln (Q^2/\mu^2)]$:

$$d\alpha_j^{-1}/dt = \beta_{0j}/4\pi \,. \tag{1}$$

The constant β_{0j} depends on the gauge group and on the number and transformation properties of matter fields:

$$\beta_{0j} = \tfrac{11}{3}\, C(G_j) - \tfrac{4}{3}\, T(R_j) \,. \tag{2}$$

If the generators of the gauge group G satsify

$$[\lambda_a, \lambda_b] = \mathrm{i} f_{abc} \lambda_c \,, \tag{3}$$

then $C(G)$ is defined by

$$f_{abc} f_{dbc} = \delta_{ad} C(G) \,. \tag{4}$$

Denoting the representation of G spanned by the fermions by R and the generators in the representation R by Λ_a, the definition of $T(R)$ is the following:

$$\mathrm{Tr}\,(\Lambda_a \Lambda_b) = \delta_{ab} T(R) \,. \tag{5}$$

Let us assume that the standard $SU(3)_c \otimes SU(2)_v \otimes U(1)_v$ interactions are first unified in SU(5) at some energy scale M_X which coincides with the energy scale of the quark and lepton substructure. (The equality of these two energy scales is plausible because the superheavy SU(5) gauge bosons mediate the proton decay processes in the same way as subconstituent interchange does [1].) Below M_X we have the effective gauge symmetry $SU(3)_c \otimes SU(2)_V \otimes SU(2)_H \otimes U(1)_Y$. Above M_X we have $SU(5) \otimes SU(2)_H \otimes SU(7)_{sc}$ up to the grand unification point M_{GU}. Above M_{GU} the full $SU(7) \otimes SU(7)$ gauge symmetry is effective. For definiteness let us put $M_X \equiv 2.7 \times 10^{14}$ GeV corresponding to the recent best estimate of the SU(5) unification point [6]. (See also refs. [7,8]. Note that in ref. [6] two-loop contributions are also included, whereas here only one-loop equations are considered for a qualitative orientation.) Besides we assume that $SU(7) \otimes SU(7)$ grand unification is at the Planck mass $M_{GU} = M_P = 1.2 \times 10^{19}$ GeV. The resulting picture of the coupling constant variation is given in fig. 2.

A general consequence of the scheme is that the value of the coupling constants at the SU(5) unification point $\alpha \approx 1/18$ is larger than usual [6–8]. This follows from the requirement of unification of the ordinary interactions with the subcolour. This is achieved by a larger number of SU(5) families. In fig. 2 the three $SU(2)_H$ singlet families in the SU(7) 3528-plet are put below $M_0 = 2M_W \equiv 1.6 \times 10^2$ GeV, where the integration of the renormalization group equations (1) is started. The other four SU(5) families from the SU(7) 2940-plet, which are doublets in $SU(2)_H$, are put to the mass scale $M_2 = 5.6 \times 10^3$ GeV. (Of course,

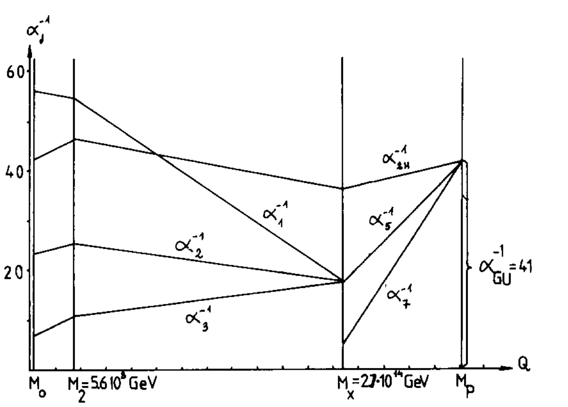

Fig. 2. The variation of the coupling constants in the SU(7) ⊗ SU(7) model with $SU(7)_{sc}$ subcolour.

in reality the $SU(2)_H$ doublet SU(5) families may be spread out in some range around M_2. This does not, however, change the qualitative picture.) M_2 is chosen in such a way that the QCD coupling constant $\alpha_s \equiv \alpha_3$ starts from $\alpha_3^{-1} = 7$ at M_0 [6,8] and meets the SU(5) unification point at M_X. The coupling constant of $SU(7)_{sc}$ subcolour is started at M_X from the value $\alpha_7^{-1} = 5$. (This is taken here generally as the "critical value" of the coupling constant in unbroken gauge theories where the abrupt transition to confinement occurs.)

The interesting consequence of this scheme is that the presently known three SU(5) families are singlets under $SU(2)_H$. They can participate in horizontal weak interactions at most only through a small mixing with the high lying $SU(2)_H$ doublet families. This explains nicely the present absence of any experimental evidence for horizontal interactions.

Once M_0, M_2, M_X and M_{GU} are fixed the change of the coupling constants for all the interactions in fig. 2 are already uniquely determined. The values of the constants β_{0i} in eq. (1) for the different energy ranges are collected in table 1. The values of $\alpha^{-1} \approx 56$ and $\alpha_2^{-1} \approx 24$ at M_0 give from the one-loop SU(5) relations [3]

$$\alpha_{em}^{-1} = \tfrac{5}{3}\alpha_1^{-1} + \alpha_2^{-1} , \qquad \sin^2\theta_W = \tfrac{1}{6} + \tfrac{5}{6}\alpha_{em}/\alpha_3 , \tag{6}$$

$\alpha_{em}(M_0^2) \approx 1/117$ for the electromagnetic coupling constant and $\sin^2\theta_W \approx 0.20$ for the Weinberg angle. The former value is somewhat too large compared to the right one $\alpha_{em}(M_0^2) \approx 1/130$ [7]. Two-loop corrections and contributions of possible low mass scalars (dynamical Higgs mesons) are, however, neglected here. The present picture is, therefore, only qualitative.

The qualitative picture of grand unification with $SU(5)_{sc}$ subcolour is basically similar to the case of $SU(7)_{sc}$. The only essential difference is that in the SU(7) factor containing $SU(5)_{sc}$ there is room for some other interaction besides $SU(5)_{sc}$. A natural choice is an unbroken SU(2) factor which is unified with $SU(5)_{sc}$ at superhigh energies. Being a smaller group its coupling constant becomes large more slowly, that is at much smaller energies than $M_X \approx 10^{14}$ GeV where the $SU(5)_{sc}$ coupling becomes large. It is tempting to identify this SU(2) with the "hypercolour"

Table 1
The constants β_{0j} in eq. (1) for the model with $SU(7)_{sc}$ subcolour.

Group		$M_0 < M < M_2$	$M_2 < M < M_X$	$M_X < M < M_{GU}$
$U(1)_Y$	β_{01}	−4	−28/3	
$SU(2)_V$	β_{02}	10/3	−2	
$SU(2)_H$	β_{02H}	22/3	−8/3	8/3
$SU(3)_C$	β_{03}	7	5/3	
$SU(5)$	β_{05}			41/3
$SU(7)_{sc}$	β_{07}			21

("technicolour") $SU(2)_{hc}$ [9,10] having a confinement radius about 10^{-4} GeV$^{-1} \equiv M_{hc}^{-1}$. (This order of magnitude is needed for the W and Z masses in $SU(2)_v \otimes U(1)_y$.) The pseudoscalar Goldstone bosons due to the spontaneous symmetry breaking of global chiral $SU(2)_{hc}$ symmetry could play the role of dynamical Higgs mesons needed for the breaking of $SU(2)_v \otimes U(1)_y$ [9,10].

Let us assume that the subcolour $SU(5)_{sc}$ and hypercolour $SU(2)_{hc}$ couplings are equal at some energy $M_7 > M_X$. Above M_7 there is the $SU(7) \supset SU(5)_{sc} \otimes SU(2)_{hc}$ symmetry. Its coupling constant becomes equal to the coupling constants of $SU(5) \otimes SU(2)_H$, contained in the other SU(7) factor, at some still higher grand unification point which is identified also here, for definiteness, with the Planck mass: $M_{GU} = M_P$. As before, SU(5) unification is taken to coincide with the energy scale of the subcomponent structure at $M_X = 2.7 \times 10^{14}$ GeV. This fixes the whole scheme apart from the quark–lepton family content below M_X. A simple assumption is that there are now four standard SU(5) families below $M_0 = 2M_W$ (where the renormalization broup equations are started). Two of them are scalars in $SU(2)_H$ with different spin structures and there is one $SU(2)_H$ doublet [corresponding, respectively, to the cases (2b) and (1a) above]. In order that the SU(5) grand unification point meets at M_X with the QCD coupling it is necessary to assume here that the V + A partners of the four standard V − A SU(5) families occur in the intermediate energy range at M_+ between M_0 and M_X. (In the previous $SU(7)_{sc}$ case the V + A families were put to M_X.) Putting again $\alpha_3(M_0^2)^{-1} = 7$ and $\alpha^{-1} = 5$ as the critical point for the transition to confinement for $SU(5)_{sc}$ and $SU(2)_{hc}$, M_+ turns out to be $M_+ = 8.7 \times 10^5$ GeV. The resulting picture for the coupling constant variation is given in fig. 3. The values of the constants β_{0j} in eq. (1) for the different energy ranges are given in table 2. The obtained values for $\alpha_1^{-1} \approx 56$ and $\alpha_2^{-1} \approx 23$ at M_0 correspond in eq. (6) again to $\alpha_{em}^{-1} \approx 117$ and $\sin^2\theta_W \approx 0.20$. (There is no change compared to fig. 2 because eq. (6) holds in SU(5) up to the one-loop level independently of the number of fermion families.)

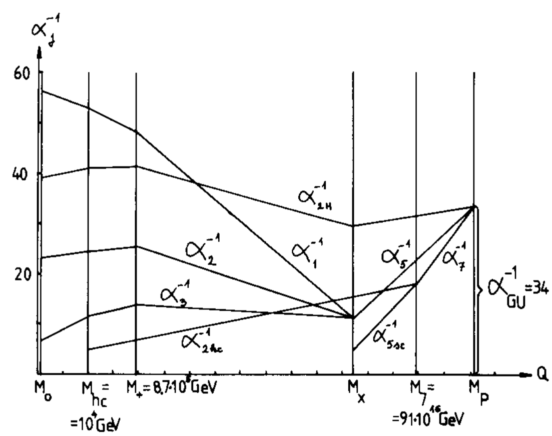

Fig. 3. The variation of the coupling constants in the SU(7) ⊗SU(7) model with $SU(5)_{sc}$ subcolour.

There are, of course, also other possibilities for the symmetry breaking patterns in SU(7) ⊗ SU(7). In particular, it is not necessary to have an intermediate SU(5) unification of $SU(3)_c \otimes SU(2)_v \otimes U(1)_y$. In the $SU(5)_{sc}$ model it is possible, for instance, to consider the unification point of $SU(5)_{sc} \otimes SU(2)_{hc}$ at $M_7 = 9.1 \times 10^{16}$ GeV simultaneously as a grand unification for the whole SU(7) ⊗ SU(7): $M_{GU} \equiv M_7 < M_P$. The SU(5) symmetry can be broken already at this point to $SU(3)_c \otimes SU(2)_v \otimes U(1)_y$. This scheme leads at M_0 to $\alpha_{em}^{-1}(M_0^2) \approx 137$ and $\sin^2\theta_W \approx 0.20$. The V + A families come out in this case near $M_+ = 3.2$

Table 2
The constants β_{0j} in eq. (1) for the model with $SU(5)_{sc}$ subcolour.

Group		$M_0 < M < M_{hc}$	$M_{hc} < M < M_+$	$M_+ < M < M_X$	$M_X < M < M_7$	$M_7 < M < M_{GU}$
$U(1)_Y$	β_{01}	−16/3	−20/3	−12		
$SU(2)_V$	β_{02}	2	2/3	−14/3		
$SU(2)_H$	β_{02H}	7/3	1	−4	8/3	8/3
$SU(3)_c$	β_{03}	17/3	13/3	−1		
SU(5)	β_{05}				41/3	41/3
$SU(5)_{sc}$	β_{05sc}				41/3	
SU(7)	β_{07}					21

$\times\ 10^6$ GeV. What is obviously missing yet is the understanding of the mechanism of symmetry breaking in these models.

Another more technical point where the above discussion can be extended concerns the spin structure of the composite states. Assumption (2) in the introduction was used here in a stronger form. Namely, it was required that the $Sl(2, C) = SU(2)_l \otimes SU(2)_r$ symmetry can be extended to a conformal SU(2, 2) symmetry. For $k \geqslant 2$ this is not always necessary. The consequence is that sometimes there are more possibilities for the wave functions.

It is a pleasure to thank G. Kramer, B. Schrempp and F. Schrempp for helpful discussions.

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