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INITIAL STATE RADIATION FOR e^+e^- ANNIHILATION INTO JETS

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A method is described which enables one to take into account initial state radiation to the process $e^+e^- \to$ hadrons. It treats exactly both the photon emission and the angular distribution of the produced particles, once this angular distribution is given for the non-bremsstrahlung case. The method is such that it can easily be used in conjunction with any Monte Carlo program for the production of hadrons.

1. Introduction

With the advent of quantum chromodynamics one can construct models [1] for the production of jets in e^+e^- annihilation. Usually the ingredients of such a model are translated into a Monte Carlo program which simulates the production of the hadronic events in e^+e^- annihilation. These programs generate two, three, or possibly four jets consisting of light and heavy quarks and gluons, and contain a mechanism to fragment these partons into hadrons.

In order to minimize the uncertainties in comparing these models to experimental data, the radiative corrections should be built in. There are two reasons to restrict these radiative corrections to initial state radiation only. In the first place, from a comparison to mu-pair production the initial state radiation is expected to be the dominant contribution for QCD relevant quantities like R and thrust [2]. In the second place, only the initial state radiation can be taken into account in a model-independent way.

The standard way in which the experimentalists tend to treat the initial state radiation is based on a method introduced by Bonneau and Martin [3], augmented possibly with the hadronic vacuum polarization [4]. This method can be used to generate the 4-momenta of the bremsstrahlung photons, once the total hadronic cross section $\sigma(s')$ for e^+e^- annihilation is known as a function of the hadronic c.m. energy $\sqrt{s'}$. Moving to the hadronic c.m. frame one would then like to generate the hadrons, as given by the model. The model usually generates particle momenta with respect to the beam axis. However, in the hadronic c.m.s., there are now two beam axes, namely of the electron and positron, respectively. So one has to extend the treatment of ref. [3] such that one knows how to apply the jet model with respect to these axes. In practice, one often makes the additional assumption that the photon is emitted along the beam direction, such that in the hadronic c.m. frame one just has one beam axis. However, this assumption is not necessary since the problem can be treated exactly.

In other words, the standard treatment only considers $\sigma(s')$ in the equations, whereas one now requires formulae containing a multidifferential hadronic cross-section formula. Then it can be applied to the 2-, 3- and 4-jet angular distributions and any other multidifferential cross section one wants to consider.

For non-hadronic final states it is preferable to include final state radiation as well. The simulation of events then becomes more involved [2].

The outline of the paper is as follows. In sect. 2 we give the bremsstrahlung formulae applicable to differential cross sections, and the virtual corrections. Sect. 3 then outlines the way events can be generated. The appendices collect some calculational details.

2. The radiative cross section

Consider N-particle production in e^+e^- annihilation via one-photon exchange. The differential cross section for unpolarized beams is obtained from

$$d\sigma^0 = (2\pi)^4 \delta\left(p_+ + p_- - \sum_{i=1}^{N} q^i \right) \frac{e^4}{2s} \sum |M^0|^2 d\rho\,, \tag{2.1}$$

where p_+, p_-, q^i denote the 4-momenta of the e^+e^- and the hadrons and $d\rho$ denotes the invariant phase-space volume for an N-particle final state. Choosing some set of independent variables (2.1) leads to a multidifferential cross section of the type

$$\frac{d\sigma^0}{d\boldsymbol{q}_1\, d\boldsymbol{q}_2 \ldots} = f(\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots; \hat{\boldsymbol{R}}, s)\,, \tag{2.2}$$

where

$$Q = \sum_i q_i\,, \qquad P = p_+ + p_-\,,$$

$$R = p_+ - p_-\,, \qquad s = P^2 = 4E^2\,,$$

$$\hat{\boldsymbol{R}} = \boldsymbol{R}/|\boldsymbol{R}|\,. \tag{2.3}$$

From (2.2) the 4-momenta q^i can be generated in the hadronic c.m.s. (which now is also the lab system) with respect to the axis $\hat{\boldsymbol{R}}$, in this case the positron direction.

The square of the matrix element is obtained by contracting the lepton tensor $L^0_{\mu\nu}$ with the hadron tensor $H_{\mu\nu}$:

$$\sum |M^0|^2 = \frac{1}{s^2} L^0_{\mu\nu}(p_+, p_-) H^{\mu\nu}(q^1, \dots q^N), \tag{2.4}$$

where

$$L^0_{\mu\nu}(p_+, p_-) = p_{+\mu} p_{-\nu} + p_{+\nu} p_{-\mu} - (p_+ \cdot p_-) g_{\mu\nu}. \tag{2.5}$$

The tensor $H_{\mu\nu}$ depends on the particular model. In general it can be written as a linear combination of form factors, depending on invariants, multiplied with certain tensors. These tensors take the form

$$a^i_\mu a^j_\nu = \left[q^i_\mu - \frac{(q^i \cdot Q)}{Q^2} Q_\mu \right] \left[q^j_\nu - \frac{(q^j \cdot Q)}{Q^2} Q_\nu \right], \tag{2.6}$$

$$\eta_{\mu\nu} = g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2}, \tag{2.7}$$

where the requirement of current conservation has been imposed. In the following it is convenient to rewrite the lepton tensor as

$$L^0_{\mu\nu} = \tfrac{1}{2}(Q_\mu Q_\nu - R_\mu R_\nu - s g_{\mu\nu}). \tag{2.8}$$

In (2.4) the first term of (2.8) does not contribute. Moreover a simplification occurs through the vanishing of R_0 in the hadronic c.m.s.

When bremsstrahlung is emitted from the initial state, we find for the process

$$e^+e^- \to \gamma(k) + N \text{ hadrons}, \tag{2.9}$$

the cross section

$$d\sigma^1 = \frac{\alpha}{4\pi^2} \frac{s'k}{s} dk\, d\Omega_\gamma (2\pi)^4 \, \delta\left(p_+ + p_- - k + \sum_{i=1}^{N} q^i \right) \frac{e^4}{2s'} \sum |M^1|^2 d\rho, \tag{2.10}$$

where k denotes both the photon energy and its 4-momentum, Ω_γ is the solid angle of the photon and

$$s' = (p_+ + p_- - k)^2 = Q^2 = 4E(E-k). \tag{2.11}$$

The square of the bremsstrahlung matrix element takes the form

$$\sum |M^1|^2 = \frac{1}{(s')^2} L^1_{\mu\nu}(p_+, p_-, k) H^{\mu\nu}(q^1, \dots q^N). \tag{2.12}$$

The complete expression for $L^1_{\mu\nu}$ is given in the appendix. Omitting terms containing Q one obtains the part relevant for (2.12):

$$L_1^{\mu\nu} = -\tfrac{1}{2}(h_- R_-^{\mu} R_-^{\nu} + h_+ R_+^{\mu} R_+^{\nu} + h_0 g^{\mu\nu}), \tag{2.13}$$

where

$$R_- = p_- - p_+ + k, \qquad R_+ = p_+ - p_- + k, \tag{2.14}$$

and the functions h are given in appendix A.

In the hadronic c.m.s., L_1 contains the directions $\hat{\boldsymbol{R}}_-, \hat{\boldsymbol{R}}_+$, which are the electron and positron directions in this frame. As is shown in the appendix, the form (2.13) allows us to relate the differential cross section for the bremsstrahlung process (2.9) to that of the lowest order process:

$$\frac{\mathrm{d}\sigma^1}{\mathrm{d}k\,\mathrm{d}\Omega_\gamma\,\mathrm{d}\boldsymbol{q}_1\,\mathrm{d}\boldsymbol{q}_2\ldots} = \frac{\alpha}{4\pi^2 s}\left[g_- f(\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots; \hat{\boldsymbol{R}}_-, s') + g_+ f(\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots; \hat{\boldsymbol{R}}_+, s')\right]. \tag{2.15}$$

In this formula the hadronic momenta $\boldsymbol{q}_i$ are considered in the hadronic c.m.s., whereas the photon energy and solid angle are taken in the lab system. The bremsstrahlung cross section is the (incoherent) sum of two multidifferential cross sections evaluated in the hadronic c.m.s. with respect to the directions of e^- and e^+. The weight of the two distributions in the sum depends on the photon variables k, Ω_γ in the lab system:

$$g_\pm = \frac{-m^2 k s'}{K_\mp^2} + k\frac{(s' + 2K_\mp)^2}{2K_+K_-}, \tag{2.16}$$

where

$$K_\mp = p_\mp \cdot k. \tag{2.17}$$

When the multidifferential cross section is integrated over the hadron variables we obtain

$$\frac{\mathrm{d}\sigma^1}{\mathrm{d}k\,\mathrm{d}\Omega_\gamma} = \frac{\alpha}{4\pi^2 s}(g_- + g_+)\sigma^0(s') = \frac{\alpha k}{\pi^2 s}\left[-\frac{m^2 s'}{4K_-^2} - \frac{m^2 s'}{4K_+^2} + \frac{s^2 + s'^2}{8K_+K_-} - 1\right]\sigma^0(s'), \tag{2.18}$$

where $\sigma^0(s')$ is the total hadronic cross section, obtained from (2.2). This is the result of Bonneau and Martin, written in a form which was given for mu pairs in ref. [2].

As is well-known, the cross section (2.18) or (2.15) integrated over the full photon phase space diverges. This infrared divergence cancels if one adds the virtual correction to the lepton part L^0. So we consider soft bremsstrahlung isotropically emitted up to a photon energy k_1 and add the virtual corrections to (2.2). The lowest order cross section plus first-order correction then takes the form

$$\frac{d\sigma^1}{dq_1\, dq_2 \ldots} = \frac{d\sigma^0}{dq_1\, dq_2 \ldots}(1 + \delta_A + \delta_\mu + \delta_\tau + \delta_{\text{had}}), \tag{2.19}$$

where the corrections δ originate from soft bremsstrahlung, vertex correction and electron vacuum polarization

$$\delta_A = \beta \ln\frac{k_1}{E} + \frac{2\alpha}{\pi}\left[\tfrac{13}{12}\ln\frac{s}{m_e^2} + \tfrac{1}{6}\pi^2 - \tfrac{14}{9}\right], \tag{2.20}$$

with

$$\beta = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right), \tag{2.21}$$

and originate from the vacuum polarization due to the μ, τ and hadrons.

The latter can be taken from the numerical evaluation [4] of a dispersion integral. The others depend on m_μ or m_τ according to

$$\delta_{\mu,\tau} = \frac{2\alpha}{\pi}\left(\tfrac{1}{3}\ln\frac{s}{m_{\mu,\tau}^2} - \tfrac{5}{9}\right). \tag{2.22}$$

3. The Monte Carlo simulation of radiative events

We now discuss the simulation of radiative events, once one has a procedure for the generation of non-radiative events. Since the model for the hadronic events is known, the dependence of σ^0 on s' is known. Assume a dependence of the form

$$\sigma^0(s') = c/s'. \tag{3.1}$$

It will be seen in the following that this assumption is not essential.

From the differential cross section for photon emission, the photon spectrum can be obtained. In the lab system we use $\boldsymbol{p}_+$ as the z-axis and denote the polar and azimuthal angle of the photon by η and ψ. Performing the full azimuthal integration and then integrating over η we find from (2.18)

$$\frac{d\sigma^1}{dk} = \frac{\alpha}{2\pi s k}\sigma^0(s')\left[-\frac{4m^2 s'}{s}J_1 - 4k^2 J_2 + s\left(1 + \frac{(s')^2}{s^2}\right)J_3\right], \tag{3.2}$$

where

$$J_1 = \frac{1}{e-\cos\eta} - \frac{1}{e+\cos\eta}, \qquad J_2 = \cos\eta, \qquad J_3 = \ln\left(\frac{e+\cos\eta}{e-\cos\eta}\right), \tag{3.3}$$

with

$$e = p_{+0}/|\,\boldsymbol{p}_+|\,. \tag{3.4}$$

Inserting the boundaries $(-1, +1)$ in the $\cos\eta$ integral we find

$$J_1 = \frac{s}{m^2}, \qquad J_2 = 2, \qquad J_3 = 2\ln\frac{s}{m_e^2}, \tag{3.5}$$

which leads to the photon spectrum

$$\frac{\mathrm{d}\sigma^1}{\mathrm{d}k} = \frac{\alpha}{\pi}\sigma_0(s')\left(\ln\frac{s}{m_e^2} - 1\right)\left[1 + \left(\frac{s'}{s}\right)^2\right]\frac{1}{k}\,. \tag{3.6}$$

For not too small k-values this spectrum can be used to generate photon energies. When one is interested in generating the non-radiative and radiative events at the same time, one has to make the transition to soft photons. This can be done in two ways.

The first method chooses some value k_1 below which the photon energy can be neglected. The total cross section for the region below k_1 can be obtained from (2.19). The total cross section from k_1 up to a maximum photon energy $k_{\max}$ can be obtained from (3.6). These two cross sections are used as weights to choose the two regions. In the soft photon region one then generates hadrons according to $\mathrm{d}\sigma^0/\mathrm{d}\boldsymbol{q}_1\,\mathrm{d}\boldsymbol{q}_2\ldots$ with respect to $\hat{\boldsymbol{R}}$. In the hard photon region one generates photon energies according to (3.6). In this approach the value of k_1 should be chosen in such a way that the correction δ in (2.19) is not too large (and negative).

The second approach avoids the division in two regions by noting that for large negative corrections one should exponentiate the leading log part of the correction. Instead of

$$\sigma^1(k<k_1) = \sigma^0(s)\left(1 + \beta\ln\frac{k_1}{E} + \delta_{\mathrm{AR}}\right), \tag{3.7}$$

we take

$$\sigma^1(k<k_1) = \sigma^0(s)(1+\delta_{\mathrm{AR}})\left(\frac{k_1}{E}\right)^{\beta}, \tag{3.8}$$

where

$$\delta_{\mathrm{AR}} = \frac{2\alpha}{\pi}\left[\tfrac{13}{12}\ln\frac{s}{m_e^2} + \tfrac{1}{6}\pi^2 - \tfrac{14}{9}\right] + \delta_\mu + \delta_\tau + \delta_{\mathrm{had}}\,. \tag{3.9}$$

Although this is strictly speaking only correct for small k_1/E, we now extend this to the hard bremsstrahlung case in the following way. Calling

$$\sigma^1(k<k_2)=\sigma^1(k<k_1)+\int_{k_1}^{k_2}\mathrm{d}k\frac{\mathrm{d}\sigma^1}{\mathrm{d}k}=F(k_2)+\sigma^0(s)\beta\ln\frac{k_2}{E}\,, \tag{3.10}$$

with $\sigma^1(k<k_1)$ as given by (3.7), we now replace (3.10) by

$$\sigma^1(k<k_2)=F(k_2)\left(\frac{k_2}{E}\right)^{\beta}\equiv G(k_2)\,. \tag{3.11}$$

This reproduces for small k_2 values, eq. (3.8) and for the maximum k-value, eq. (3.10). In the case of a $1/s'$ behaviour of the hadronic cross section $F(k)$ can easily be obtained and reads

$$F(k)=\sigma^0(s)\left(1+\delta_{\mathrm{AR}}-\tfrac{1}{2}\beta\ln\left(1-\frac{k}{E}\right)-\tfrac{1}{2}\beta\frac{k}{E}\right). \tag{3.12}$$

The integrated spectrum (3.11) can now be used to generate the k-values. Generating a random number $\zeta\in[0,G(k_{\max})]$ and solving numerically

$$\zeta=G(k) \tag{3.13}$$

gives k. Since we also know for a fixed value k the primitive function of the $\cos\eta$ integration, i.e., eq. (3.2), we can in the same way generate the $\cos\eta$ value. The azimuthal angle finally is a random number between 0 and 2π.

Having generated in this manner the photon 4-momentum we can then calculate the weights g_+ and g_- in eq. (2.15). Choosing then the axis $\hat{\boldsymbol{R}}_+$ or $\hat{\boldsymbol{R}}_-$ with a probability determined by the weights g_+ and g_-, we generate the hadron momenta in the hadron c.m.s. with respect to the chosen axis. Finally we have to transform the momenta back to the lab system.

In the case that the energy dependence (3.1) does not hold, one has to evaluate $F(k)$ anew. One case, which is of practical interest, namely that of heavy quarks is given in appendix B. If the integrals cannot be done analytically the first method should be applied to generate a photon energy directly from eq. (3.6).

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Appendix A

For completeness we give the full lepton tensor with bremsstrahlung:

$$L_1^{\mu\nu} = -\tfrac{1}{2}\left[h_- R_-^\mu R_-^\nu + h_+ R_+^\mu R_+^\nu + h_0 g^{\mu\nu} + h_1 Q^\mu Q^\nu + h_2 (R_-^\mu Q^\nu + R_-^\nu Q^\mu) + h_3 (R_+^\mu Q^\nu + R_+^\nu Q^\mu)\right], \tag{A.1}$$

where

$$h_\pm = -\frac{m^2}{K_\mp^2} + \frac{s'}{2K_+K_-},$$

$$h_0 = -\frac{m^2 s'}{K_-^2} - \frac{m^2 s'}{K_+^2} + \frac{(s'+2K_-)^2 + (s'+2K_+)^2}{2K_+K_-},$$

$$h_1 = \frac{m^2}{K_-^2} + \frac{m^2}{K_+^2} - \frac{s}{K_+K_-},$$

$$h_{2,3} = -\frac{K_{+,-}}{K_+K_-}. \tag{A.2}$$

The following kinematical relations are useful:

$$R_\mp \cdot Q = 2K_\pm, \tag{A.3}$$

$$R_\mp^2 = -s' - 4K_\pm,$$

from which we have in the hadronic c.m.s.,

$$\boldsymbol{R}_\mp^2 = \frac{(s'+2K_\pm)^2}{s'}, \tag{A.4}$$

$$h_0 = h_- \boldsymbol{R}_-^2 + h_+ \boldsymbol{R}_+^2. \tag{A.5}$$

In the hadronic c.m.s. we find for the no bremsstrahlung case for typical terms in $\Sigma|M^0|^2$—those obtained from (2.6) and (2.7), respectively:

$$\sum |M^0|^2 \sim -\frac{1}{2s^2}\left[(\boldsymbol{R}\cdot\boldsymbol{q}^i)(\boldsymbol{R}\cdot\boldsymbol{q}^j) + s\boldsymbol{q}^i\cdot\boldsymbol{q}^j\right]$$

$$= -\frac{1}{2s}\left[(\hat{\boldsymbol{R}}\cdot\boldsymbol{q}^i)(\hat{\boldsymbol{R}}\cdot\boldsymbol{q}^j) + \boldsymbol{q}^i\cdot\boldsymbol{q}^j\right], \tag{A.6}$$

and

$$\sum |M^0|^2 \sim -\frac{1}{s}. \tag{A.7}$$

In the bremsstrahlung case the same terms in $H^{\mu\nu}$ now give rise to

$$\sum |M^1|^2 \sim -\frac{1}{2(s')^2}\left[h_- \boldsymbol{R}_-^2 (\hat{\boldsymbol{R}}_- \cdot \boldsymbol{q}^i)(\hat{\boldsymbol{R}}_- \cdot \boldsymbol{q}^j) + h_+ \boldsymbol{R}_+^2 (\hat{\boldsymbol{R}}_+ \cdot \boldsymbol{q}^i)(\hat{\boldsymbol{R}}_- \cdot \boldsymbol{q}^j) + h_0 \boldsymbol{q}^i \cdot \boldsymbol{q}^j\right]$$

$$= -\frac{1}{2s'}\left\{\frac{h_- \boldsymbol{R}_-^2}{s'}\left[(\hat{\boldsymbol{R}}_- \cdot \boldsymbol{q}^i)(\hat{\boldsymbol{R}}_- \cdot \boldsymbol{q}^j) + \boldsymbol{q}^i \cdot \boldsymbol{q}^j\right] + \frac{h_+ \boldsymbol{R}_+^2}{s'}\left[(\hat{\boldsymbol{R}}_+ \cdot \boldsymbol{q}^i)(\hat{\boldsymbol{R}}_+ \cdot \boldsymbol{q}^j) + \boldsymbol{q}^i \cdot \boldsymbol{q}^j\right]\right\}, \qquad \text{(A.8)}$$

and

$$\sum |M^1|^2 \sim \frac{1}{2(s')^2}\left[h_- \boldsymbol{R}_-^2 + h_+ \boldsymbol{R}_+^2 - 3h_0\right]$$

$$= -\frac{1}{s'}\left[\frac{h_- \boldsymbol{R}_-^2}{s'} + \frac{h_+ \boldsymbol{R}_+^2}{s'}\right]. \qquad \text{(A.9)}$$

Comparing (A.6), (A.7) with (A.8) and (A.9) we find the weights

$$g_\pm = h_\pm \boldsymbol{R}_\pm^2 k, \qquad \text{(A.10)}$$

where the definition (2.15) for $g_\pm$ and the normalisation (2.10) have been used.

It should be noted that for the calculation of L_0 and L_1 the ultra-relativistic limit has been used, i.e., terms of order m^2/E^2 have been neglected, except where denominators can take similar values.

Appendix B

For the production of a heavy quark pair (or a τ-pair) the assumed $1/s'$ dependence of $\sigma^0(s')$ may not be correct, even at PETRA/PEP energies. Calling the quark or lepton mass M, one has

$$\frac{\mathrm{d}\sigma^0}{\mathrm{d}\Omega}(s') = \frac{3e_\mathrm{q}^2\alpha^2}{4s'}\left(1 - \frac{4M^2}{s'}\right)^{1/2}\left(1 + \cos^2\theta + \frac{4M^2}{s'}\sin^2\theta\right), \qquad \text{(B.1)}$$

where e_q is the fractional quark charge and θ denotes the scattering angle. The total cross section for massive quark pair production now reads

$$\sigma_\mathrm{q}^0(s') = \frac{4\pi e_\mathrm{q}^2\alpha^2}{s'}\left(1 - \frac{4M^2}{s'}\right)^{1/2}\left(1 + \frac{2M^2}{s'}\right). \qquad \text{(B.2)}$$

The first factor in (B.2) is just the total cross section for light quark pair production

$$\sigma^0(s') = \frac{4\pi e_q^2\alpha^2}{s'} . \tag{B.3}$$

Expanding the M^2/s' dependent factors in (B.2) we see, that deviations from (B.3) are of the order $6(M^2/s')^2$. For the b-quarks for instance, it implies that deviations of more than 4% arise when the emitted photon energy is larger than $\frac{2}{3}$ of the beam energy E, for $E = 15$ GeV.

If one wants to consider photons with larger energies one should replace (3.12) by

$$F(k) = \sigma_q^0(s)(1 + \delta_{AR} + H(k)), \tag{B.4}$$

where

$$H(k) = \beta\left\{ -\ln\left(1 - \frac{k}{E}\right) + 2\ln\frac{2a}{a+y} + \frac{1}{(2 + 4M^2/s)a}\left[\ln\left(1 - \frac{k}{E}\right) - 2\ln\frac{1+a}{1+y} - \frac{k}{E}y + \left(3 + \frac{4M^2}{s}\right)(y - a) - \tfrac{1}{3}(y^3 - a^3)\right]\right\}, \tag{B.5}$$

where

$$a = \left(1 - \frac{4M^2}{s}\right)^{1/2}, \qquad y = \left(1 - \frac{4M^2}{s'}\right)^{1/2}. \tag{B.6}$$

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