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ON THE ROUGHENING TRANSITION IN ABELIAN LATTICE GAUGE THEORIES

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We study the width of the confining string between static quarks in abelian lattice gauge theories using strong coupling expansions. We consider gauge groups Z_n and U(1) in 3 and 4 dimensions. This extends previous work with Lüscher, where SU(2) and Z_2 were studied. In $\nu = 3$ dimensions we find evidence for a roughening transition. It is characterized by a divergence of the string width for an infinitely far separated quark–antiquark pair, while the string tension remains non-zero. In $\nu = 4$ dimensions for the abelian groups we do not have evidence for a roughening transition away from a phase transition.

1. Introduction and summary of results

In a recent work with Lüscher [1] the width σ^2 of a chromo-electric flux tube between widely separated static quarks in pure gauge theory has been studied. The strong coupling expansion for σ^2 in four-dimensional SU(2) lattice gauge theory indicates that for values of the coupling parameter β above 1.9 the width σ^2 diverges with the separation between the quarks. This phenomenon is a kind of surface roughening transition. The surface which fills out the euclidean Wilson loop then fluctuates so strongly that its transversal width diverges when the loop size goes to infinity. A related phenomenon occurs in the three-dimensional Ising model, where the width of a magnetic domain wall diverges at a roughening temperature well below the critical point [2]. By duality this implies a roughening transition for the three-dimensional Z_2 gauge theory. Using the same method as for SU(2) we also observed this transition [1]. On the other hand for Z_2 in $\nu = 4$ dimensions we found a roughening temperature which appears to be equal to the self-dual point, where the theory is supposed to undergo a first-order phase transition.

In this paper we extend our study of the roughening phenomenon to the abelian gauge groups Z_n, all n, and U(1) in $\nu = 3$ and 4 dimensions. These models are briefly reviewed in sect. 2. Using the same methods as in ref. [1], the high-temperature series for the string width σ^2 in the case of an infinite separation between the static quarks are calculated up to 12th order. Based on these series we estimate the inverse roughening temperatures β_R, where σ^2 diverges. Our results are discussed in detail in sect. 3.

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In $\nu = 3$ dimensions we find values of β_R well below the phase transition point β_c for all groups under consideration. This indicates the existence of a roughening transition. On the other hand for $\nu = 4$ the obtained values of β_R are so near the lower phase transition points β_c that it is not possible to decide whether they are equal or not.

The roughening transition in three-dimensional lattice gauge theories has also been investigated independently by A. and E. and P. Hasenfratz [3]. Their conclusions are similar to ours. The possible importance of roughening transitions in gauge theories has also been emphasized by Itzykson [4].

2. The models

We study euclidean gauge theories with gauge group $G = Z_n$ or U(1) on a hypercubical lattice in $\nu = 3$ and 4 dimensions. The gauge field variables $U(b) \in G$ are attached to the links b of the lattice. The product of the variables $U(b)$ for the four links b on the boundary of an elementary plaquette p is denoted by $U(p)$. We use the standard Wilson action [5]

$$L = \tfrac{1}{2}\beta \sum_{p} (U(p) + \bar{U}(p)), \qquad \beta = \frac{1}{g^2}, \tag{2.1}$$

where the sum extends over all unoriented plaquettes.

In $\nu = 3$ dimensions the Z_2, Z_3 and Z_4 lattice gauge theories are mapped by duality [6, 7] onto the corresponding Z_n spin systems, which are known to undergo a single phase transition at certain critical temperatures [8]. The corresponding critical couplings for the gauge theories are

$$\begin{aligned} &Z_2\colon \beta_{2c} = 0.76\,, \\ &Z_3\colon \beta_{3c} = 1.09\,, \qquad (\nu = 3)\,, \\ &Z_4\colon \beta_{4c} = 1.52 = 2\beta_{2c}\,. \end{aligned} \tag{2.2}$$

For Z_n, $n \geqslant 5$ in $\nu = 3$ dimensions Monte Carlo studies [9] show a single phase transition. The critical couplings β_{nc} seem to tend to $\beta = \infty$ as $n \to \infty$. For $\beta < \beta_c$ the theories confine static quarks, while for $\beta > \beta_c$ this is not the case. For U(1), $\nu = 3$ no transition has been found [9]. The model apparently confines at all values of β.

In $\nu = 4$ dimensions the Z_2, Z_3 and Z_4 lattice gauge theories are self-dual [6, 7]. They are supposed to undergo a first-order transition at their self-dual couplings [10]. From self-duality one finds

$$\begin{aligned} &Z_2\colon \beta_{2c} = 0.44\,, \\ &Z_3\colon \beta_{3c} = 0.67\,, \qquad (\nu = 4)\,, \\ &Z_4\colon \beta_{4c} = 0.88 = 2\beta_{2c}\,. \end{aligned} \tag{2.3}$$

For $n \geqslant 5$ in $\nu = 4$ dimensions the Z_n lattice gauge theory is supposed to have two phase transitions [10, 11]. As n increases the higher transition coupling β_c moves to infinity, while the lower β_c tends to a certain finite limit. This is equal to the transition coupling of the U(1) model which is supposed to have a single phase transition [10–12] from a confining to a spinwave phase. Monte Carlo studies [10] indicate that the lower transition couplings for these models are all around $\beta = 1$:

$$Z_n,\ n \geqslant 5;\ \mathrm{U}(1):\ \beta_c \approx 1\,, \qquad (\nu = 4)\,. \tag{2.4}$$

3. Roughening temperatures

For small values of β pure lattice gauge theories are known to confine static quarks by a potential which rises linearly for large separations. The slope of the potential is the string tension α. In ref. [1] the width σ^2 of the flux tube connecting a static quark–antiquark pair has been defined through the electric field energy density between them. Its high-temperature expansion can be calculated using the same graphs which appear in the expansion of the string tension α [13].

The natural expansion parameters in high-temperature series are the coefficients $0 \leqslant a_r(\beta) < 1$ in the Fourier expansion,

$$\exp(\beta \operatorname{Re} U) = N(\beta)\Big(1 + \sum_{r \neq 0} a_r(\beta)\chi_r(U)\Big)\,. \tag{3.1}$$

The sum extends over all inequivalent non-trivial irreducible representations of the group G, labelled by r. χ_r denotes the corresponding characters and $N(\beta)$ is a normalization factor. The inequivalent irreducible representations for Z_n and U(1) are

$$U \to U^r = \chi_r(U)\,, \tag{3.2}$$

where $r = 0, 1, \ldots, n-1$ for Z_n and $r \in \mathrm{Z}$ for U(1).

Using orthogonality relations for characters one finds [14]

$$a_r(\beta) = c_r(\beta)/c_0(\beta)\,, \tag{3.3}$$

where

$$c_r(\beta) = \int_G \mathrm{d}U U^{-r} \exp(\beta \operatorname{Re} U)\,, \tag{3.4}$$

$\mathrm{d}U$ is the Haar measure on G.

In the following we write

$$x = a_1(\beta)\,, \qquad y = a_2(\beta)\,, \qquad z = a_3(\beta)\,. \tag{3.5}$$

In particular,

$$\begin{aligned} &\text{for } Z_2\text{: } x = \tanh\beta\,,\\ &\text{for } Z_3\text{: } x = y = (1-\exp(-\tfrac{3}{2}\beta))/(1+2\exp(-\tfrac{3}{2}\beta))\,,\\ &\text{for } Z_4\text{: } x = z = \tanh\tfrac{1}{2}\beta\,, \qquad y = x^2\,,\\ &\text{for } Z_5\text{: } y = z\,,\\ &\text{for U(1): } a_r(\beta) = I_r(\beta)/I_0(\beta)\,,\\ &(I_r = \text{modified Bessel function})\,. \end{aligned} \tag{3.6}$$

First we consider the high-temperature series for the string tension α. For gauge groups Z_2, Z_3 and U(1) in $\nu = 3$ dimensions they have been given by Drouffe up to 16th order [15] (see also [16–18])*. For Z_2, $\nu = 4$ the series has been calculated in [16, 17], for Z_3, $\nu = 4$ in [18]. For $n \geq 4$ and U(1) the series are up to 12th order: for $\nu = 3$:

$$\begin{aligned} -\alpha = \ln x &+ 2x^4 + 2x^4y + 10x^8 + 9x^{10} - 8x^8y + 12x^6y^2 + 2y^5 + \tfrac{314}{3}x^{12}\\ &- 50x^{10}y + 20x^8y^2 + 8x^4y^4 + x^9z - 2(2-\delta_{n4})y^6 + 2(1-\delta_{n4})y^5zx^{-1}\,; \end{aligned} \tag{3.7}$$

for $\nu = 4$:

$$\begin{aligned} -\alpha = \ln x &+ 4x^4 + 4x^4y + 56x^8 + 54x^{10} + 56x^8y + 24x^6y^2 + 4y^5\\ &+ \tfrac{3628}{3}x^{12} - 92x^{10}y + 76x^8y^2 + 16x^4y^4 + 6x^9z - 4(2-\delta_{n4})y^6\\ &+ 4(1-\delta_{n4})y^5zx^{-1}\,. \end{aligned} \tag{3.8}$$

The series can be expanded in terms of x. The resulting expansions are summarised in tables 1, 2. In figs. 1 and 2 we plot the curves corresponding to (3.7) and (3.8) for Z_2 to Z_5 and U(1)**. High-temperature series for the string tension of several hamiltonian lattice gauge theories have been calculated in ref. [19]. In $\nu = 3$ dimensions the curves go to zero below the critical couplings, whereas the true string tension is supposed to vanish at the critical coupling. This effect might be associated with the roughening transition [3], which we are going to discuss. In $\nu = 4$ dimensions the series appear to converge to non-zero values for α at the lower transition couplings. This indicates that the transition is a first-order one.

Next we discuss the high-temperature series for the string width σ^2. Tables 3, 4 contain the series in terms of x for all groups under consideration up to x^{12}. For Z_4 and Z_2 the series in x for σ^2 (as well as for α) coincide in both $\nu = 3$ and $\nu = 4$. In $\nu = 3$ dimensions this is due to the fact that the Z_4 model decouples into two independent Z_2 models [7, 20]. The possibility that the same is true for $\nu = 4$ is under investigation.

* The 14th order term for Z_2, $\nu = 3$ in [16] differs from the one in [15].

** The curves for Z_n, $n \geq 6$ and U(1) are practically indistinguishable.

TABLE 1

String tension series in $\nu = 3$ dimensions

$$\alpha = -\ln x - \sum_{k \geq 4} \alpha_k x^k$$

Group \ k	α_k: 4	5	6	7	8	9	10	11	12
Z_2, Z_4	2	0	2	0	10	0	16	0	$\frac{242}{3}$
Z_3	2	2	0	0	22	−7	29	6	$\frac{428}{3}$
Z_5	2	0	1	$\frac{1}{3}$	$\frac{31}{3}$	$\frac{1}{6}$	$\frac{395}{48}$	$\frac{2777}{2016}$	$\frac{174557}{2016}$
Z_6	2	0	1	0	$\frac{125}{12}$	0	$\frac{401}{48}$	0	$\frac{24955}{288}$
Z_7	2	0	1	0	$\frac{31}{3}$	$\frac{1}{60}$	$\frac{397}{48}$	$\frac{1}{40}$	$\frac{124139}{1440}$
Z_8	2	0	1	0	$\frac{31}{3}$	0	$\frac{5957}{720}$	0	$\frac{124147}{1440}$
Z_9	2	0	1	0	$\frac{31}{3}$	0	$\frac{397}{48}$	$\frac{1}{2520}$	$\frac{124139}{1440}$
Z_{10}	2	0	1	0	$\frac{31}{3}$	0	$\frac{397}{48}$	0	$\frac{1737947}{20160}$
Z_n, $n \geq 11$ U(1)	2	0	1	0	$\frac{31}{3}$	0	$\frac{397}{48}$	0	$\frac{124139}{1440}$

TABLE 2

String tension in $\nu = 4$ dimensions

Group \ k	α_k: 4	5	6	7	8	9	10	11	12
Z_2, Z_4	4	0	4	0	56	0	144	0	$\frac{3616}{3}$
Z_3	4	4	0	0	80	62	130	20	$\frac{5968}{3}$
Z_5	4	0	2	$\frac{2}{3}$	$\frac{170}{3}$	$\frac{1}{3}$	$\frac{2123}{24}$	$\frac{16889}{1008}$	$\frac{1208429}{1008}$
Z_6	4	0	2	0	$\frac{341}{6}$	0	$\frac{2129}{24}$	0	$\frac{173179}{144}$
Z_7	4	0	2	0	$\frac{170}{3}$	$\frac{1}{30}$	$\frac{2125}{24}$	$\frac{1}{20}$	$\frac{862619}{720}$
Z_8	4	0	2	0	$\frac{170}{3}$	0	$\frac{31877}{360}$	0	$\frac{862627}{720}$
Z_9	4	0	2	0	$\frac{170}{3}$	0	$\frac{2125}{24}$	$\frac{1}{1260}$	$\frac{862619}{720}$
Z_{10}	4	0	2	0	$\frac{170}{3}$	0	$\frac{2125}{24}$	0	$\frac{12076667}{10080}$
Z_n, $n \geq 11$ U(1)	4	0	2	0	$\frac{170}{3}$	0	$\frac{2125}{24}$	0	$\frac{862619}{720}$

The series for σ^2 are all increasing with x. They start with x^4 so that they are in fact short series. We look for a divergence of σ^2 with the help of different methods. Firstly, as in [1] we study the inverse string width measured in a length scale given by the string tension, i.e., the quantity

$$(\sigma^2 \cdot \alpha)^{-1} .$$

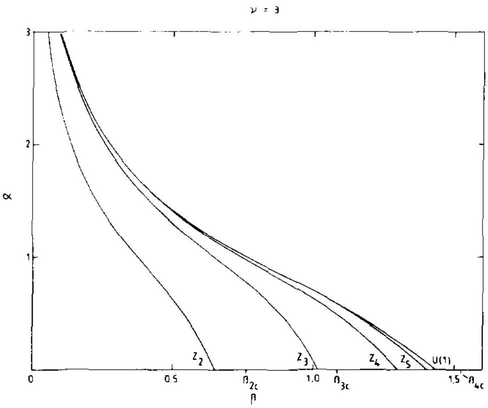

Fig. 1. The string tension for groups Z_2, Z_3, Z_4, Z_5, U(1) in $\nu=3$ dimensions as given by the high-temperature series eq. (3.7) and refs. [13, 15–18].

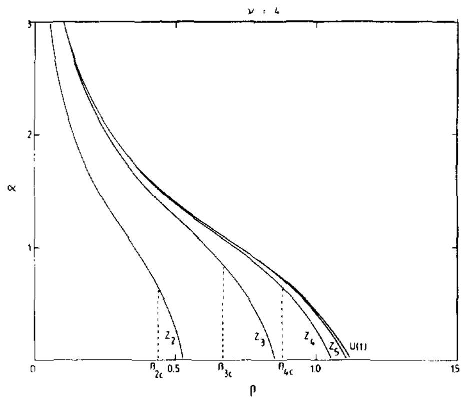

Fig. 2. The string tension for groups Z_2, Z_3, Z_4, Z_5, U(1) in $\nu=4$ dimensions as given by the high-temperature series eq. (3.8) and refs. [13, 16–18].

We calculate its asymptotic expansion in x. A zero of $(\sigma^2\alpha)^{-1}$ then indicates that σ^2 diverges there and yields a value for the inverse roughening temperature. For Z_2 in 3 dimensions the value obtained [1] is in good agreement with the Ising model calculations [2], which supports the reliability of the procedure.

Secondly, we look for poles in different Padé approximants to the series for $F(x)$, where

$$\sigma^2(x) \sim x^4 F(x)\,. \tag{3.9}$$

We consider poles for real x between 0 and 1. The [0, 8] approximants corresponding to the inverse series $F^{-1}(x)$ are also included. Because the series for Z_n, $n \geqslant 6$ are numerically practically the same as for U(1), they are not discussed separately.

As mentioned above, the series are short. Results from Padé methods are only reliable if they are stable for different approximants. This is indeed the case for Z_2 in

TABLE 3

Series for the string width in $\nu = 3$ dimensions

$$\sigma^2 = \sum_{k \geqslant 4} \sigma_k x^k$$

Group \ k	σ_k 4	5	6	7	8	9	10	11	12
Z_2, Z_4	2	0	6	0	40	0	148	0	854
Z_3	2	2	4	0	64	−8	196	48	1154
Z_5	2	0	5	$\frac{1}{3}$	$\frac{121}{3}$	$\frac{1}{6}$	$\frac{3127}{24}$	$\frac{1529}{504}$	$\frac{47955}{56}$
Z_6	2	0	5	0	$\frac{485}{12}$	0	$\frac{1565}{12}$	0	$\frac{493223}{576}$
Z_7	2	0	5	0	$\frac{121}{3}$	$\frac{1}{60}$	$\frac{391}{3}$	$\frac{1}{40}$	$\frac{615937}{720}$
Z_8	2	0	5	0	$\frac{121}{3}$	0	$\frac{46921}{360}$	0	$\frac{615941}{720}$
Z_9	2	0	5	0	$\frac{121}{3}$	0	$\frac{391}{3}$	$\frac{1}{2520}$	$\frac{615937}{720}$
Z_{10}	2	0	5	0	$\frac{121}{3}$	0	$\frac{391}{3}$	0	$\frac{17246237}{20160}$
Z_n, $n \geqslant 11$	2	0	5	0	$\frac{121}{3}$	0	$\frac{391}{3}$	0	$\frac{615937}{720}$
U(1)									

TABLE 4

Series for the string width in $\nu = 4$ dimensions

Group \ k	σ_k 4	5	6	7	8	9	10	11	12
Z_2, Z_4	4	0	12	0	120	0	552	0	4700
Z_3	4	4	8	0	168	72	600	208	6524
Z_5	4	0	10	$\frac{2}{3}$	$\frac{362}{3}$	$\frac{1}{3}$	$\frac{5623}{12}$	$\frac{5897}{252}$	$\frac{389201}{84}$
Z_6	4	0	10	0	$\frac{725}{6}$	0	$\frac{2813}{6}$	0	$\frac{1335719}{288}$
Z_7	4	0	10	0	$\frac{362}{3}$	$\frac{1}{30}$	$\frac{1406}{3}$	$\frac{1}{20}$	$\frac{1667377}{360}$
Z_8	4	0	10	0	$\frac{362}{3}$	0	$\frac{84361}{180}$	0	$\frac{1667381}{360}$
Z_9	4	0	10	0	$\frac{362}{3}$	0	$\frac{1406}{3}$	$\frac{1}{1260}$	$\frac{1667377}{360}$
Z_{10}	4	0	10	0	$\frac{362}{3}$	0	$\frac{1406}{3}$	0	$\frac{46686557}{10080}$
Z_n, $n \geqslant 11$	4	0	10	0	$\frac{362}{3}$	0	$\frac{1406}{3}$	0	$\frac{1667377}{360}$
U(1)									

$\nu = 3$ dimensions where a long series is available [2], which yields the same results as extracted from the low-order terms.

Table 5 contains the obtained values for x_R, and the corresponding β_R, for different groups and for $\nu = 3$ and 4. First we discuss the case $\nu = 3$. Gauge group Z_2 was considered already in (1). Due to the above-mentioned factorization property

TABLE 5

Roughening points [values of x_R (β_R)]

$\nu = 3$ dimensions

Group \ [L, M]	x_R (β_R) from Padé for σ^2 [4, 4]	[3, 4]	[2, 4]	[3, 5]	[2, 6]	[1, 7]	[0, 8]	x_R (β_R) from $(\sigma^2\alpha)^{-1}$
Z_2	0.459 (0.50)		0.461 (0.50)		0.459 (0.50)		0.455 (0.49)	0.460 (0.50)
Z_3	0.460 (0.85)	0.435 (0.80)	0.449 (0.82)	0.443 (0.81)	0.443 (0.81)	0.423 (0.78)	no real pole	no real pole
Z_4	0.459 (0.99)		0.461 (1.00)		0.459 (0.99)		0.455 (0.99)	0.460 (0.99)
Z_5	0.459 (1.03)	0.460 (1.03)	0.458 (1.03)	0.459 (1.03)	0.458 (1.03)	0.467 (1.05)	0.456 (1.02)	0.458 (1.03)
U(1)	0.458 (1.03)		0.459 (1.04)		0.458 (1.03)		0.455 (1.03)	0.459 (1.04)

$\nu = 4$ dimensions:

Group \ [L, M]	x_R (β_R) from Padé for σ^2 [4, 4]	[3, 4]	[2, 4]	[3, 5]	[2, 6]	[1, 7]	[0, 8]	x_R (β_R) from $(\sigma^2\alpha)^{-1}$
Z_2	0.394 (0.42)		0.405 (0.43)		0.401 (0.42)		0.392 (0.41)	0.401 (0.43)
Z_3	0.391 (0.72)	0.383 (0.70)	0.394 (0.72)	0.388 (0.71)	0.388 (0.71)	0.382 (0.70)	no real pole	no real pole
Z_4	0.394 (0.83)		0.405 (0.86)		0.401 (0.85)		0.392 (0.83)	0.401 (0.85)
Z_5	0.395 (0.86)	0.407 (0.89)	0.407 (0.89)	0.407 (0.89)	0.401 (0.87)	0.408 (0.89)	0.393 (0.85)	0.399 (0.87)
U(1)	0.395 (0.86)		0.407 (0.89)		0.402 (0.88)		0.393 (0.86)	0.400 (0.87)

[7] the Z_2 results also hold for Z_4. They indicate a roughening transition at

$$Z_4,\ \nu = 3:\ \beta_R \approx 1.00\,, \tag{3.10}$$

well below the critical point.

For U(1) and Z_n, $n \geq 5$ we obtain

$$\nu = 3:\ \beta_R \approx 1.03\,, \tag{3.11}$$

which is far away from the critical point.

The case of Z_3 is somewhat delicate. In contrast to Z_2, Z_4 or U(1) the Z_3 series also contains non-negligible odd orders and the coefficients behave somewhat irregularly. The inverse series $F^{-1}(x)$ as well as $(\sigma^2\alpha)^{-1}$ up to the 8th order do not have zeros for real x between 0 and 1. Instead they show a minimum around $x \approx 0.41$ corresponding to $\beta \approx 0.75$. Up to the next lower orders on the other hand, these series do have a zero near $\beta \approx 0.75$ (7th order) and $\beta \approx 0.80$ (6th order). The Padé approximants show a pole around $\beta \approx 0.8$. With some reservations one might conclude the existence of a roughening transition at

$$Z_3,\ \nu = 3:\ \beta_R \approx 0.8\,. \tag{3.12}$$

Now we come to $\nu = 4$ dimensions. As mentioned above our series for Z_2 and Z_4 are identical. Z_2 has been studied in ref. [1]. The obtained values for β_R are all very near the transition coupling β_c. For U(1) and Z_n, $n \geq 5$ the values are

$$\nu = 4:\ \beta_R \approx 0.87\,. \tag{3.13}$$

The transition couplings are not known exactly for these groups. The range, where β_c is estimated from Monte Carlo calculations [10], is consistent with the above value for β_R. For Z_3 we encounter the same phenomenon as in 3 dimensions. $(\sigma^2\alpha)^{-1}$ as well as $F(x)^{-1}$ only show a minimum near $\beta \approx 0.8$. The Padé approximants, however, all have poles around $\beta \approx 0.7$. This value is slightly above the critical point $\beta_c = 0.67$ and shows that Z_3 has to be handled with special care.

4. Conclusions

The high-temperature expansions for the string width σ^2 suggest that three-dimensional lattice gauge theories with gauge groups Z_n or U(1) possess a roughening transition. It is characterized by a divergence of σ^2 at a roughening point β_R below the critical point. As stressed before [1, 3] the roughening phenomenon is probably not a deconfining transition, that is, the string tension is supposed not to vanish at β_R. We observe that the corresponding values x_R are very much the same for all groups including Z_3 (see [3] for a discussion of this point).

In 4 dimensions the obtained values for β_R are near the transition couplings and do not strictly allow one to decide whether Z_n and U(1) lattice gauge theories have a roughening transition distinct from the known phase transition.

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Note added in proof

After submission of this work we received a paper by C. Itzykson, M. Peskin and J.B. Zuber entitled Roughening of Wilson's surface, Saclay Dph-T/94, in which the roughening transition in SU(2) and Z_2 lattice gauge theories is investigated.

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