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SYMMETRY-BREAKING ASPECTS OF THE ROUGHENING TRANSITION IN GAUGE THEORIES

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An infinitely long chromoelectric flux tube breaks translation invariance in transverse directions spontaneously. We argue that the associated Goldstone bosons live effectively in 1 + 1 dimensions and therefore, by the Mermin-Wagner-Coleman theorem, destabilize the flux tube. A large class of effective lagrangians describing the long-wavelength fluctuations of (finite) flux tubes is furthermore shown to give rise to a quark potential $V(L) = \alpha L + \beta + \gamma L^{-1} + O(L^{-2})$, where γ is a *universal* constant.

1. Introduction

It has recently been observed [1–3] that chromoelectric flux tubes roughen when stretched: instead of approaching a well localized straight cigar shaped region, the support of the chromoelectric field energy density due to the presence of a static quark-antiquark pair broadens indefinitely with the quark separation L. The purpose of the present article is to discuss some features of the roughening phenomenon, which are independent of the detailed form of the underlying dynamics, but rather are characteristic to the peculiar geometrical situation and to the quantum mechanical nature of the problem.

The central idea, which has previously been formulated in an analogous physical context by Günther, Nicole and Wallace [4], is the following. Suppose it would be possible to create an infinitely long straight flux tube by pulling a quark-antiquark pair infinitely far apart. As demonstrated by the existence of the Nielsen-Olesen vortex [5], such a flux tube can be perfectly stable classically. Quantum mechanically, however, it is unstable for the following reason. If the flux tube as a whole makes a transverse fluctuation with some wavelength λ, the cost in energy is only $\varepsilon(\lambda) = 2\pi/\lambda$ (for large λ). These, of course, are the Goldstone modes associated with the spontaneously broken transverse translational symmetry. Since they exist only along the flux tube, they live effectively in a 1 + 1 dimensional world. By the

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Mermin-Wagner-Coleman theorem [6], the amplitudes of such Goldstone fluctuations diverge with λ so that the flux tube gets completely delocalized. In other words, the assumption that an infinitely long quantum mechanical flux tube of finite width exists, is self-contradictory.

The above argument does not apply to lattice gauge theories, because translation symmetry is only discrete there. An infinitely long flux tube does therefore not necessarily imply the presence of massless modes. In fact, for very strong coupling such flux tubes do exist [7]. As has been pointed out [1–3], however, these infinitely long flux tubes delocalize for coupling constants g^2 smaller than some g_R^2. By the above, we thus can consider the roughening transition a transition, where continuous translational symmetry is effectively restored*.

Not much can be said about the gluon field energy distribution, when the quark-antiquark separation L is finite, except, of course, that its transversal width must diverge for $L \to \infty$. It is conceivable, on the other hand, that the gluon field is confined to a thin tube (string) connecting the quark with the antiquark, which undergoes quantum mechanical fluctuations without losing its identity. The width of the string is thought to be related to the glueball mass. This picture is plausible for lattice gauge theories with a coupling constant g^2 just below the critical value g_R^2, where long flux tubes are rough but where the bulk correlation length is still small. Whether it is a valid description of the continuum theory is an open question. Still, we may take it as an ansatz, write down an effective theory for the collective motion of the string and see what the consequences are.

To illustrate the argumentation above that an infinitely long flux tube is unstable quantum mechanically, we calculate (sect. 2) the spectrum of fluctuations around a Bloch wall in 2 + 1 dimensions. It would be more appropriate, of course, to do this with the Nielsen-Olesen vortex, but a Bloch wall is mathematically much simpler than a vortex and yet has the crucial property of being infinitely extended in 1 + 1 dimensions and localized in the transverse direction. In sect. 3 we write down a class of effective lagrangians describing the collective motion of a hypothetical thin flux tube. The existence of massless modes in these theories is then shown to be the unifying root of the roughening effect and the power corrections to the linear quark potential. In sect. 4, we demonstrate by a few examples that the leading correction (the term proportional to L^{-1}) is universal, i.e., independent of the parameters in the effective lagrangian. Sect. 5 contains some concluding remarks.

2. Fluctuations around a Bloch wall in 2 + 1 dimensions

We here consider an Ising-like ferromagnet described by the action**

$$S = \frac{1}{2m} \int d^3x \left\{ \partial_\mu \phi \, \partial^\mu \phi - \left(\phi^2 - m^2 \right)^2 \right\} . \tag{1}$$

* The situation is somewhat reminiscent of the transition from the low temperature to the coulombic phase in Z_N (N large) gauge theories, where the discrete Z_N symmetry becomes effectively a continuous U(1) symmetry.

** The metric is $g_{00} = -g_{11} = -g_{22} = 1$; instead of x^0, x^1, x^2 we shall also use the symbols t, y, z.

The familiar Bloch wall solution to the field equations is

$$\phi_{\rm cl}(x) = m \operatorname{tgh} mz\,, \tag{2}$$

which is independent of t and y and well localized in the transverse direction z.

Up to second order, the action for a fluctuation

$$\phi(x) = \phi_{\rm cl}(x) + \eta(x) \tag{3}$$

around a Bloch wall is

$$S = \text{constant} + \frac{1}{2m}\int \mathrm{d}^3x\, \eta(x)\cdot \Delta^{\phi}\eta(x)\,. \tag{4}$$

The fluctuation operator Δ^{ϕ} is given by

$$\Delta^{\phi} = -\partial_0^2 + \partial_1^2 + (\partial_2 - 2\phi_{\rm cl})(\partial_2 + 2\phi_{\rm cl})\,. \tag{5}$$

Its eigenfunctions separate according to

$$\Delta^{\phi}\eta = \left(k_0^2 - k_1^2 - \varepsilon\right)\eta\,, \tag{6a}$$

$$\eta(x) = \mathrm{e}^{i(k_0x^0 + k_1x^1)}\psi(x^2)\,, \tag{6b}$$

$$-(\partial_2 - 2\phi_{\rm cl})(\partial_2 + 2\phi_{\rm cl})\psi(x^2) = \varepsilon\psi(x^2)\,. \tag{6c}$$

The momentum $k = (k^0, k^1)$ is arbitrary and the reduced eigenvalues ε can be determined explicitly by making a clever substitution (ref. [8], p. 1651 ff), which reduces eq. (6c) to a hypergeometric equation. The outcome is as follows. There are two discrete eigenvalues

$$\psi_0(z) = m(\operatorname{ch} mz)^{-2}, \qquad \varepsilon = 0\,, \tag{7a}$$

$$\psi_1(z) = m \operatorname{sh} mz (\operatorname{ch} mz)^{-2}, \qquad \varepsilon = 3m^2\,, \tag{7b}$$

as well as a line of continuous eigenvalues

$$\psi(z;k_2) = m\,\mathrm{e}^{ik_2z}\left\{3(\operatorname{tgh} mz)^2 - 3i\frac{k_2}{m}\operatorname{tgh} mz - \frac{k_2^2}{m^2} - 1\right\},$$

$$\varepsilon = k_2^2 + 4m^2\,, \qquad -\infty < k_2 < \infty\,. \tag{7c}$$

The eigenvalues E of the fluctuation operator Δ^{ϕ} therefore fall into three classes:

$$E = k_0^2 - k_1^2, \tag{8a}$$

$$E = k_0^2 - k_1^2 - 3m^2, \tag{8b}$$

$$E = k_0^2 - k_1^2 - k_2^2 - 4m^2. \tag{8c}$$

The bulk (8c) of the spectrum comes from spin waves scattering off the Bloch wall. Their mass is $2m$, which is equal to the mass of the fundamental fluctuations around the classical vacua $\phi = \pm m$. The modes (8b) are exponentially localized in the transverse direction z for all times t: they represent bounded travelling waves along the Bloch wall. They too are massive with a mass equal to $m\sqrt{3}$. Class (8a), finally, also represents bounded travelling waves along the Bloch wall, but these modes are massless. From eqs. (6b) and (7a) we see furthermore that the corresponding fluctuations $\phi(x)$ have the general form

$$\begin{aligned} \phi(x) &= \phi_{\mathrm{cl}}(x) + \xi(x^0, x^1) m (\operatorname{ch} m x^2)^{-2} \\ &= \phi_{\mathrm{cl}}\left(x^0, x^1, x^2 + \xi(x^0, x^1)\right) + \mathrm{O}(\xi^2), \end{aligned} \tag{9}$$

i.e., $\phi(x)$ is simply a locally translated Bloch wall. In other words, the modes (8a) are the Goldstone modes associated with the spontaneously broken translation symmetry. The crucial point here is that they are well separated from the rest of the spectrum by a mass gap. This is what makes them behaving effectively like Goldstone bosons in 1 + 1 dimensions so that the Mermin-Wagner-Coleman theorem applies: there are no infinitely long Bloch walls in a *quantized* ferromagnet in 2 + 1 dimensions.

3. Effective theories for fluctuating thin flux tubes

As explained in sect. 1, the glue field in the presence of a static quark-antiquark pair at a finite distance L can perhaps be described by a fluctuating thin tube containing the chromoelectric flux, which flows from the quark to the antiquark. This means that the degrees of freedom of the glue field can be divided into two classes containing those, which describe the state inside the thin flux tube, respectively those describing its position in space. Suppose the quarks are located at $\boldsymbol{x} = (0,0,0)$ and $\boldsymbol{x} = (L,0,0)$. The position of the string can then be specified by a two-component vector field

$$\boldsymbol{\xi}(x^0, x^1), \qquad 0 \leqslant x^1 \leqslant L, \qquad \boldsymbol{\xi}(x^0, 0) = \boldsymbol{\xi}(x^0, L) = 0, \tag{10}$$

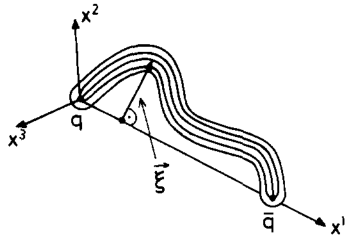

Fig. 1. Thin flux tube at a fixed time x^0. The position vector ξ is parallel to the (x^2, x^3) plane.

representing the deflection of the thin flux tube from its rest position $x = (x^0, x^1, 0, 0)$, $0 \leqslant x^1 \leqslant L$ (see fig. 1)*. In what follows we assume that the internal degrees of freedom of the flux tube are frozen and concentrate on the resulting effective dynamics of the "order parameter" field ξ.

We do not know, of course, what the effective action for ξ is. However, we may expect it to have the following general properties:

(i) It should be local, i.e.,

$$S_{\mathrm{eff}} = \int_{0<x^1<L} \mathrm{d}^2x\, \mathcal{L}(x), \tag{11}$$

where the Lagrange density $\mathcal{L}(x)$ is a function of $\xi(x)$ and its derivatives.

(ii) $\mathcal{L}(x)$ must be invariant under the following symmetry operations:

(a) Poincaré transformations in the (x^0, x^1) plane;

(b) O(2) rotations and translations of the field vector ξ.

In particular, the last property implies that $\mathcal{L}(x)$ does not depend on $\xi(x)$, but on its derivatives $\partial_\mu \xi(x), \partial_\mu \partial_\nu \xi(x), \ldots (\mu, \nu = 0, 1)$ only. We are thus left with

$$S_{\mathrm{eff}} = \int_{0<x^1<L} \mathrm{d}^2x \Big\{ a\partial_\mu \xi \cdot \partial^\mu \xi + b\partial_\mu \partial^\mu \xi \cdot \partial_\nu \partial^\nu \xi$$
$$+ c(\partial_\mu \xi \cdot \partial^\mu \xi)^2 + d(\partial_\mu \xi \cdot \partial_\nu \xi)(\partial^\mu \xi \cdot \partial^\nu \xi) + \cdots \Big\}. \tag{12}$$

Here, a, b, c and d are constants and the terms included are all those having a dimension (= total number of derivatives) smaller or equal to four.

The effective action (12) is not renormalizable. This is not a catastrophe, however, because the high frequency oscillations of ξ are to be cut off anyhow: when ξ wiggles with a wavelength which is not much larger than the diameter of the thin flux tube, the internal degrees of freedom of the tube are excited and the description of the state in terms of ξ breaks down. Thus, when doing calculations

* "Overhangs" of the string are excluded here. Within the perturbative analysis presented below, overhangs are irrelevant anyhow.

with S_{eff}, a systematic cutoff must be introduced. Significant predictions for the low-frequency properties of the flux-tube states can then safely be obtained. In this regime, all terms in the effective action of dimension larger or equal to four are irrelevant: when the corresponding vertices are inserted in a Feynman graph, its infrared behaviour improves. We thus arrive at the important conclusion that the gaussian action

$$S_{\mathrm{eff}}^{0} = a\int_{0<x^1<L} \mathrm{d}^2x\, \partial_\mu \xi \cdot \partial^\mu \xi \tag{13}$$

can be expected to give an at least qualitatively correct description of the long-wavelength fluctuations of the thin flux tube*.

The most important feature of the effective flux tube theory is that the ξ field is massless. For example, the roughening effect is due to this fact: if one computes the width σ of the ground-state wave function in the gaussian approximation (13), one finds (ref. [3], sect. 3.2) that the long-wavelength fluctuations of ξ make a large contribution:

$$\sigma^2 \sim \frac{1}{2\pi a} \ln L, \qquad (L \to \infty). \tag{14}$$

Another manifestation of the zero-mass fluctuations of the string is the appearance of power corrections to the linear quark potential [10]

$$V(L) = \alpha L + \beta + \gamma L^{-1} + \mathrm{O}(L^{-2}), \qquad (L \to \infty). \tag{15}$$

Namely, $V(L)$ is equal to the ground-state energy of the effective ξ-field theory (up to an additive term proportional to L), which would be a linear function of L with only exponentially small corrections, if there was a mass gap. The constants α and β in eq. (15) depend on the parameters $a, b, \ldots$ in the effective action (12), but, as we shall argue in sect. 4, γ does not depend on them and can therefore be exactly evaluated in the gaussian approximation [10]**:

$$\gamma = -\tfrac{1}{12}\pi. \tag{16}$$

It is this kind of universality, which ultimately makes the effective field theory approach quantitatively useful, although we have no theoretical means to determine the coupling constants.

* We emphasize that the basis for this conclusion is strictly perturbation theoretic. Still, perturbation theory has recently been shown to be asymptotic (at least for the free energy) in a $(\nabla\phi)^4$ theory on a lattice [9].

** For a flux tube in d space-time dimensions, this number must be multiplied by $\frac{1}{2}(d-2)$.

Recall finally that a mass term for $\xi(x)$ has been dismissed because of translation symmetry in field space [requirement (iib) above]. This symmetry is explicitly broken by the boundary condition $\xi(x^0,0)=\xi(x^0,L)=0$, which implies that the ground-state expectation value of ξ is zero. Making L large thus amounts to an attempt to create a state with *spontaneous* translational symmetry breaking. As has been argued before (sects. 1, 2), such a state cannot exist and it is avoided here, because the string roughens indefinitely as $L\to\infty$ [eq. (14)].

4. Universality of the Coulomb potential correction to the linear quark potential

Superficially, one might argue that γ [eq. (15)] must be universal, because the coupling constants multiplying interaction terms of dimension δ carry a dimension $(\text{length})^{\delta-2}$ so that they cannot, in perturbation theory, contribute to a dimensionless number. However, the ξ-field theory cannot live without a cutoff and this might well upset the naive dimensional analysis. Rather than a general proof, we shall now present some sample calculations to demonstrate the universality of γ.

The ground-state energy $V(L)$ of the ξ-field theory is most conveniently calculated in the euclidean path integral formalism. As a cutoff we use a lattice $\Lambda=\mathbb{Z}^{2}$*. $V(L)$ is then given by

$$V(L)=-\lim_{T\to\infty}\frac{1}{T}\ln Z, \tag{17}$$

$$Z=\int\prod_{z_1=1}^{L-1}\prod_{z_2=1}^{T-1}\mathrm{d}^2\xi(z)\exp(-S_{\mathrm{eff}}[\xi]). \tag{18}$$

The effective action appearing here is the euclidean version of expression (12) suitably latticized. For concreteness, we shall only consider the following two cases:

$$S_{1,2}=S_0+S^{\mathrm{I}}_{1,2}, \tag{19a}$$

$$S_0=-a\sum_{z^1=1}^{L-1}\sum_{z^2=1}^{T-1}\xi(z)\cdot\Delta\xi(z), \tag{19b}$$

$$S^{\mathrm{I}}_1=b\sum_{z^1=1}^{L-1}\sum_{z^2=1}^{T-1}\Delta\xi(z)\cdot\Delta\xi(z), \tag{19c}$$

$$S^{\mathrm{I}}_2=c\sum_{z^1=0}^{L-1}\sum_{z^2=0}^{T-1}\left[\partial_\mu\xi(z)\cdot\partial_\mu\xi(z)\right]^2. \tag{19d}$$

* Henceforth, all distances are measured in units of the lattice spacing, which is set equal to one for convenience.

The notation here is as follows. ∂_μ is the lattice derivative,

$$\partial_\mu f(z) = f(z+\hat{\mu}) - f(z),$$

$$\partial_\mu^* f(z) = f(z) - f(z-\hat{\mu}), \tag{20}$$

where $\hat{\mu}$ denotes the unit vector in the positive μ-direction. The lattice laplacian then is

$$\Delta = \partial_\mu^* \partial_\mu = \partial_\mu \partial_\mu^* . \tag{21}$$

Furthermore, the following boundary conditions have been assumed [cf. eq. (10)]:

$$\xi(0, z_2) = \xi(L, z_2) = 0, \tag{22a}$$

$$\xi(z_1, 0) = \xi(z_1, T) = 0. \tag{22b}$$

Consider first the case of the gaussian action S_1. Decomposing the field into normal modes,

$$\xi(z) = 2(T \cdot L)^{-1/2} \sum_p \eta(p) \sin p_1 z_1 \sin p_2 z_2, \tag{23a}$$

$$p_1 = \frac{\pi}{L}, \frac{2\pi}{L}, \ldots, (L-1)\frac{\pi}{L}, \tag{23b}$$

$$p_2 = \frac{\pi}{T}, \frac{2\pi}{T}, \ldots, (T-1)\frac{\pi}{T}, \tag{23c}$$

we have

$$S_1 = \sum_p [\eta(p)]^2 \left(aE_p + bE_p^2\right), \tag{24}$$

$$E_p = 4 - 2\cos p_1 - 2\cos p_2 . \tag{25}$$

It follows that

$$Z = (\pi)^{(L-1)(T-1)} \prod_p \left(aE_p + bE_p^2\right)^{-1}, \tag{26}$$

and hence

$$V(L) = -(L-1)\ln\pi + \frac{1}{\pi}\sum_{p_1} \int_0^\pi \mathrm{d}p_2 \ln\left(aE_p + bE_p^2\right)$$

$$= -(L-1)\ln\pi + \sum_{p_1} \left[f(p_1) + g(p_1) \right],$$

$$f(p_1) = \frac{1}{\pi} \int_0^\pi \mathrm{d}p_2 \ln(4 - 2\cos p_1 - 2\cos p_2), \tag{27a}$$

$$g(p_1) = \frac{1}{\pi}\int_0^{\pi} \mathrm{d}p_2 \ln[a + b(4 - 2\cos p_1 - 2\cos p_2)]\,. \tag{27b}$$

To determine the large-L behaviour of $V(L)$, we use Euler's sum formula (e.g., ref. [11], p. 46ff):

Lemma: Suppose $h(p)$, $0 \leqslant p \leqslant \pi$ is a $(2k+1)$-times continuously differentiable function. Then, as $L \to \infty$,

$$\sum_{\nu=1}^{L-1} h\left(\nu\frac{\pi}{L}\right) = \frac{L}{\pi}\int_0^{\pi} \mathrm{d}p\, h(p) - \tfrac{1}{2}(h(0) + h(\pi))$$

$$- \sum_{\mu=1}^{k} \left(\frac{\pi}{L}\right)^{2\mu-1} \frac{B_{2\mu}}{(2\mu)!}\left[h^{(2\mu-1)}(0) - h^{(2\mu-1)}(\pi)\right]$$

$$+ \mathrm{O}(L^{-2k-1})\,, \tag{28}$$

where the $B_{2\mu}$'s are the Bernoulli numbers (ref. [12], sect. 9.6).

Now, $g(p)$ is obviously differentiable in the interval $[0, \pi]$ and

$$g^{(2\mu-1)}(0) = g^{(2\mu-1)}(\pi) = 0\,, \qquad (\mu = 1, 2, \dots)\,.$$

It follows that up to terms smaller than any power of L^{-1}

$$\sum_{p_1} g(p_1) = \frac{L}{\pi}\int_0^{\pi} \mathrm{d}p\, g(p) - \frac{1}{2}(g(0) + g(\pi))\,.$$

In particular, there is no contribution to γ and this coefficient is therefore independent of the parameters a and b.

The contribution to $V(L)$ involving $f(p)$ is what we would have obtained from the free field action S_0 alone. The lemma above cannot immediately be applied here, because $f(p)$ is not obviously differentiable at $p = 0$: when the integrand is differentiated it becomes singular at $p_1 = p_2 = 0$. However, we may first evaluate the integral explicitly (ref. [12], sect. 4.224):

$$f(p) = \ln\left\{1 + 2\sin\tfrac{1}{2}p\sqrt{1 + \sin^2\tfrac{1}{2}p} + 2\sin^2\tfrac{1}{2}p\right\}. \tag{29}$$

This representation is manifestly C^{∞} in $[0, \pi]$ and

$$f^{(2\mu-1)}(\pi) = 0\,, \qquad (\mu = 1, 2, \dots)\,,$$

$$f'(0) = 1\,.$$

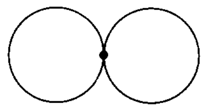

Fig. 2. First-order correction to $V(L)$ in the theory with action S_2.

It thus follows from the lemma that

$$\sum_{p_1} f(p_1) = \frac{L}{\pi}\int_0^{\pi} \mathrm{d}p f(p) - \tfrac{1}{2}\ln(3+2\sqrt{2}\,)$$

$$-\tfrac{1}{12}\pi L^{-1} + \mathrm{O}(L^{-3}), \tag{30}$$

which proves eq. (16).

Next, consider the more complicated action S_2 [eq. (19)]. To first order in c, the correction to the quark potential comes from a graph with two non-overlapping loops (fig. 2):

$$\Delta V^{(1)}(L) = \frac{c}{4a^2}\sum_{z_1=0}^{L-1}\left\{2\left[\partial_\mu G_L(x,y)\overleftarrow{\partial}_\nu\right]^2 + \left[\partial_\mu G_L(x,y)\overleftarrow{\partial}_\mu\right]^2\right\}_{x=y=z}. \tag{31}$$

Here, G_L denotes the Green function of the lattice laplacian in the strip $0 \leqslant z_1 \leqslant L$:

$$-\Delta G_L(x,y) = \delta_{x,y} \qquad \text{for } 1 \leqslant x_1 \leqslant L-1,$$

$$G_L(x,y) = 0, \qquad \text{for } x_1 = 0, L. \tag{32}$$

Explicitly [cf. eq. (23b)],

$$G_L(x,y) = \frac{2}{L}\sum_{p_1}\int_{-\pi}^{\pi}\frac{\mathrm{d}p_2}{2\pi}\sin p_1 x_1 \sin p_1 y_1\, \mathrm{e}^{ip_2(x_2-y_2)}$$

$$\times(4 - 2\cos p_1 - 2\cos p_2)^{-1}. \tag{33}$$

Inserting this expression into eq. (31), performing the p_2 integrals and applying Euler's sum formula would straightforwardly confirm that there is no correction to γ to first order in c. Since this is not particularly illuminating, we shall follow a position-space method, which was suggested to the author by Symanzik [13].

The idea is to express the Green function $G_L(x, y)$ by the infinite-volume Green function

$$G(x,y)=\int_{-\pi}^{\pi}\frac{\mathrm{d}^2p}{(2\pi)^2}\left(\mathrm{e}^{ip\cdot(x-y)}-1\right)(4-2\cos p_1-2\cos p_2)^{-1}. \tag{34}$$

Namely, we have

$$\partial_\mu G_L(x,y)\overleftarrow{\partial}_\nu=\sum_{n=-\infty}^{\infty}\left\{\partial_\mu G(x_1-y_1+2nL;x_2-y_2)\overleftarrow{\partial}_\nu\right.$$

$$\left.-\,\partial_\mu G(x_1+y_1+2nL;x_2-y_2)\overleftarrow{\partial}_\nu\right\}. \tag{35}$$

The sum here is absolutely convergent, because

$$\left|\partial_\mu G(x,y)\overleftarrow{\partial}_\nu\right|<\text{constant}\cdot\left(1+(x-y)^2\right)^{-1}. \tag{36}$$

It follows that

$$\partial_\mu G_L(x,y)\overleftarrow{\partial}_\nu\big|_{x=y=z}=\tfrac{1}{2}\delta_{\mu\nu}-R_{\mu\nu}(z_1), \tag{37a}$$

$$R_{\mu\nu}(z_1)=\sum_{n=-\infty}^{\infty}\left\{\partial_\mu\partial_\nu^* G(2nL;0)(1-\delta_{n,0})\right.$$

$$\left.+\,\partial_\mu(\delta_{\nu,1}\partial_1-\delta_{\nu,2}\partial_2^*)G(2z_1+2nL;0)\right\}. \tag{37b}$$

The crucial point is that the defect $R_{\mu\nu}$ is small for $L\to\infty$: either the terms in eq. (37b) are constant and of order L^{-2}, or they fall off as z_1^{-2} or $(z_1-L)^{-2}$ away from the boundary $z_1=0, L$. Inserting eq. (37) into eq. (31) yields

$$\Delta V^{(1)}(L)=\frac{c}{4a^2}\left\{2L+\sum_{z_1=0}^{L-1}\left[-4R_{\mu\mu}(z_1)+2R_{\mu\nu}(z_1)R_{\mu\nu}(z_1)+\left(R_{\mu\mu}(z_1)\right)^2\right]\right\}.$$

From the estimate (36) (which also holds if ∂_ν is replaced by ∂_ν^*) and eq. (37b), it is easy to infer that the terms quadratic in $R_{\mu\nu}$ do not contribute to γ. Furthermore, using

$$-\Delta G(x,y)=\delta_{x,y}, \tag{38}$$

we have

$$\sum_{z_1=0}^{L-1} R_{\mu\mu}(z_1) = \sum_{z_1=0}^{L-1} \sum_{n=-\infty}^{\infty} (\partial_1^2 - \partial_2^*\partial_2) G(2z_1 + 2nL; 0)$$

$$= \sum_{z_1=-\infty}^{\infty} (\partial_1^2 - \partial_2^*\partial_2) G(2z_1; 0),$$

which is independent of L. Summing up, we have shown that

$$\Delta V^{(1)}(L) = \frac{c}{2a^2} L + \text{constant} + \mathrm{O}(L^{-2}) . \tag{39}$$

The above proof of universality of γ to first order in c carries over to all other admissible interactions. That higher-order graphs do not contribute either is plausible, because diagrams with overlapping loops are less singular in the infrared than the simple graph of fig. 2. In fact, the proof technique used above also works for higher-order graphs, but we shall not go into any of the rather involved details here. We only remark that the (discrete) Lorentz invariance of the effective action is essential for the proof.

5. Conclusions

The main results obtained are

(i) that infinitely long localized flux tubes cannot exist in a (continuum) Yang-Mills theory;

(ii) that the coefficient γ of the Coulomb potential correction to the linear quark potential at large separations is universal within a wide class of effective theories describing the vibrations of thin flux tubes.

Whereas the first result requires very little input (and could in fact be made rigorous in an axiomatic framework), the second assumes the picture of a fluctuating thin flux tube with frozen internal degrees of freedom. This is also implicit in the SOS-type models of roughening and is probably correct in lattice gauge theories with a coupling constant g^2 just below the roughening point g_R^2. At least in these cases, we thus expect γ to be equal to the universal value (16), a prediction, which is consistent with recent Monte Carlo [14] and strong coupling [15]* calculations.

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* γ is sensitive to a violation of (discrete) Lorentz invariance. The velocity of light must therefore be carefully renormalized in hamiltonian lattice gauge theories in order to obtain the correct value for γ.

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