ORDER α_s^2 CORRECTION TO JET CROSS SECTIONS IN e^+e^- ANNIHILATION

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We calculate the Sterman–Weinberg type cross section for three jets and the $O(\alpha_s^2)$ correction to the thrust distribution $\sigma^{-1}\,d\sigma/dT$. The effect on $\sigma^{-1}\,d\sigma/dT$ is sizeable (in the $\overline{\rm MS}$ scheme) but not in conflict with the expected convergence of the perturbation series.

The virtual photon produced in e^+e^- annihilation is one of the cleanest and most powerful probes of the short-distance structure of hadrons. For example, there is no ambiguity due to target structure functions, and the presently available energies allow to probe distances smaller than 10^{-15} cm. This makes PETRA and PEP a laboratory in which perturbative QCD can be subjected to significant tests.

A first prediction of QCD is het existence of multi-jet events originating in hard gluon bremsstrahlung [1]. This has been nicely confirmed by the recent PETRA data [2], and the observed properties of those events seem to be in good (qualitative) agreement with QCD. So far there is, however, precious little evidence for the more fundamental aspects of the theory such as its non-abelian gauge structure (triple-gluon coupling) which only shows up in order α_s^2 and higher. And even the present apparent consistency of theory and experiment could prove to be accidental supposing that higher-order corrections are large. On top of that, any quantitative analysis of the data, i.e., the determination of $\alpha_s(Q^2)$ or, equivalently, the scale parameter of the strong interactions Λ and comparison with other processes has meaning only if higher-order contributions are included.

A first step in this direction has been the calculation of the $O(\alpha_s^2)$ correction to the total cross section [3]. In the $\overline{\rm MS}$ renormalization scheme [4] it was found

$$\sigma = \sigma_0\,\{1 + \alpha_s(Q^2)/\pi + [\alpha_s(Q^2)/\pi]^2(1.986 - 0.115\,N_f)\}\,, \tag{1}$$

where [5]

$$\alpha_s(Q^2) = 4\pi[\beta_0 \ln Q^2/\Lambda^2 + \beta_1/\beta_0\,\ln(\ln Q^2/\Lambda^2)]^{-1}\,, \quad \beta_0 = 11 - \tfrac{2}{3}N_f, \quad \beta_1 = 102 - \tfrac{38}{3}N_f\,. \tag{2}$$

σ_0 refers to the Born cross section, and N_f is the number of flavours.

The main result of this calculation is that higher-order corrections to the total cross section are indeed small. The $\overline{\rm MS}$ renormalization scheme also minimizes higher-order corrections to various other processes [6] which one might take as indication that the perturbation expansion converges well. From the more physical point of view the next to leading QCD correction to the total cross section has, however, little significance. It has an effect of only less than half a percent on the zeroth order background which, in addition, can totally be absorbed into the $O(\alpha_s)$

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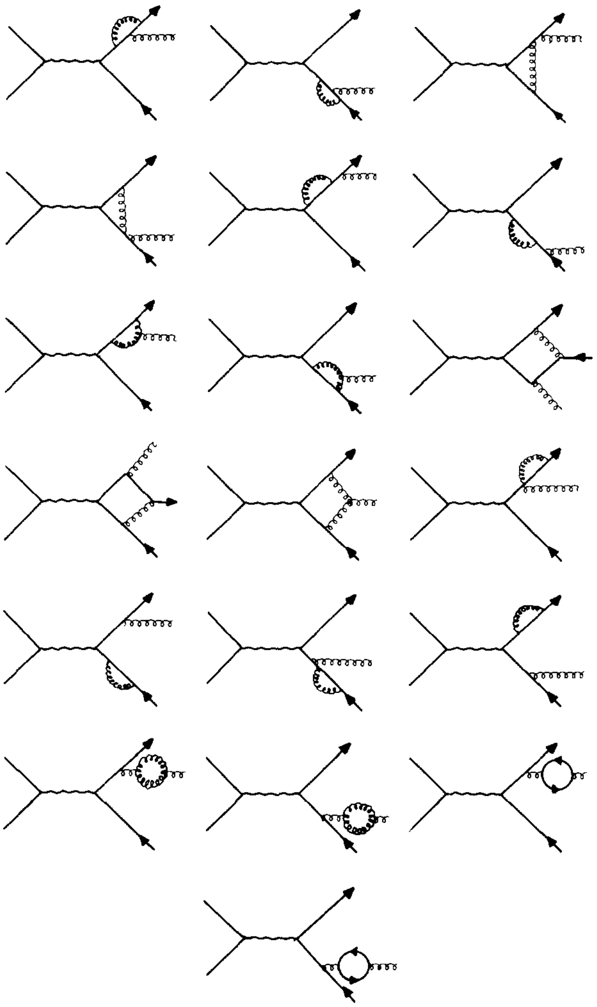
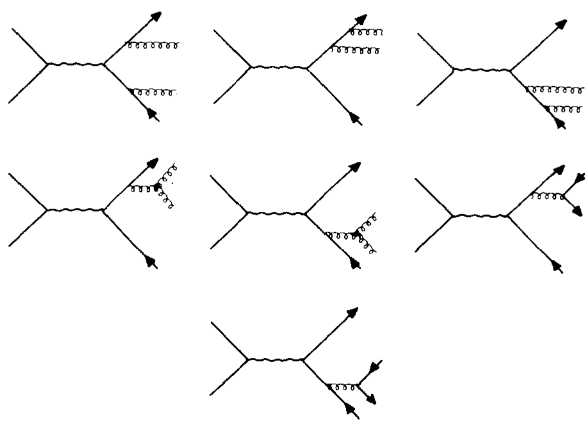


(a)



(b)

Fig. 1. Order α_s^2 three-jet diagrams (a) interfering with the tree graphs and four-jet diagrams (b).

contribution by replacing (TS total subtraction scheme)

$$\Lambda_{\overline{\rm MS}} \to \Lambda_{\rm TS} = \exp\,[(4/\beta_0)(1.986 - 0.115 N_{\rm f})]\Lambda_{\overline{\rm MS}} \underset{N_{\rm f}=5}{=} 1.44\,\Lambda_{\overline{\rm MS}}\,. \qquad (3)$$

This would require to measure the total cross section (absolutely) with unprecedented accuracy.

In this letter we shall go one step further and calculate the order α_s^2 correction to the three-jet structure. Due to the limited format we will not go into any details of the calculation but refer the reader to the long write-up [7].

The corresponding three- and four-jet diagrams are shown in fig. 1. Individually, the loop-corrected three-jet diagrams (fig. 1a) and the four-jet diagrams (fig. 1b) are infrared singular. When combined to properly defined cross sections, which are insensitive to the emission of soft and/or collinear quanta, those singularities are supposed to cancel as in QED. Such cross sections are, e.g., the Sterman–Weinberg [8] type cross section for three jets (three jets with energy and angular resolution) and the thrust [9] distribution $\sigma^{-1}\,\mathrm{d}\sigma/\mathrm{d}T$ which we will discuss here.

The jet cross section can be written

$$\mathrm{d}\sigma^{\mathrm{jet}} \underset{T<1}{=} \mathrm{d}\sigma^{3\text{-jet}}(\epsilon,\delta) + \mathrm{d}\sigma^{4\text{-jet}}(\epsilon,\delta)\,, \tag{4}$$

where $\mathrm{d}\sigma^{3\text{-jet}}(\epsilon,\delta)$ is the cross section for three-jet events which have all but a fraction 2ϵ of the total energy distributed within three separated cones of (full) opening angle δ, and $\mathrm{d}\sigma^{4\text{-jet}}(\epsilon,\delta)$ is the proper four-jet cross section with jet energies larger than $\epsilon Q/2$ and relative jet angles larger than δ. Two-jet events have been eliminated by requiring $T<1$. Clearly, $\mathrm{d}\sigma^{\mathrm{jet}}$ is independent of any particular choice of ϵ and δ. The proper four-jet cross section $\mathrm{d}\sigma^{4\text{-jet}}(\epsilon,\delta)$ is infrared finite (to order α_s^2) and can be computed along the lines of ref. [10]. The calculation of $\mathrm{d}\sigma^{3\text{-jet}}(\epsilon,\delta)$ proceeds as follows.

The loop diagrams (fig. 1a) are calculated in n dimensions. The infrared (ultraviolet) singularities then appear as poles $\sim 1/\lambda^2$ and $\sim 1/\lambda$ ($\sim 1/\lambda$) in $\lambda = n-4$. We perform renormalization in the $\overline{\mathrm{MS}}$ scheme which corresponds to the subtraction of the ultraviolet poles and the associated factors of $(\ln 4\pi - \gamma)$. The radiative corrections to the three-jet diagrams (fig. 1b) are also calculated in n dimensions. The region of one soft and/or collinear parton emission gives rise to poles $\sim 1/\lambda^2$ and $\sim 1/\lambda$ as well which cancel against the infrared poles of the loop diagrams as they should. After the poles have vanished the limit $\lambda \to 0$ is taken. All calculations are performed in the Feynman gauge.

In intermediate stages of the calculation expressions become quite lengthy. The final result assumes, however, the relatively compact form

$$\frac{1}{\sigma}\frac{\mathrm{d}^2\sigma^{3\text{-jet}}(\epsilon,\delta)}{\mathrm{d}x_1\,\mathrm{d}x_2} = \frac{2}{3}\frac{\alpha_s(Q^2)}{\pi}\frac{\sigma_0}{\sigma}\Bigg[\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}\Bigg\{1 - \frac{\alpha_s(Q^2)}{\pi}\Bigg[\left(\frac{4}{3}\ln\frac{\epsilon}{x_1} + \frac{4}{3}\ln\frac{\epsilon}{x_2} + 3\ln\frac{\epsilon}{x_3} + \frac{19}{4} - \frac{N_\mathrm{f}}{6}\right)\ln\frac{1-\cos\delta}{2}$$

$$-\ln\epsilon\left(3\ln\frac{1-\cos\theta_{13}}{2} + 3\ln\frac{1-\cos\theta_{23}}{2} - \frac{1}{3}\ln\frac{1-\cos\theta_{12}}{2}\right) - \left(\frac{4}{3}\frac{\epsilon}{x_1} + \frac{4}{3}\frac{\epsilon}{x_2} + 3\frac{\epsilon}{x_3}\right)\ln\frac{1-\cos\delta}{2}$$

$$+\left(\frac{11}{2} - \frac{N_\mathrm{f}}{3}\right)\ln x_3 + R(x_1,x_2)\Bigg]\Bigg\} + \frac{\alpha_s(Q^2)}{\pi}F(x_1,x_2)\Bigg] + \mathrm{O}(\epsilon) + \mathrm{O}(\delta^2)\,, \tag{5}$$

where

$$R(x_1,x_2) = \Bigg\{\ln x_3\left(3\ln\frac{1-x_1}{x_3} - \tfrac{3}{2}\ln x_1\right) - \tfrac{1}{6}\ln(1-x_3)\,[\ln x_1 + \ln(1-x_1)]$$

$$-\ln x_1\,[2\ln x_1 + \tfrac{4}{3}\ln(1-x_1) - \tfrac{3}{2}\ln(1-x_2) + 2] + \tfrac{3}{2}\,\mathcal{L}_2\left(\frac{1-\cos\theta_{13}}{2}\right) - \tfrac{4}{3}\mathcal{L}_2(1-x_1) + (x_1 \leftrightarrow x_2)\Bigg\} \tag{6}$$

$$+\tfrac{3}{2}\ln\frac{1-x_1}{x_3}\ln\frac{1-x_2}{x_3} + \tfrac{1}{3}\ln x_3\ln(1-x_3) + \tfrac{1}{6}\ln x_1\ln x_2 - \tfrac{1}{6}\,\mathcal{L}_2\left(\frac{1-\cos\theta_{12}}{2}\right) + \tfrac{1}{3}\,\mathcal{L}_2(1-x_3) + \tfrac{13}{18}N_\mathrm{f} + \tfrac{2}{3}\pi^2 - \tfrac{44}{3}\,,$$

$$F(x_1,x_2) = \frac{x_3^2}{(1-x_1)(1-x_2)}\{\tfrac{1}{12}\,[\ln(1-x_3)\ln(1-x_1) - \ln x_1\ln(1-x_1) - \mathcal{L}_2(1-x_1) + (x_1 \leftrightarrow x_2)]$$

$$-\tfrac{1}{6}\ln x_3\ln(1-x_3) - \tfrac{1}{6}\mathcal{L}_2(1-x_3) + \tfrac{1}{12}N_\mathrm{f} + \tfrac{1}{36}\pi^2 - \tfrac{11}{12}\} + \frac{x_3}{(1-x_1)(1-x_2)}\{\tfrac{1}{6}[-\ln(1-x_3)\ln(1-x_1)$$

$$+\ln x_1\ln(1-x_1) + \mathcal{L}_2(1-x_1) + (x_1 \leftrightarrow x_2)] + \tfrac{1}{3}\ln x_3\ln(1-x_3) + \tfrac{1}{3}\mathcal{L}_2(1-x_3) - \tfrac{1}{18}\pi^2 + \tfrac{5}{6}\}$$

$$+\tfrac{1}{6}[\ln(1-x_3)\ln(1-x_1) - \ln x_1\ln(1-x_1) + 16\ln(1-x_1) - \mathcal{L}_2(1-x_1) + (x_1 \leftrightarrow x_2)]$$

$$-\tfrac{1}{3}\ln x_3\ln(1-x_3) - \tfrac{1}{3}\mathcal{L}_2(1-x_3) + \tfrac{1}{18}\pi^2 + \tfrac{1}{6}$$

$$+\tfrac{1}{12}\left[\frac{x_1^2-x_2^2}{(1-x_1)(1-x_2)}(\ln(1-x_3)\ln(1-x_1) - \ln x_1\ln(1-x_1) - \mathcal{L}_2(1-x_1)) + (x_1 \leftrightarrow x_2)\right]$$

$$-\tfrac{2}{3}\left[\frac{1-x_1}{x_1}+\{4(1-x_2)-\tfrac{17}{4}(1-x_1)\}\frac{\ln(1-x_1)}{x_1}+(1-x_1)(1-x_2)\frac{\ln(1-x_1)}{x_1^2}+(x_1\leftrightarrow x_2)\right]$$

$$-\frac{1}{3}\frac{1-x_3}{x_3}\left[1+\frac{1+x_3}{x_3}\ln(1-x_3)\right], \tag{7}$$

$$\mathcal{L}_2(x)=-\int_0^x dz\,\frac{\ln(1-z)}{z}, \tag{8}$$

and $x_i = 2E_i/Q$ $(x_1+x_2+x_3=2)$ denotes the (scaled) energy of the quark (x_1), antiquark (x_2) and gluon jet (x_3), respectively. The cosines of the angles between the jet axes are given by

$$\cos\theta_{ij}=1-(2/x_ix_j)(x_i+x_j-1). \tag{9}$$

The $O(\epsilon)$ and $O(\delta^2)$ terms are computed numerically.

The leading contribution $(\epsilon, \delta^2 \ll 1)$ of eq. (5) agrees with the leading logarithm calculation of Smilga and Vysotsky [11] who found [‡1]

$$\frac{1}{\sigma}\frac{d^2\sigma^{3\text{-jet}}(\epsilon,\delta)}{dx_1dx_2}=\frac{2}{3}\frac{\alpha_s(Q^2)}{\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}\left\{1-\frac{\alpha_s(Q^2)}{\pi}\left[\left(\frac{4}{3}\ln\frac{\epsilon}{x_1}+\frac{4}{3}\ln\frac{\epsilon}{x_2}+3\ln\frac{\epsilon}{x_3}+\frac{19}{4}-\frac{N_f}{6}\right)\ln\frac{\delta^2}{4}\right.\right.$$

$$\left.\left.-\ln\epsilon\left(3\ln\frac{1-\cos\theta_{13}}{2}+3\ln\frac{1-\cos\theta_{23}}{2}-\frac{1}{3}\ln\frac{1-\cos\theta_{12}}{2}\right)\right]\right\}. \tag{10}$$

For $\epsilon, \delta^2 \ll 1$, but $[\alpha_s(Q^2)/\pi]\ln(\epsilon/x_i)\ln(\delta^2/4)$ still small against one, i.e., in the perturbative regime this is, however, found not to be a good approximation.

The running coupling constant in eq. (5) is evaluated at Q^2 which provides the natural scale when all jet angles are large. But when approaching the two-jet limit the four momentum squared, which determines the strength of the quark–gluon coupling, can become much smaller. This mismatch of scales will result in large logarithms which, in certain kinematical regimes might render the perturbation series nonconvergent.

The leading logarithmic correction can directly be read off from eq. (5):

$$-\tfrac{2}{3}[\alpha_s(Q^2)/\pi]^2(\sigma_0/\sigma)[(x_1^2+x_2^2)/(1-x_1)(1-x_2)](\tfrac{11}{2}-\tfrac{1}{3}N_f)\ln x_3\,. \tag{11}$$

The logarithms in $R(x_1, x_2)$ and $F(x_1, x_2)$ are finite in the limit $x_1, x_2 \to 1$ or cancel against [‡2] $-3\ln x_3 \ln\frac{1}{2}(1-\cos\delta)$ [cf. eq. (5)]. As can readily be seen, (11) may be absorbed into the strong coupling constant by the change of scales

$$Q^2\to x_3^2Q^2,\quad \alpha_s(Q^2)\to\alpha_s(x_3^2Q^2)\,. \tag{12}$$

That is to say, if the three-point vertex is renormalized at $x_3^2Q^2$ rather than Q^2 the loop corrections give rise to logarithms which exactly cancel (11). This absorption mechanism will also appear in higher orders so that better convergence of the perturbation series can be expected if $d\sigma^{3\text{-jet}}(\epsilon, \delta)$ is expanded in powers of $\alpha_s(x_3^2Q^2)$.

The thrust distribution $\sigma^{-1}\,d\sigma/dT$ can be obtained from eq. (5) and $d\sigma^{4\text{-jet}}(\epsilon, \delta)$ by a simple phase space integration. The jet cones of the three-jet cross section $d\sigma^{3\text{-jet}}(\epsilon, \delta)$ must, however, not overlap. This can be avoided by taking δ sufficiently small.

In our numerical calculations we have taken $\alpha_s(Q^2)/\pi = 0.05$ which corresponds to $\Lambda = 0.3$ GeV at $Q = 30$ GeV and for $N_f = 5$ based on the two-loop expression (2). The thrust distribution is shown in fig. 2 for the $\overline{\text{MS}}$ and TS

‡1 Note that ref. [11] contains a misprint (A.V. Smilga, private communication) and that our δ is defined differently.

‡2 Note that the angles between the jet axes must not be smaller than δ.

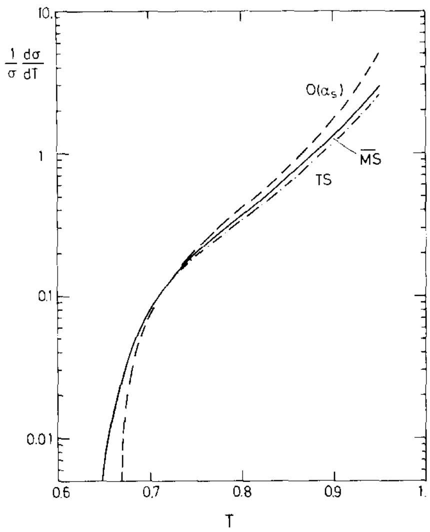


Fig. 2. Thrust distribution $\sigma^{-1}\,d\sigma/dT$ to order α_s^2 in the $\overline{\text{MS}}$ and TS subtraction scheme for $\alpha_s(Q^2)/\pi = 0.05$. Also shown is the order α_s contribution (same α_s).

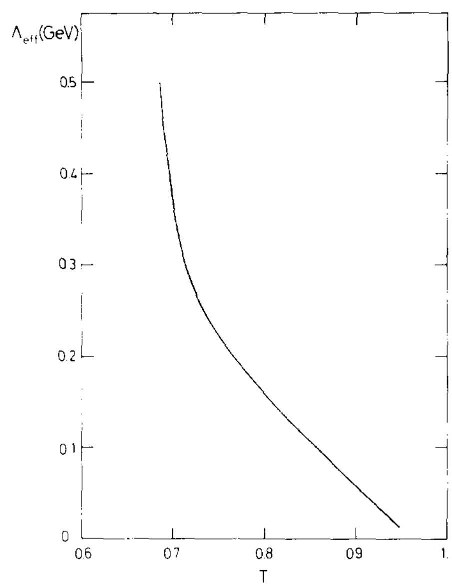


Fig. 3. The effective scale parameter Λ (Q = 30 GeV, $\Lambda_{\overline{\text{MS}}}$ = 0.3 GeV, N_f = 5).

subtraction scheme together with the order α_s contribution. The TS scheme reduces the higher-order corrections in e^+e^- annihilation to an absolute minimum.

The shape of the thrust distribution changes quite a bit (≈30% at $T = 0.9$) if order α_s^2 corrections are included. At very low thrust values $\sigma^{-1}\,d\sigma/dT$ will get enhanced. This is entirely due to four-jet production. At larger thrust values the (order α_s^2) three-jet cross section takes over which is negative and, finally, at very large T will render $\sigma^{-1}\,d\sigma/dT$ negative (not drawn anymore in fig. 2). This is exactly what we expect from the leading logarithm calculation [12][‡3]

$$\sigma^{-1}d\sigma/dT \underset{T\to 1}{\simeq} -\tfrac{32}{9}[\alpha_s(Q^2)/\pi]^2\,|\ln(1-T)|^3/(1-T)\,. \tag{13}$$

The seeming negative thrust distribution near $T = 1$ will be cured by higher order corrections.

If the order α_s^2 correction were absorbed into the $O(\alpha_s)$ contribution similar to (3), or if the experimentalists would fit their data by the $O(\alpha_s)$ distribution allowing for a thrust dependent α_s, one would find an effective Λ which is no longer constant but varies with T. This is shown in fig. 3. Obviously, the region $3^{-1/2} \leqslant T \leqslant \frac{2}{3}$, which is only populated by four-jet events, cannot be cast into a three-jet distribution ($T \geqslant 2/3$). As a result, Λ_{eff} will go to infinity as $T \to 2/3$. But well above $T = 2/3$ fig. 3 indicates quite nicely how Λ and the strong coupling constant will change by including higher-order corrections. For instance, the fit to the lowest-order distribution would give $\Lambda \approx 0.05$ GeV at high thrust and $\Lambda \approx 0.5$ GeV at low thrust instead of a universal $\Lambda_{\overline{\text{MS}}} = 0.3$ GeV.

We conclude that, in the $\overline{\text{MS}}$ (and TS) scheme, the perturbation series for multijet production is well behaved (for $T \lesssim 0.95$). On the other hand, the effect of higher orders is still large enough to be detected which provides an unique window into the dynamics of hadrons at short distances.

While this paper was written we received a preprint by Ellis et al. [13] who have performed a similar calculation. We do not agree, however, with their conclusions.

‡3 This can also be read off from eq. (5) noticing that in the $T \to 1$ limit ϵ and δ are limited to $\epsilon \lesssim 2(1-T)$ and $\frac{1}{2}(1-\cos\delta) \lesssim 2(1-T)$.

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