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ON THE ROUGHENING TRANSITION IN NON-ABELIAN LATTICE GAUGE THEORIES

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We estimate the location $\beta_R = (g_R^2 N)^{-1}$ of the recently discussed roughening transition for SU(N) lattice gauge theories in $\nu = 3$ and 4 dimensions. The obtained values always lie in the crossover region where the string tension starts to deviate significantly from the leading strong coupling behaviour. $\beta_R(N)$ increases as N increases, tending to a finite limit (≈ 0.38 for $\nu = 4$) as $N \to \infty$. Results for the solid on solid model (SOS) approximation are also presented and shown to give a good qualitative description; however, group theoretical effects seem to be important for quantitative estimates.

1. Introduction

It has recently been realized [1–3] that a surface roughening transition, well known previously in the Ising model in $\nu = 3$ dimensions [4] which is dual to the $\nu = 3$ Z_2 lattice gauge theory, should occur in all gauge theories in $\nu \geq 3$ dimensions. A manifestation of the phenomenon is a divergence in the width of the flux tube connecting an infinitely separated static quark-antiquark pair for couplings $\beta \geq \beta_R \equiv (g_R^2 N)^{-1}$ while the string tension remains finite.

If the quark and antiquark lie on a line parallel to one of the lattice axes then the string width is zero for $\beta = 0$. For high temperatures just above the roughening point, i.e., $\beta \lesssim \beta_R$ a physical picture of a wildly fluctuating string which keeps its identity is probably reasonable. The situation in the continuum limit $\beta \to \infty$ may, however, be much more complicated. Nevertheless arguments can be made that a string roughening takes place independent of the detailed form of the underlying dynamics due to the special geometrical situation and the quantum mechanical nature of the problem. Indeed following ideas of Günther, Nicole and Wallace [5],

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Lüscher [6] has shown that the assumption that an infinitely long quantum mechanical flux tube of finite width exists in the continuum theory is self-contradictory. In lattice gauge theories it can then be argued that an alternative characterization of the roughening transition is the effective restoration of translational symmetry in the sense that non-translation invariant states do not exist for $\beta > \beta_R$. In refs. [1–3, 7] the roughening points in abelian and SU(2) lattice gauge theories have been estimated. Itzykson, Peskin and Zuber [1] consider an indicator for the transition which effectively measures the degree of translational symmetry restoration. In ref. [3] a string width is defined and is used as an indicator for the transition. It is satisfying that for the groups analysed so far, the roughening points estimated by the two methods are in good quantitative agreement.

In ref. [7] we extended the studies of ref. [1] to abelian groups Z_N, U(1). The present work further extends the study to the non-abelian groups SU(N). The models are defined in sect. 2. We have calculated the string tension and string width in the high-temperature expansion to 12th order. Results for the string tension have been presented previously for SU(2) [8–10] and SU(3) [8, 11, 12]. Only the results for the limit $N = \infty$ are new*. The roughening point for SU(2) in $\nu = 4$ was considered in [1, 3]. As in this case the roughening points always occur in the crossover region where the string tension starts to deviate significantly from the leading high-temperature behaviour.

Our results are discussed in some detail in sect. 3. We have also calculated the string width in the solid on solid (SOS) model approximation [15, 1, 2]. It is indeed true that this approximation qualitatively describes the roughening phenomenon but it seems that group theoretical details are important for a precise determination of the roughening point. For $\nu = 4$ we find that the roughening points increase with N and tend to a finite limit $\beta_R \sim 0.38$, well below the single plaquette integral singularity [16] at $\beta = 0.5$.

2. The models

We study euclidean gauge theories with gauge group SU(N) on a hypercubical lattice in $\nu = 3$ and 4 dimensions. The gauge field variables $U(\mathrm{b}) \in \mathrm{SU}(N)$ are attached to the links b of the lattice. The ordered product of the variables $U(\mathrm{b})$ for the four links b on the boundary of an elementary plaquette p is denoted by $U(\mathrm{p})$. We use the standard Wilson action [17],

$$L = \sum_{\mathrm{p}} L[U(\mathrm{p})], \qquad \text{with} \qquad L[U] = \frac{1}{g^2}(\mathrm{Tr}\, U + \mathrm{Tr}\, U^+), \tag{1}$$

where the sum extends over all unoriented plaquettes.

* The string tension for $N = \infty$ has also been studied by Brower [13]. The analogous calculation for hamiltonian lattice theories has been made in ref. [14].

For strong coupling g^2N the pure lattice gauge theories are known to confine static quarks by a potential which increases linearly for large separations. The slope of the potential is the string tension α. In ref. [3] a width of the flux tube connecting a static quark-antiquark pair was defined through the distribution of the chromoelectric field energy density between them. Its high-temperature expansion can be calculated using the same graphs which appear in the expansion of the string tension α [10].

The natural expansion parameters in high-temperature series are the coefficients $0 \leqslant a_r(g^2) < 1$ in the Fourier expansion

$$\exp L[U] = \mathcal{N}(g^2)\Big(1 + \sum_{r\neq 0} d_r a_r(g^2)\chi_r(U)\Big). \tag{2}$$

The sum extends over all inequivalent non-trivial irreducible representations of the group SU(N), labelled by r. d_r and χ_r denote the corresponding dimensions and characters respectively and $\mathcal{N}(g^2)$ is a normalization factor. Using orthogonality relations, we have

$$d_r a_r = d_r a_{\bar{r}} = \frac{\int \mathrm{d}U \chi_r(U) \exp L[U]}{\int \mathrm{d}U \exp L[U]}, \tag{3}$$

where $\mathrm{d}U$ is the Haar measure on SU(N).

In table 1 we list the representations that occur in our analysis, their dimensions, and notations for the corresponding Fourier coefficients a_r. The Fourier coefficient for the fundamental representation is denoted by u. It is possible to expand all other coefficients as power series in u and convenient to express the string tension and string width as power series in u alone. One obtains*

$$-\alpha = \ln u + \sum_{n\geqslant 4} \alpha_n u^n, \tag{4}$$

$$\sigma^2 = \sum_{n\geqslant 4} \sigma_n u^n. \tag{5}$$

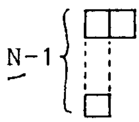
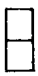
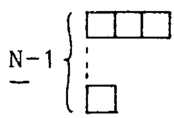
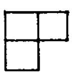
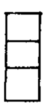
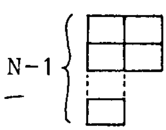
The coefficients α_n, σ_n have a finite limit as $N \to \infty$ [18]. To discuss the case $N \to \infty$ it is convenient to introduce the variable β:

$$\beta \equiv \frac{1}{g^2 N}. \tag{6}$$

The one-plaquette integral u has been evaluated explicitly as a function of β in the

* For $\nu = 2$: $\alpha_n = \sigma_n = 0$ for all n.

TABLE 1

Representation r	Dimension d_r	Notation for corresponding Fourier coefficient a_r
	N	u
	$d_1=\frac{1}{2}N(N+1)$	v_1
	$d_2=N^2-1$	v_2
	$d_3=\frac{1}{2}N(N-1)$	v_3
	$D_1=\frac{1}{6}N(N+1)(N+2)$	w_1
	$D_2=\frac{1}{2}(N-1)N(N+2)$	w_2
	$D_3=\frac{1}{3}N(N^2-1)$	w_3
	$D_4=\frac{1}{6}N(N-1)(N-2)$	w_4
	$D_5=\frac{1}{2}(N-2)N(N+1)$	w_5

Special cases: $N=4$: $w_3=w_5, w_4=u$,
$N=5$: $w_4=v_3$.

limit $N \to \infty$, β fixed by Gross and Witten [16]. It has a point of non-analyticity at $\beta = 0.5$ and is given by

$$u \underset{\substack{N\to\infty \\ \beta \text{ fixed}}}{=} \begin{cases} \beta, & \text{for } \beta < 0.5, \\ 1 - \dfrac{1}{4\beta}, & \text{for } \beta > 0.5. \end{cases} \tag{7}$$

In the limit $N \to \infty$ the other Fourier coefficients (see table 1) also take on a simple form [19] ($u < 0.5$)

$$d_i v_i(N,u) = V_i u^2 + O\big((2u)^N\big),$$

$$D_i w_i(N,u) = W_i u^3 + O\big((2u)^N\big), \tag{8}$$

where the coefficients V_i, W_i are given by

$$V_i = \lim_{N\to\infty} \frac{d_i}{N^2}, \qquad W_i = \lim_{N\to\infty} \frac{D_i}{N^3}. \tag{9}$$

A remark about the strong coupling expansion for $N = \infty$ is here in order. One often sees in the literature the statement that 'planar' graphs dominate the strong coupling expansion in the limit $N \to \infty$. Which strong coupling graphs are included as planar is, however, hardly, if ever, defined. This has led some confusion in the literature. See, for example, refs. [20, 21].

If one considers only *self-avoiding* surfaces with the topology of a disc as being planar, as seems to be the case in Weingarten's paper [20], the above statement is not true. Already in 10th order in the strong coupling expansion for the string tension one finds graphs which correspond to non-self-avoiding surfaces but contribute in the limit $N \to \infty$. (For example consider graph no. 11 of ref. [10], which is essentially equivalent to Weingarten's counterexample.) Graphs of this type produce the main difference between the coefficients for SU(∞) and SOS approximation, which is not negligible (see table 3).

On the other hand these graphs correspond to surfaces which are planar in the sense that it is possible to deform them into flat surfaces with trivial topology. For this purpose one has to consider carefully how the different plaquettes are sewn together through group contractions [22] in the integration procedure. A corresponding remark in the case of Weingarten's graph has been made by Förster [21].

Finally we discuss the solid on solid model (SOS) approximation [15, 1, 2]. For $\nu = 3$ it associates with each surface without overhangs or islands bounded by a planar loop $\mathcal{C}$ an energy

$$H^{\text{SOS}} = 2J \sum_{i,j} \left(|h_{i,j} - h_{i+1,j}| + |h_{i,j} - h_{i,j+1}| \right), \tag{10}$$

where i,j denote points of the 2d lattice in the plane of $\mathcal{C}$, and the surface is specified by the heights $h_{i,j} = 0, \pm 1, \pm 2, \ldots$. The free energy of the SOS model is the string tension in Z_2 gauge theory in the approximation of omitting surfaces with overhangs and islands (and leaving out the leading term $\ln u$). For other groups and also $\nu = 4$ we define the solid on solid approximation by a similar procedure omitting also all group theoretical structure and identifying the variable $\exp(-2J) = u$.

3. Results and discussion

First we consider the high-temperature series for the string tension α. For gauge groups SU(2), SU(3) in $\nu = 3$ dimensions they have been given by Drouffe [8] to 16th order (see also ref. [9]). The results for SU(2), SU(3) in $\nu = 4$ dimensions have been given in refs. [10–12]. The results to 12th order for $N \geqslant 4, \nu = 3, 4$ are summarized in table 2. In table 3 we exhibit the coefficients $\alpha_n, n \leqslant 12$, defined in eq. (4) for $N = 2, 3, \infty$ and the corresponding curves are plotted in figs. [1, 2]. We have not bothered to explicitly derive the coefficients for all N. However, as found by Kogut and Shigemitsu [14] the behaviour in N (at least to low orders in u considered here) is smooth and the $N = \infty$ limit approached rather rapidly. In table 3 we also include the results of the SOS approximation and see that only qualitative features of the series are reproduced.

The coefficients σ_n appearing in the expansion for the string width [eq. (5)] are given in table 4 for $N = 2, 3, \infty$ and for the SOS approximation. We seek a signal for the divergence at a roughening point u_R by (i) looking for a pole in the Padé approximants to σ^2, (ii) seeking a zero in the series for $(\sigma^2\alpha)^{-1}$. The results of this analysis are summarized in table 5.

To draw any conclusions one has to assume that the low-order Padés give the roughening point quite accurately, as seems to be the case for Z_2 [4]. For the case of SU(2) ($\nu = 3, 4$) and SU(∞) ($\nu = 4$) the values appearing in table 5 are stable and reasonable estimates for the roughening points can be made. For the case of $N = 3$, however, it is difficult to make a reliable estimate for u_R since our Padé table is not at all stable. The instability is due mainly to the large negative coefficient σ_{11}. The case $N = 3$, however, is not expected to be an exception in that we anticipate the roughening phenomenon to occur, but more work is required to establish the roughening points with more certainty.

From our analysis we find that the roughening transition occurs in all cases at 'the crossover region' where the string tension starts to deviate significantly from the leading high temperature behaviour. This is also indicated in figs. 1, 2. The roughening points $\beta_R(N)$ increase as N increases approaching a value $\beta_R(\infty)$ well below (at least for $\nu = 4$) the point of non-analyticity 0.5 in $u(\infty, \beta)$. The fact that this non-analyticity produces a sharp turnover in the string tension in $\nu = 2$ dimensions when plotted as a function of β has led to the speculation that the

TABLE 2
String tension to 12th order for SU(N), $N > 4$

$\nu = 3$

$$\alpha = \ln u + 2u^4 + 2u^4(d_1v_1 + d_2v_2 + d_3v_3) - 4(N^2-1)u^6 + 10u^8$$

$$+ 2(d_1v_1^5 + d_2v_2^5 + d_3v_3^5) + 12u^6(d_1v_1^2 + d_2v_2^2 + d_3v_3^2)$$

$$- 8u^8(d_1v_1 + d_2v_2 + d_3v_3) - 12(N^2-2)u^{10}$$

$$+ 2N^{-1}u^{-1}(d_1v_1^5[D_1w_1 + D_2w_2 + D_3w_3] + d_2v_2^5[D_2w_2 + D_5w_5]$$

$$+ d_3v_3^5[(1-\delta_{N4})(D_3w_3 + D_4w_4) + D_5w_5])$$

$$- 4d_1^2v_1^6 - 2d_2^2v_2^6 - 2(2-\delta_{N4})d_3^2v_3^6 + 8u^4(d_1v_1^4 + d_2v_2^4 + d_3v_3^4)$$

$$+ \frac{12(N+2)}{N(N+1)^2} d_1^2 d_2 u^6 v_1^2 v_2 + \frac{24}{(N^2-1)} d_1 d_2 d_3 u^6 v_1 v_2 v_3 + \frac{4(N^2-2)}{(N^2-1)^2} d_2^3 u^6 v_2^3$$

$$+ \frac{12(N-2)}{N(N-1)^2} d_2 d_3^2 u^6 v_2 v_3^2 + 24u^8(d_1v_1^2 + d_2v_2^2 + d_3v_3^2)$$

$$- 4u^8(d_1v_1 + d_2v_2 + d_3v_3)^2 + Nu^9(D_1w_1 + 3D_2w_2 + 2D_3w_3 + D_4w_4 + 3D_5w_5)$$

$$- (18N^2 + 32)u^{10}(d_1v_1 + d_2v_2 + d_3v_3) + \left(32N^4 + \tfrac{218}{3}\right)u^{12}$$

$\nu = 4$

$$\alpha = \ln u + 4u^4 + 4u^4(d_1v_1 + d_2v_2 + d_3v_3) - 8(N^2-1)u^6 + 56u^8$$

$$+ 4(d_1v_1^5 + d_2v_2^5 + d_3v_3^5) + 24u^6(d_1v_1^2 + d_2v_2^2 + d_3v_3^2)$$

$$+ 56u^8(d_1v_1 + d_2v_2 + d_3v_3) - 24(7N^2-10)u^{10}$$

$$+ 4N^{-1}u^{-1}(d_1v_1^5[D_1w_1 + D_2w_2 + D_3w_3] + d_2v_2^5[D_2w_2 + D_5w_5]$$

$$+ d_3v_3^5[(1-\delta_{N4})(D_3w_3 + D_4w_4) + D_5w_5])$$

$$- 8d_1^2v_1^6 - 4d_2^2v_2^6 - 4(2-\delta_{N4})d_3^2v_3^6 + 16u^4(d_1v_1^4 + d_2v_2^4 + d_3v_3^4)$$

$$+ \frac{24(N+2)}{N(N+1)^2} d_1^2 d_2 u^6 v_1^2 v_2 + \frac{48}{(N^2-1)} d_1 d_2 d_3 u^6 v_1 v_2 v_3 + \frac{8(N^2-2)}{(N^2-1)^2} d_2^3 u^6 v_2^3$$

$$+ \frac{24(N-2)}{N(N-1)^2} d_2 d_3^2 u^6 v_2 v_3^2 + 48u^8(d_1v_1^2 + d_2v_2^2 + d_3v_3^2)$$

$$+ 28u^8(d_1v_1 + d_2v_2 + d_3v_3)^2 + 6Nu^9(D_1w_1 + 3D_2w_2 + 2D_3w_3 + D_4w_4 + 3D_5w_5)$$

$$- 4(51N^2 - 28)u^{10}(d_1v_1 + d_2v_2 + d_3v_3) + \left(240N^4 - 352N^2 + \tfrac{3964}{3}\right)u^{12}$$

TABLE 3
String tension coefficients α_n [defined in eq. (4)]

$\nu = 3$					α_n				
Model \ n	4	5	6	7	8	9	10	11	12
SU(2)	2	0	0	0	$\frac{34}{3}$	0	$-\frac{1012}{405}$	0	$\frac{122882}{1215}$
SU(3)	2	6	-5	-18	$\frac{319}{4}$	$-\frac{1149}{20}$	$\frac{1567797}{10240}$	$-\frac{2428911}{4096}$	$\frac{132197339}{122880}$
SU(∞)	2	0	4	0	10	0	76	0	$\frac{530}{3}$
SOS	2	0	4	0	10	0	24	0	$\frac{194}{3}$

$\nu = 4$					α_n				
Model \ n	4	5	6	7	8	9	10	11	12
SU(2)	4	0	0	0	$\frac{176}{3}$	0	$\frac{10936}{405}$	0	$\frac{1532044}{1215}$
SU(3)	4	12	-10	-36	$\frac{391}{2}$	$\frac{1131}{10}$	$\frac{2550837}{5120}$	$-\frac{5218287}{2048}$	$\frac{285551579}{61440}$
SU(∞)	4	0	8	0	56	0	344	0	$\frac{4588}{3}$
SOS	4	0	8	0	56	0	208	0	$\frac{3196}{3}$

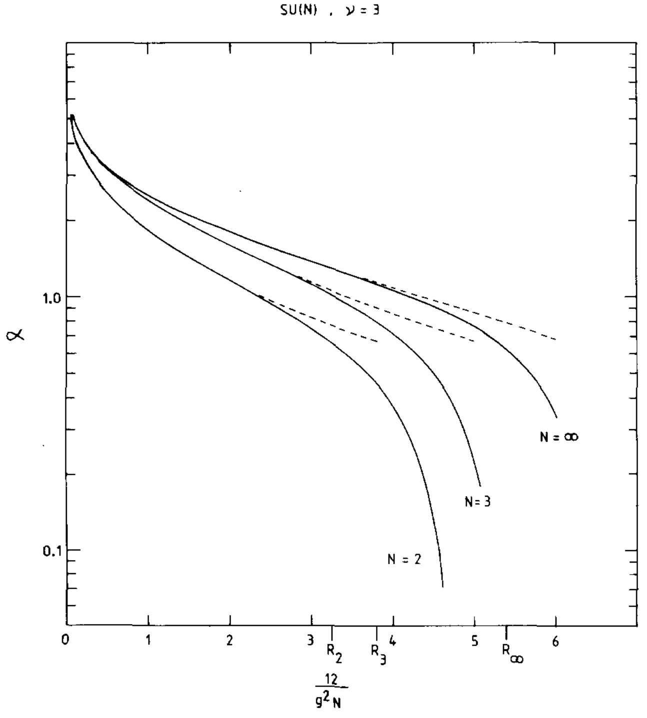


Fig. 1. The string tension α as a function of $\beta = (g^2N)^{-1}$ for SU(N) lattice gauge theory in $\nu = 3$ dimensions, $N = 2, 3, \infty$. The solid lines show the results of 12th-order strong coupling expansions, the dashed lines represent the lowest-order strong coupling curves. The estimated roughening points R_N are also indicated on the β-axis.

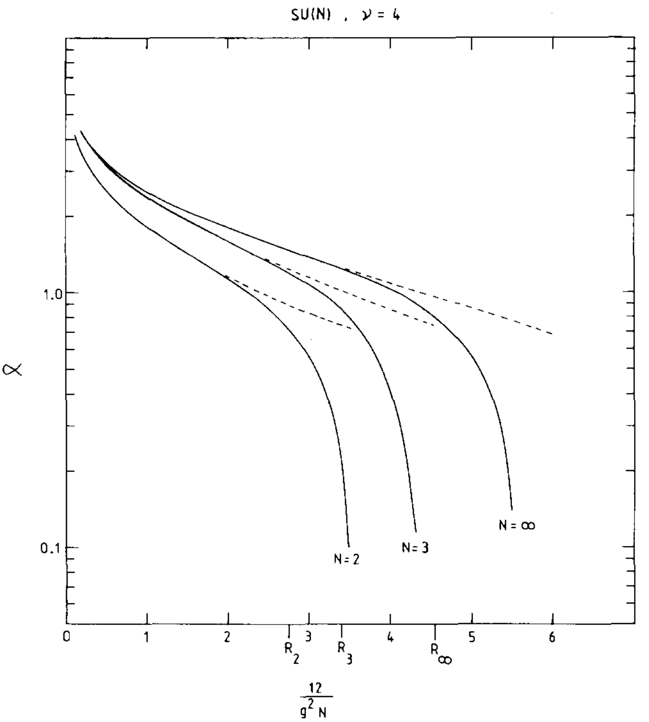


Fig. 2. Same as fig. 1, but for $\nu = 4$.

TABLE 4
String width coefficients σ_n [defined in eq. (5)]

$\nu = 3$	σ_n								
Model \ n	4	5	6	7	8	9	10	11	12
SU(2)	2	0	4	0	$\frac{124}{3}$	0	$\frac{43048}{405}$	0	$\frac{1054442}{1215}$
SU(3)	2	6	−1	−18	$\frac{583}{4}$	$-\frac{1209}{20}$	$\frac{2374581}{5120}$	$-\frac{299571}{256}$	$\frac{79729799}{20480}$
SU(∞)	2	0	8	0	40	0	272	0	1166
SOS	2	0	8	0	40	0	168	0	766

$\nu = 4$	σ_n								
Model \ n	4	5	6	7	8	9	10	11	12
SU(2)	4	0	8	0	$\frac{368}{3}$	0	$\frac{150896}{405}$	0	$\frac{5650204}{1215}$
SU(3)	4	12	−2	−36	$\frac{663}{2}$	$\frac{1431}{10}$	$\frac{3275701}{2560}$	$-\frac{476211}{128}$	$\frac{140596359}{10240}$
SU(∞)	4	0	16	0	120	0	896	0	5588
SOS	4	0	16	0	120	0	656	0	3988

TABLE 5
Roughening points [values of u_R, $\beta_R = 1/g_R^2 N$]

$\nu = 3$	u_R (β_R) from Padé for σ^2							u_R (β_R) from $(\sigma^2\alpha)^{-1}$
Model \ $[L, M]$	[4,4]	[3,4]	[2,4]	[3,5]	[2,6]	[1,7]	[0,8]	
SU(2)	0.459		0.459		0.459		0.457	0.461
	(0.27)		(0.27)		(0.27)		(0.27)	(0.27)
SU(3)	0.555	0.382	0.473	0.422	0.425	$(0.359 \pm i0,1)$	0.277	0.277
	(0.47)	(0.30)	(0.38)	(0.33)	(0.34)	(0.28)	(0.22)	(0.22)
SU(∞)	0.420		0.543		0.429		0.460	0.475
	(0.42)		(0.54)		(0.43)		(0.46)	(0.48)
SOS	0.475		0.470		0.475		0.473	0.484
$\nu = 4$	u_R (β_R) from Padé for σ^2							u_R (β_R) from $(\sigma^2\alpha)^{-1}$
Model \ $[L, M]$	[4,4]	[3,4]	[2,4]	[3,5]	[2,6]	[1,7]	[0,8]	
SU(2)	0.397		0.410		0.404		0.395	0.401
	(0.22)		(0.23)		(0.23)		(0.22)	(0.23)
SU(3)	0.461	0.340	0.411	0.372	0.375	$(0.371 \pm i0,1)$	0.272	0.273
	(0.37)	(0.27)	(0.32)	(0.29)	(0.29)	(0.29)	(0.22)	(0.22)
SU(∞)	0.365		0.366		0.383		0.385	0.393
	(0.37)		(0.37)		(0.38)		(0.38)	(0.39)
SOS	0.411		0.407		0.416		0.412	0.426

origin of the sharp turnover observed in $\nu = 4$ might be associated with analogous singularities. From a physical point of view, however, it would be surprising if the analytic behaviour of the one plaquette integral had anything to do with the roughening phenomenon and our value of $u_R(\infty)$ indeed supports the intuition that it does not.

With regard to the solid on solid approximation the results indicate that the approximation is indeed good qualitatively but quantitatively the group theory does introduce important additional effects.

In conclusion the existence of a roughening phenomenon in lattice gauge theories with arbitrary gauge group seems well established. The implications of this phenomenon for deductions about the continuum limit are not so well understood although they are thought to be drastic. Indeed, it has been speculated [1, 2] that the string-tension has a non-analytic behaviour at β_R

$$\alpha \sim \text{analytic} + \text{const} \cdot e^{-\text{const.}/|\beta - \beta_R|^{1/2}} \tag{11}$$

If this is the case then β_R is a natural boundary for the extrapolation of strong coupling expansions unless the strength of the non-analytic piece can be shown to be negligibly small. In particular some doubt is shed on any estimates of continuum quantities such as the scale parameter Λ using strong coupling expansions [10–12, 14]. On the other hand if the crossover from strong to weak coupling behaviour occurs rapidly for the quantities under consideration, as it seems to be the case [23], a mild non-analyticity might perhaps not affect much these estimates.

We thank F. Green and M. Lüscher for discussions.

Note added in proof

After completion of this work we became aware of the work of Drouffe and Zuber [24]. We thank these authors for pointing out an error in our results for the string tension for the groups Z_3 and SU(3) in $\nu = 4$ dimensions in ref. [12]. The corrected result appears in table 3.

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