

PRODUCTION OF CHARGED HYPERPIONS IN e^+e^- ANNIHILATION

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We study the production of charged hyperpions—the pseudo-Goldstone bosons of the hypercolor scenario—in e^+e^- annihilation. Rate estimates and decay distributions based on various decay mechanisms are presented and compared with the background from the usual e^+e^- annihilation processes.

1. Introduction

The idea of implementing the Higgs mechanism in a dynamical way, without the use of spin-0 fields, has lately received considerable attention [1]. In the simplest version of one of such scenarios, variously called hypercolor or technicolor [2]^{*}, one introduces a doublet of hyperquarks having a global $SU(2)_L \times SU(2)_R$ symmetry, which is broken spontaneously to the $SU(2)_{L+R}$ symmetry. As a consequence of spontaneous symmetry breaking, one generates a triplet of Goldstone bosons which are absorbed by the W^\pm and Z^0 : this is how the W^\pm and Z^0 get their mass. The Goldstone bosons themselves disappear and the residual $SU(2)$ symmetry ensures that the relation $m_W/m_Z \cos \theta = 1$ holds, where θ is the Glashow-Weinberg-Salam angle.

In order to give masses to the fundamental fermions, quarks and leptons, it is necessary to extend the simplest hypercolor scheme. In the extended technicolor

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^{*} We shall use the terms hypercolor and technicolor interchangeably and apologize for this mild source of confusion. We shall use capital letters to denote hyper/technifermions as opposed to small letters for ordinary fermions.

scheme (ETC) [3, 4], one is left with a large number of Goldstone bosons, which acquire masses due to the ordinary $SU(3)_C \times SU(2)_L \times U(1)$ interactions as well as through the extended technicolor interactions. In one such minimal ETC model [5], one is left with a quartet of 0^- color singlet pseudo-Goldstone bosons (PGB) $\pi'^+, \pi'^-, \pi'^3, \pi'^0$ ^{**}, together with color triplets and octets. These PGB's should be produced in ordinary interactions if their masses are not very large. Though the natural scale of the technicolor interaction is $\Lambda_T \sim 1$ TeV [1–3], the PGB π'^{\pm} are predicted not to get masses in the lowest order in α^{**} , the electromagnetic fine structure constant [4, 5]. The natural mass scale of $m_{\pi'^{\pm}}$ is thus $m_{\pi'^{\pm}}^2 \sim \alpha^2 \Lambda_T^2$, giving $m_{\pi'^{\pm}} \sim O(10 \text{ GeV})$. Explicit model-dependent calculations [5, 6] show that the contribution of ETC forces does not change the order of magnitude estimate for $m_{\pi'^{\pm}}$.

Our purpose in this paper is to study the production in e^+e^- annihilation of a pair of color singlet charged PGB's, π'^{\pm} and their subsequent decays. Some of these considerations as well as the production mechanisms of the neutral PGB, π'^0, π'^3 have been reported in ref. [7] and compared with the canonical Higgs scenario.

We calculate the π'^{\pm} related signals and compare them to the backgrounds anticipated from the usual e^+e^- interactions. In particular, we calculate the semileptonic decays $\pi'^{\pm} \rightarrow \pi'^0 (\ell^{\pm} \nu_{\ell}, q\bar{q}')$ and the second-order electroweak decays $\pi'^{\pm} \rightarrow \gamma + (\ell^{\pm} \nu_{\ell}, q\bar{q}')$. The former can compete favorably with the expected dominant semiweak decays $\pi'^{\pm} \rightarrow f'\bar{f}'$, if (i) $m_{\pi'^0} < m_{\pi'^{\pm}}$ and (ii) the $\pi'f'\bar{f}'$ couplings turn out to be much smaller than anticipated from the naive Goldberger-Treiman type arguments [1]. Even if the semifermionic decays of π'' 's are suppressed, the details of the purely fermionic decays $\pi'^{\pm} \rightarrow f'\bar{f}'$, $\pi'^0 \rightarrow f\bar{f}$ are expected to be quite *different* from the Higgs decays $\phi^0 \rightarrow f\bar{f}$, $\phi^{\pm} \rightarrow f'\bar{f}'$. The decay modes of π'' 's may provide a unique window into the realm of technicolor—in much the same way that the decay $\pi^0 \rightarrow 2\gamma$ signaled the presence of color interactions.

The paper is organized as follows. In sect. 2 we study the process $e^+e^- \rightarrow \pi'^+\pi'^-$ followed by the expected semiweak decays of the π'^{\pm} . Signatures based on these decays are studied and compared with the normal e^+e^- background. In sect. 3 we turn to somewhat unconventional and probably rare decays of the π'' 's, namely the semileptonic decays. Sect. 4 contains a summary of our results and a discussion.

* The underlying chiral symmetry of the minimal Peskin model is $SU(8)_L \times SU(8)_R$ broken to $SU(8)_{L+R}$. One has then 63 Goldstone bosons, three of which disappear and give masses to the W^{\pm} and Z^0 . There are 60 residual bosons, 4 of which are color singlets, 24 color triplets and 32 color octets. For details of the spectroscopy and mass estimates see refs. [5, 6]. The considerations in this paper do not depend on the details of the Peskin model and are more general.

** The neutral PGB π'^0, π'^3 do not receive any mass from the $SU(2)_L \times U(1)$ forces to all orders in α . Their mass arises entirely from the ETC forces and hence is model dependent. However, we do not believe that the Pati-Salam bosons [9] which would induce the decay $K_L \rightarrow \mu e$ have anything to do with the mass generation of π'^0 and π'^3 , as has been assumed in ref. [6].

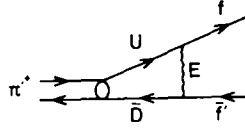


Fig. 1. Extended technicolor boson induced decays of the technipion π'^{\pm} .

2. The process $e^+e^- \rightarrow \pi'^+\pi'^-$

The cross section for the process $e^+e^- \rightarrow \pi'^+\pi'^-$ may be expressed in terms of its contribution to $R^{(1)}$:

$$\Delta R = \sigma(e^+e^- \rightarrow \pi'^+\pi'^-)/\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{1}{4} \left(1 - \frac{4m_{\pi'}^2}{s}\right)^{3/2},$$

where s is the (c.m. energy)² and the factor $\frac{1}{4}$ is due to the spin-0 nature of π'^{\pm} . Of course, the production of a pair of charged Higgs boson ϕ^{\pm} is also given by the same expression.

What are the signatures of π'^{\pm} 's in e^+e^- annihilation experiments and how could one distinguish π'^{\pm} from ϕ^{\pm} ?

The decays of π'^{\pm} are *in general* model dependent. In most ETC models, the techniquark combination making up the π'^{\pm} is orthogonal to the charged Goldstone boson which gives mass to W^{\pm} [4, 5]. Thus, π'^{\pm} cannot decay via a single W^{\pm} exchange alone. There are then three possible decay mechanisms which we shall presently explore. These decay schemes in decreasing order of importance are:

(i) π'^{\pm} decays via the so-called extended techniboson (or sideways boson, the name given by Eichten and Lane [4]) as shown in fig. 1.

This will give rise to final states

$$\pi'^{\pm} \rightarrow q\bar{q}, \ell^{\pm} \nu_{\ell}, \quad (\ell = e, \mu, \tau). \tag{1}$$

Final states in (1) look *formally* very similar to the W^{\pm} induced modes [1], though the *details* are expected to be quite different. One still expects the helicity pattern which is well known from the decays of π^{\pm} . However, the naive color counting estimates [8] used very often while discussing the charged Higgs decays $\phi^{\pm} \rightarrow f\bar{f}$, are *not* expected to hold. Relative rates in (1) depend on the color and extended technicolor representation of π'^{\pm} and hence are *a priori* undetermined.

(ii) If $m_{\pi'} < m_{\pi}$, then π'^{\pm} could decay semileptonically as shown in fig. 2:

$$\pi'^{\pm} \rightarrow \pi'^0 + W_{\text{vir}}^{\pm} \rightarrow \ell^{\pm} \nu_{\ell}, q\bar{q}' \tag{2}$$

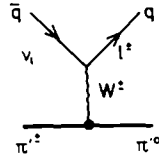


Fig. 2. Lowest order semileptonic decays of the technipions π'^{\pm} .

The branching ratio for (2) may not be small though (2) is of order G_F^2 in contrast to (1), which formally is semiweak and hence of order G_F .

(iii) π'^{\pm} decays via the second-order electroweak process shown in fig. 3 [15]:

$$\pi'^{\pm} \rightarrow \gamma W_{\text{vir}}^{\pm} \rightarrow \ell^{\pm} \nu_{\ell}, q\bar{q}' \quad (\ell = e, \mu, \tau). \quad (3)$$

We consider the decay mechanism (iii) highly unlikely unless the following two circumstances hold:

(a) There is another mechanism to give masses to the fermions and the effective $f \rightarrow F$ coupling is exceedingly small, or the dominant coupling of the U and D techniquarks is to u and d quarks;

(b) $m_{\pi'^{\pm}} < m_{\pi'^0}$.

If (a) holds but $m_{\pi'^0} < m_{\pi'^{\pm}}$, then the dominant decay modes of π'^0 and π'^{\pm} would be $\pi'^0 \rightarrow 2$ gluons, and $\pi'^{\pm} \rightarrow 2\gamma$. This circumstance would produce very exciting final states in the decays of toponium J_T , for example $J_T \rightarrow \pi'^{\pm} + \gamma \rightarrow 3\gamma$!

2.1. ETC DECAYS OF π'^{\pm}

To be definite, we assume that the quarks, leptons and technifermions all transform as left-handed doublets and right-handed singlets under $SU(2)_L \times U(1)$.

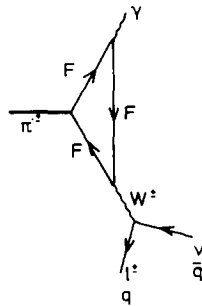


Fig. 3. Second-order electroweak decays of the technipions π'^{\pm}

The most general form of the effective four-Fermi interaction is then [4]

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{\text{ETC}} &= G_E \sum_{i,j=1}^n \sum_{r=1}^{n_T} \sum_{K=1}^{N_c} \left[\bar{u}_{iL} \gamma_\mu U_K^{(r)} + \bar{d}_{iL} \gamma_\mu D_K^{(r)} \right] \\
&\times \left[\Gamma_{ij(r)}^u \bar{U}_{KR}^{(r)} \gamma^\mu u_{jR} + \Gamma_{ij(r)}^d \bar{D}_{KR}^{(r)} \gamma^\mu d_{jR} \right] + \left[\bar{\nu}_{iL} \gamma^\mu U_K^{(r)} + \bar{e}_{iL} \gamma^\mu D_K^{(r)} \right] \\
&\times \left[\Gamma_{ij(r)}^e \bar{D}_{KR}^{(r)} \gamma^\mu e_{jR} \right] + \text{h.c.} \tag{4}
\end{aligned}$$

The indices appearing in eq. (4) have the following meanings. There are n generations of quarks and leptons (index i, j), n_T generations of technifermions (index r), K is the technicolor index (dimension N_c) and color indices are not shown. The matrices $\Gamma^{u,d,e}$ are determined by the ETC gauge boson couplings between ordinary fermions and technifermions and by the mass matrix of the ETC gauge bosons. In other words $\Gamma_{ij}^{u,d,e}$ are generalized Cabibbo matrices. The scale of the coupling constant, G_E is set by the requirement that the ordinary fermion masses are generated by E boson exchanges. This gives

$$(m_{LR}^u)_{ij} = \sum_{r,k} G_E \Gamma_{ij(r)}^u \langle 0 | \bar{U}_{KR}^{(r)} U_{KL}^{(r)} | 0 \rangle \approx (1 \text{ TeV})^3 G_E. \tag{5}$$

With the help of the lagrangian (4) we evaluate the process (1). We shall write the amplitude for the case when both f and f' are quarks (the amplitude for leptons is obtained using analogous manipulations):

$$\begin{aligned}
m &= \langle \bar{f} \bar{f}' | \mathcal{L}_{\text{eff}}^{\text{ETC}} | \pi'^+ \rangle \\
&= \langle \bar{f} \bar{f}' | G_E \left[\Gamma_{ij}^{(d)} (\bar{D}_R \gamma^\mu f'_{jR}) (\bar{f}_{iL} \gamma_\mu U_L) + \Gamma_{ij}^{u*} (\bar{f}_R \gamma_\mu U_R) (\bar{D}_L \gamma_\mu f'_{jL}) \right] | \pi'^+ \rangle \\
&= G_E \langle 0 | \bar{D} \gamma_5 U | \pi'^- \rangle \left\langle \bar{f} \bar{f}' \left| \left[\bar{f}_i \Gamma_{ij}^d \frac{(1-\gamma_5)}{2} f'_j + \bar{f}_i \Gamma_{ij}^{u*} \frac{(1+\gamma_5)}{2} f_j \right] \right| 0 \right\rangle, \tag{6}
\end{aligned}$$

where use has been made of Fierz transformation and we have assumed that π'^{\pm} are pseudoscalar particles. We shall now use (5) to express G_E in terms of quark masses. We shall take $r=1$ for simplicity (or equivalently assume that each quark receives mass from the condensate of a single technifermion pair $\langle \bar{U}U \rangle_0$, $\langle \bar{D}D \rangle_0$, etc.). Then

$$G_E = \frac{m_i^{(q)}}{\Gamma_{ii}^{(q)}} \frac{1}{\langle \bar{U}U \rangle}, \quad q = u, d, \dots$$

and we have tacitly assumed

$$\langle \bar{Q}_K Q_{K'} \rangle_0 = \delta_{r,r'} \delta_{K,K'} \langle \bar{U} U \rangle.$$

We can now express (6) as

$$m(\pi'^+ \rightarrow f_i \bar{f}_j) = (-i) \frac{1}{\langle \bar{U} U \rangle} \langle 0 | \bar{D} \gamma_5 U | \pi'^+ \rangle \times \left\langle \text{ff} \left[\left[-m_i^d R_{ij} \frac{(1-\gamma_5)}{2} + m_i^u R_{ij} + \frac{(1-\gamma_5)}{2} \right] \right] 0 \right\rangle, \quad (7)$$

where $R_{ij} = \Gamma_{ij} / \Gamma_{ii}$. It is clear from (7) that the matrix elements for the process $\pi'^+ \rightarrow \text{ff}'$ are determined only up to, in *general* unknown, generalized Cabibbo angles. Without further ado, we write the result

$$\Gamma(\pi'^+ \rightarrow f_i \bar{f}_j) = R_{ij}^2 K_{ij}^2 \frac{(m_f + m_{f'})^2}{8\pi F_\pi^2} m_{\pi'}, \quad (8)$$

and we have used the normalization

$$\frac{1}{\langle \bar{U} U \rangle} \langle 0 | \bar{D} \gamma_5 U | \pi'^+ \rangle = \frac{2\sqrt{2}}{F_\pi}. \quad (9)$$

K_{ij} is a color-technicolor factor and depends on the final-state fermions as well as the TC/ETC representation of π'^\pm . $K_{ij}^c \Gamma_{ij}$ needs a detailed technicolor model. We shall leave the factor $K_{ij}^c \Gamma_{ij}$ ($\equiv \tilde{\Gamma}_{ij}$) unspecified, remarking that the decay modes of π'^\pm into a fermion pair differ from those of the Higgs ϕ^\pm [8]. One could express F_π^2 in terms of the Fermi coupling constant G_F through the relation [2, 3] $\sqrt{2} G_F F_\pi^2 = n_f^{-1}$, where n_f is the number of technifermion doublets ($n_f \equiv N_c \times N_T$) giving

$$\Gamma(\pi'^+ \rightarrow f_i \bar{f}_j) \simeq \tilde{\Gamma}_{ij} \frac{G_F}{4\sqrt{2}\pi} (m_f + m_{f'})^2 m_\pi n_f. \quad (10)$$

In particular,

$$\frac{\Gamma(\pi^\pm \rightarrow c\bar{s})}{\Gamma(\pi^\pm \rightarrow \tau^\pm \nu_\tau)} \approx \chi \frac{m_c^2}{m_\tau^2}, \quad (11)$$

where

$$\chi \equiv \tilde{\Gamma}_{c\bar{s}}^2 / \tilde{\Gamma}_{\tau\nu}^2.$$

This has to be contrasted with the Higgs branching ratio [8]

$$\frac{\Gamma(\phi^+ \rightarrow c\bar{s})}{\Gamma(\phi^\pm \rightarrow \tau^\pm \nu_\tau)} \approx 3 \frac{m_c^2}{m_\tau^2}. \quad (12)$$

In general, χ is a free parameter in the range $0 \leq \chi \leq \infty$ and though 3 certainly lies in the range, we could not find any realistic model published so far [5, 6] in which the color factor 3 (or a number close to it) could be reproduced. So, we still expect heavy quarks and/or heavy fermions to be the dominant decay products of π'^{\pm} but the relative branching ratios are undetermined.

We would like to digress here somewhat and discuss the situation of flavor changing neutral currents which seems to be a problem in extended technicolor theories. This is related to the elements of the generalized Cabibbo matrix $\Gamma_{ij(r)}^u$ and $\Gamma_{ij(r)}^d$ in the effective lagrangian (4) and to G_E . It is generally true that if *all* the ordinary fermions $u, d, s, c, b, t; e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$ couple to the *same* techniquarks, let us say, U and/or D , then the ETC contributions to the $K_L - K_S$ mass difference are unacceptably large*. Thus, ETC theories with unconstrained couplings are already in serious phenomenological trouble on this account [10]. However, the situation can be saved if *one* of the following two circumstances hold:

(i) The technifermion \rightarrow fermion transitions are *strictly* flavor conserving. This would necessitate technifermion family generations on the same pattern as for the ordinary fermions. Thus only couplings of the type

$$u \rightarrow U, \quad d \rightarrow D, \quad s \rightarrow S, \quad c \rightarrow C, \quad \text{etc.}$$

are allowed but

$$u \rightarrow C, \quad u \rightarrow D$$

are not. One would still generate fermion masses from the usual mass feedback mechanism, but there are *no* non-diagonal neutral currents.

(ii) One need not have identical fermion-technifermion families but the fermions of same charge and helicity receive mass from the same vacuum expectation value $\langle U\bar{U} \rangle, \langle D\bar{D} \rangle, \langle E\bar{E} \rangle, \langle N\bar{N} \rangle$ (as in the model of ref. [5], for example).

The condition (ii) is the ETC equivalent of the naturalness conditions discussed for the Higgs case by Glashow and Weinberg [11]. It guarantees diagonal neutral currents induced by a single PGB π'^0, π'^3 exchange, but the contributions of the single ETC-boson exchange diagrams as well as of the double ETC-boson exchange box diagrams are large and in disagreement with the observed mass difference. In

* In such simple ETC models, there is no suppression of the flavor-changing neutral currents since there is no techni-GIM mechanism [10]. A. Ali would like to acknowledge useful discussions with L. Susskind about the nature of fermion couplings in ETC models.

our opinion (ii) must be supplemented by an ETC analogue of the GIM mechanism [12]*. Whether one could implement the techni-GIM mechanism without any disturbing effects elsewhere is not obvious to us and its discussion here will take us far afield. Though assuming (ii) one could express Γ_{ij} in terms of the known Cabibbo angles, the specification of K_{ij} and hence χ still needs an explicit model, and we do not want to commit ourselves to any particular model at this stage.

What are the signatures of π'^{\pm} decaying via (6)? The pair production of $\pi'^+\pi'^-$ and subsequent decays into a heavy quark and/or heavy fermion pair will lead to one of the following three signatures [1, 4, 7]:

(i) $\chi \sim 1$: the favorable decay chain in this case could be

$$\begin{array}{l}
 e^-e^- \rightarrow \pi'^+\pi'^- \\
 \begin{array}{l}
 \downarrow \quad \downarrow \\
 \tau^+\nu_\tau \quad \bar{c}s \\
 \downarrow \quad \downarrow \\
 \text{hadrons} \quad \text{hadrons} \\
 \downarrow \\
 (e^+, \mu^+) + \dots
 \end{array}
 \end{array}
 \tag{13}$$

This will lead to an electron (or muon) recoiling against a hadronic jet. The background to the process

$$e^+e^- \rightarrow (e, \mu) + \text{hadrons}$$

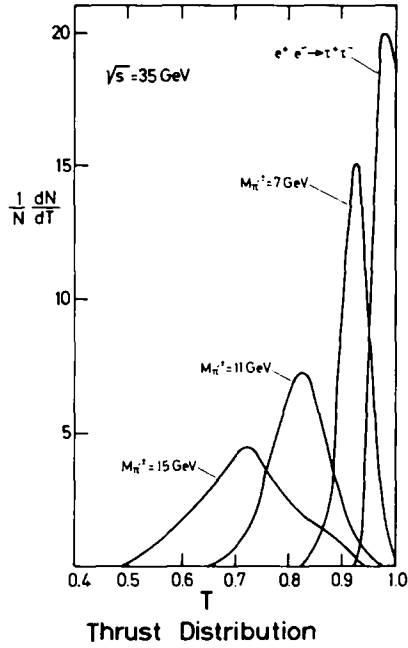
comes from the $\tau^+\tau^-$ pair production

$$\begin{array}{l}
 e^+e^- \rightarrow \tau^+\tau^- \\
 \begin{array}{l}
 \downarrow \quad \downarrow \\
 \text{hadrons} \\
 \downarrow \\
 (e^+, \mu^+) + \nu\text{'s.}
 \end{array}
 \end{array}
 \tag{14}$$

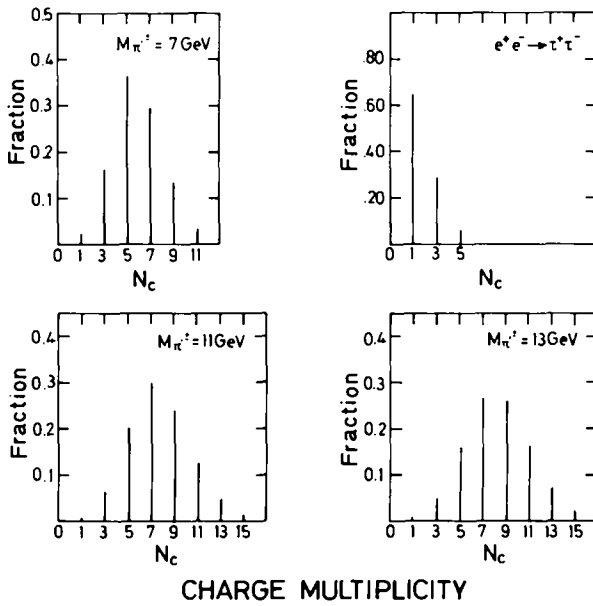
However, the topologies of the two types of events (13) and (14) are expected to be quite different. For those experiments in which one is able to measure the invariant mass of the recoiling hadrons, the τ -induced background can be removed relatively easily by demanding that the invariant mass of the hadrons m_{had} be greater than m_τ .

An equally good separation of (13) and (14) could be obtained if one could measure $\langle n_{\text{ch}} \rangle$ and $\langle n_{\text{kaons}} \rangle$, which is expected to be very different for the two sources (13) and (14). Assuming reasonable fragmentation properties of the $\bar{c}s$ quark

* We remark that the problem lies in having non-zero Cabibbo angles and acceptable flavor-changing neutral currents.



(a)



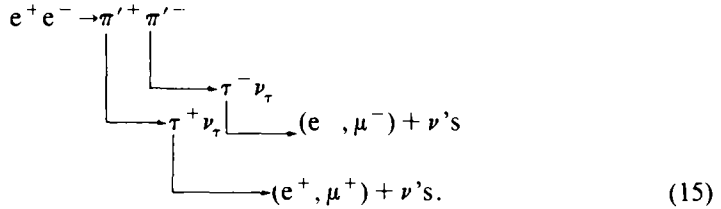
CHARGE MULTIPLICITY

(b)

Fig. 4. (a) Comparison of the normalized hadron thrust distribution from the process $e^+ e^- \rightarrow \pi^+ \pi^- \rightarrow (e, \mu) + \text{hadrons}$ with the hadron thrust distribution from $e^+ e^- \rightarrow \tau^+ \tau^- \rightarrow (e, \mu) + \text{hadrons}$. (b) Comparison of the charged multiplicity distribution of the hadronic jet recoiling against the (e, μ) from the technipion production (13) with the charged multiplicity distribution from $e^+ e^- \rightarrow \tau^+ \tau^- \rightarrow (e, \mu) + \text{anything}$.

pair [13], we plot in fig. 4 the charged multiplicity and the thrust distributions from (13) and (14) for the hadronic jet recoiling against the e (or μ). We note that a very clear signal/background separation is possible using any of the distributions $d\sigma/dm_{\text{had}}$, $d\sigma/dn_K$, $d\sigma/dn_{\text{ch}}$, $d\sigma/dT$, or other related jet distributions.

(ii) $\chi \ll 1$: the dominant decay chain in this case would be



This will give rise to “anomalous dilepton events,” very much reminiscent of the τ^\pm . However, the process (15) will differ from the τ^\pm induced background in several important details: the $e^+ e^-$, $e^+ \mu^-$, and $\mu^+ \mu^-$ events arising from (15) would be highly non-collinear and *non-coplanar* with a large missing momentum. In fig. 5 we plot the acollinearity angle distribution $(1/\sigma)d\sigma/d\cos\theta_{\ell^+ \ell^-}$ for

- (i) $\sqrt{s} = 35 \text{ GeV}$, $m_{\pi'^\pm} = 15 \text{ GeV}$;
- (ii) $\sqrt{s} = 100 \text{ GeV}$, $m_{\pi'^\pm} = 45 \text{ GeV}$.

Near the threshold the $e^+ e^-$ events are very non-collinear. Away from the threshold the acollinearity distribution gets more peaked around $\cos\theta_{e\mu} = -1$, but there is still a very clear $\tau^\pm - \pi'^\pm$ separation possible. We remark that in both the chains (13) and (15), the e (or μ) is expected to be very soft with $\langle E_{e,\mu} \rangle \simeq \frac{1}{6} E_{\text{beam}}$ — a good low energy lepton detection is at a premium!

Of course, both $\ell^+ \ell^-$ and $\ell^- \ell^-$ hadron events can also arise from a pair of heavy leptons $L^+ L^-$. However, there are two important differences between $L^+ L^-$ events

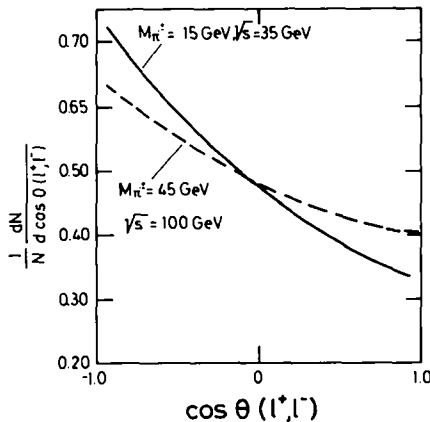


Fig. 5. Acollinearity angle distribution for the dileptons from the technipion process (15).

and the ones from a pair of technipions, $\pi'^+\pi'^-$, namely:

(1) angular distribution:

$$\frac{d\sigma}{d\Omega}(\pi'^+\pi'^-) \propto \sin^2\theta,$$

$$\frac{d\sigma}{d\Omega}(L^+L^-) \propto (1 + \cos^2\theta) + \frac{1}{\gamma} \sin^2\theta; \quad (16)$$

(2) threshold behavior:

$$\Delta R(e^+e^- \rightarrow \pi'^-\pi'^+) \sim \frac{1}{4}\beta^3,$$

$$\Delta R(e^+e^- \rightarrow L^+L^-) \sim \beta. \quad (17)$$

Asymptotically L^+L^- will contribute one unit of R as against $\frac{1}{4}$ for the $\pi'^+\pi'^-$ to the total hadronic cross section. The angular distribution of the parent (π'^+ or L^+) with respect to the e^+ (e^-) beam is retained by the lepton (e, μ) and the hadronic jet. In fig. 6 we plot the distributions $(1/\sigma)d\sigma/d\cos\theta(\mu, \text{beam})$ from (13) or (15) and compare them with the corresponding distribution from the heavy lepton process*

$$e^+e^- \rightarrow L^+L^-$$

$$(18)$$

for (a) $\sqrt{s} = 35$ GeV, $m_{\pi'^\pm} = 8$ GeV, and (b) $\sqrt{s} = 100$ GeV, $m_{\pi'^\pm} = 25$ GeV. The angular distributions from the process

$$Z^0 \rightarrow \pi'^-\pi'^+ \rightarrow (e^+, \mu^+) + \text{hadrons},$$

$$Z^0 \rightarrow L^+L^- \rightarrow (e^+, \mu^+) + \text{hadrons}$$

are very similar to the ones presented in fig. 6b.

(iii) $\chi \gg 1$: this would lead to events of the type

$$e^+e^- \rightarrow \pi'^+\pi'^- \rightarrow \text{hadrons}. \quad (19)$$

* The distributions for the heavy lepton pair were calculated using the following decay branching ratios: $(e^+ \nu_e \nu_L) - (\mu^+ \nu_\mu \nu_L) = (\tau^+ \nu_\tau \nu_L) = \frac{1}{3}(u\bar{d} \nu_L) = \frac{1}{3}(c\bar{s} \nu_L)$, a $V - A$ interaction was assumed for the $L^\pm \rightarrow \nu_L$ transition [17]. Subsequent decays of τ^- and the c-quark were implemented according to the periodic table, and quark model was used where decay modes were not known.

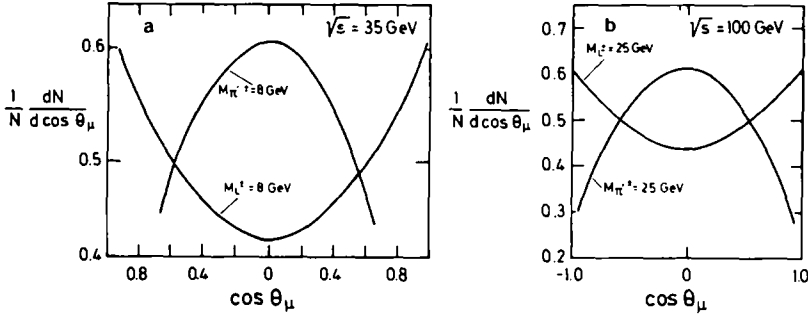


Fig. 6. Angular distribution $(1/N)d\sigma/d\cos\theta_\mu$ for the technipion process [15] and due to a heavy lepton pair process (a) $\sqrt{s} = 35$ GeV, $m_{\pi^+} = m_{\pi^-} = 8$ GeV, (b) $\sqrt{s} = 100$ GeV, $m_{\pi^+} = m_{\pi^-} = 25$ GeV.

Near the threshold (19) will give rise to almost isotropic events—large sphericity and acoplanarity. However, the rate would be rather small, as it is suppressed due to the $\frac{1}{4}\beta^3$ behavior. Away from the threshold the hadronic events (19) tend to be 2-jet-like. The acoplanarity distribution $(1/\sigma)d\sigma(\pi'^+\pi'^-)/dA$ shows this quantitatively in fig. 7. The two-jet-like behavior comes from the assumption that the π'^\pm decays are well represented by

$$\pi'^\pm \rightarrow c\bar{s}.$$

In reality, depending on $m_{\pi'^\pm}$, there will be events of the 3- and 4-jet type

$$\begin{aligned} \pi'^\pm &\rightarrow c\bar{s} + \text{gluon} \\ &\rightarrow c\bar{s} + 2 \text{ gluons.} \end{aligned} \quad (20)$$

Thus, our distributions in fig. 7 tend to underestimate somewhat the $\pi'^+\pi'^-$ induced signal for large $m_{\pi'^\pm}$ and \sqrt{s} . We seek the indulgence of the reader for not taking into account the complication from the process (20) at this stage. The background to the process (19) comes from the usual e^+e^- interactions

$$e^+e^- \rightarrow 2,3,4 \text{ jets.} \quad (21)$$

Using a Monte Carlo described in ref. [13] we have evaluated the background from (21) to the technipion signal (19). This is shown in fig. 7 for $\sqrt{s} = 35$ GeV. Judging from the analysis done in search of new quark thresholds at PETRA, we fear that the multijet background to (9) would be formidable (though perhaps not insurmountable). Establishing the technipion signal through (19) may turn out to be as difficult an enterprise as detecting a change in the total hadronic cross section in e^+e^- annihilation.

Given the indeterminacy in the relative branching ratios, we hesitate to quote expected event rates and restrict ourselves to reiterating the signatures (14), (15), and

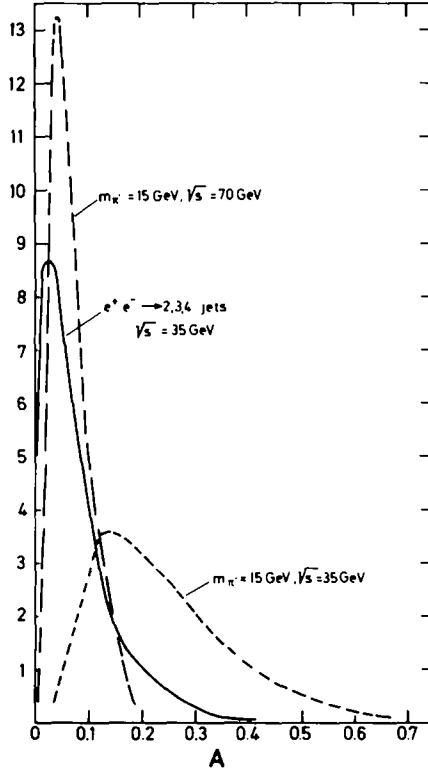


Fig. 7. Acoplanarity distribution from the technipion process (19) and the background from the $e^+e^- \rightarrow \text{hadrons}$ via (21).

(19) with the lepton and/or the hadronic jet having a $\sin^2\theta$ distribution with respect to the incoming e^+e^- beam direction.

3. Semileptonic decays of π'^{\pm}

In this section we study the lowest order semileptonic weak process (assuming $m_{\pi'^{\pm}} < m_{\pi'}$,

$$\pi'^{\pm} \rightarrow \pi'^3 + W_{\text{vir}}^{\pm} \rightarrow \ell' \nu_{\ell'}, q\bar{q} \tag{2}$$

and the second-order electroweak process [15]

$$\pi'^{\pm} \rightarrow \gamma + W_{\text{vir}}^{\pm} \rightarrow \ell' \nu_{\ell'}, q\bar{q}. \tag{3}$$

The decay (2) is analogous to the well known π_{e3}^0 decay $\pi^0 \rightarrow \pi^+ \bar{e} \nu_e$ whereas the decay (3) is analogous to the second-order electromagnetic decay $\pi^0 \rightarrow 2\gamma$. Before embarking upon any detailed calculation we would like to orient ourselves as to the orders of magnitude. The decay $\pi'^{\pm} \rightarrow \bar{f} f'$ is *formally* semiweak and is therefore proportional to $G_F m_f^2 m_{\pi'}$. In the absence of any other selection rule forbidding this decay, it is expected to be the dominant mechanism. The process (2), which is a weak process of strength G_F , is given by $\sim (G_F^2 m_{\pi'}^5 / 384 \pi^3) \times$ phase space. However, it is conceivable that the ETC induced diagram (1) is very much suppressed (for instance, if the dominant ETC decay of π'^{\pm} is $U\bar{D} \rightarrow u\bar{d}$ or if the Cabibbo angles in $U\bar{D} \rightarrow c\bar{s}$, $\tau \nu_{\tau}$ are very small). In that case the semileptonic decay $\pi'^{\pm} \rightarrow \pi'^0 + \ell' \nu_{\ell}, q\bar{q}$ would become competitive.

The amplitude for the decay (2) is given by

$$m(\pi'^{\pm} \rightarrow \pi'^3 + \ell' \nu_{\ell}) = \sqrt{\frac{1}{2}} G_F m_W^2 \langle \pi'_{(p)} | V_{\mu}^{\pm} | \pi'_{(k)} \rangle \frac{1}{(q^2 - m_W^2)} \bar{u}_{\ell} \gamma_{\mu} (1 - \gamma_5) u_{\nu}, \quad (22)$$

with the following Lorentz-covariant decomposition:

$$\langle \pi'^3(p) | V_{\mu}^{\pm} | \pi'_{(k)} \rangle = f_+(q^2)(k+p)_{\mu} + f_-(q^2)(k-p)_{\mu}, \quad (23)$$

assuming CVC

$$f_+(0) \simeq \sqrt{2}, \quad f_-(0) \simeq 0. \quad (24)$$

Neglecting q^2 dependence in the W-propagator, we have

$$\Gamma(\pi'^{\pm} \rightarrow \pi'^0 + \ell' \nu_{\ell}) = \frac{G_F^2 m_{\pi'}^5}{384 \pi^3} f(m_{\pi'^0}^2 / m_{\pi'^{\pm}}^2). \quad (25)$$

where

$$f(k) = 1 - 8k + 8k^3 - k^4 - 12k^2 \ln k.$$

One could use the simple form (25) to estimate the decay rate. However, we present $\Gamma(\pi'^{\pm} \rightarrow \pi'^0 + (\ell' \nu_{\ell} + q\bar{q}))$ in table 1, using the exact form without neglecting q^2 in the W-propagator. We would like to point out that the π'^{\pm} decay (2) scales as $m_{\pi'}^5$, as opposed to the semiweak decay $\pi'^{\pm} \rightarrow \bar{f} f'$ being $\sim m_{\pi'}^3$. If the coupling of the (U, D) technidoublet to the heavy fermion pair is suppressed, the decay $\pi'^{\pm} \rightarrow \pi'^0 + \ell' \nu_{\ell}, q\bar{q}$ could indeed be the dominant decay mode of π'^{\pm} .

What are the signatures of $\pi'^{\pm} \pi'^{\mp}$ if (2) indeed turn out to be the dominant decay modes of π'^{\pm} ? Clearly, this depends on how the π'^3 decays. We argue that in the

TABLE 1

$m_{\pi^{\prime\pm}}$ (GeV)	$\Gamma(\pi^{\prime\pm} \rightarrow \pi^{\prime 0} + \ell^{\prime} \nu_{\ell})$ (a)	$\Gamma(\pi^{\prime\pm} \rightarrow \gamma + \ell^{\prime} \nu_{\ell})$ (b)
10.0	3.3×10^{-3} eV	1.2×10^{-5} eV
20.0	11.4 eV	1.6×10^{-3} eV
30.0	0.167 keV	2.8×10^{-2} eV
40.0	0.92 keV	0.23 eV
50.0	3.3 keV	1.23 eV

(a) The entries correspond to assuming $m_{\pi^{\prime 0}} = 8$ GeV.

(b) The numbers correspond to assuming 2 techniflavors \times 3 colors \times 4 technicolors in the evaluation of the triangle diagram.

scenario in which the $\pi^{\prime\prime} \bar{f} f^{\prime}$ couplings are suppressed it may be that the $\pi^{\prime 3} \bar{f} f^{\prime}$ couplings are also suppressed. The decay mode

$$\pi^{\prime 3} \rightarrow 2\gamma$$

may then become important. Unlikely as this scenario is, we would nevertheless like to record the decay modes. Thus, one could have:

$$\begin{array}{l}
 e^+ e^- \rightarrow \pi^{\prime+} \pi^{\prime-} \\
 \quad \quad \quad \downarrow \\
 \quad \quad \quad \pi^{\prime 0} + (\ell^{\prime} \nu_{\ell}, q\bar{q}) \\
 \quad \quad \quad \quad \quad \downarrow \\
 \quad \quad \quad \quad \quad \text{hadrons}
 \end{array} \tag{26}$$

The signatures of (26) are: (i) mixed lepton-hadron events much like the ones from $b\bar{b}$ and $c\bar{c}$ production but much more spherical, (ii) a $\sin^2\theta$ dependence of the jet axis and (iii) events of the type $e^+e^- \rightarrow \ell^+\ell^- + \text{hadron jets}$ with the ℓ^+ and ℓ^- *not* originating from the hadron jet. Alternatively, one could have the following decay chain:

$$\begin{array}{l}
 e^+ e^- \rightarrow \pi^{\prime+} \pi^{\prime-} \\
 \quad \quad \quad \downarrow \\
 \quad \quad \quad \pi^{\prime 0} + (\ell^{\prime} \nu_{\ell}, q\bar{q}) \\
 \quad \quad \quad \quad \quad \downarrow \\
 \quad \quad \quad \quad \quad 2\gamma
 \end{array} \tag{27}$$

leading to the signatures

$$\begin{aligned}
 e^+ e^- &\rightarrow 4\gamma + \ell^+ \ell^- \\
 &\rightarrow 4\gamma + \ell^+ + \text{hadrons} \\
 &\rightarrow 4\gamma + \text{hadrons.}
 \end{aligned} \tag{28}$$

Invariant mass cuts can be used to reconstruct π'^0 and remove ordinary photon background.

Finally, we would like to estimate the rate for the second order electroweak process (3) shown in fig 3. Clearly, this decay mode is exceedingly unlikely, unless there is another mechanism to give fermions a mass and the ETC forces indeed are of the Pati-Salam leptoquark type [9]. In that case the effective ETC coupling constant G_E can not be expressed in terms of the known fermion masses*. An upper limit on G_E can, however, be guessed from the limit on $K_L \rightarrow \mu^+ e^-$ as was done in ref. [6] giving $G_E < 10^{-5} G_F$, where G_F is the Fermi coupling constant. The process (2) will be absent if $m_{\pi'} < m_{\pi'^0}$. The process (3) will then be the only decay mechanism for ** π' .

The amplitude for the process (3) may be expressed as

$$\left(\pi'^{\pm} \rightarrow \gamma + \begin{array}{c} W^{\pm} \\ \downarrow \ell^{\pm} \nu_f \end{array} \right) = f \left(\frac{k^2}{M^2}, \frac{p^2}{M^2}, \frac{q^2}{M^2} \right) \frac{g}{(q^2 - m_W^2)} \epsilon_{\mu\nu\sigma\lambda} k^{\mu} q^{\nu} E^{\sigma} \\ \times \bar{u}_{\ell} \gamma_{\lambda} (1 - \gamma_5) u_{\nu}, \quad (29)$$

where $g = e/\sin\theta_W$ and M is the techniquark constituent mass ~ 1 TeV. Approximating $f(k^2/M^2, p^2/M^2, q^2/M^2)$ by $f(0,0,0)$ which can be calculated [6] using the triangle diagram, we get

$$f(0,0,0) \approx \frac{n_f N_c \alpha}{6\pi F_{\pi} \sin\theta_W}. \quad (30)$$

It is now straightforward to calculate the decay width. We get (neglecting m_{ℓ})

$$\Gamma(\pi' \rightarrow \gamma \ell^{\pm} \nu_f) = \frac{G_F (\alpha/\pi)^3 m_{\pi'}}{192\sqrt{2} \pi \sin^4 \theta_W} \left(\frac{1}{3} n_f^3 n_c^2 \right) \\ \times \left[\frac{1}{9m_{\pi'}^2} \left\{ m_{\pi'}^4 - 6(m_W^2 - m_{\pi'}^2)(4m_W^2 - 3m_{\pi'}^2) \right\} \right. \\ \left. + \frac{2}{3m_{\pi'}^4} \left\{ m_{\pi'}^6 - 6m_{\pi'}^4 m_W^2 + 9m_{\pi'}^2 m_W^4 - 4m_W^6 \right\} \ln \frac{m_W^2 - m_{\pi'}^2}{m_W^2} \right]. \quad (31)$$

* In other words fermion masses receive very small contributions from leptoquark forces.

** The idea of extended technicolor was introduced originally to give masses to fermions. If the fermions get their mass from some other mechanism, the need to extend technicolor becomes very obscure. Nevertheless, the TC sector of the theory may be rich enough to admit bound states like π'^{\pm}, π'^0 . In that case the $f \rightarrow F$ transitions would be very small, and the dominant decay mechanism would be the semifermionic modes (2) and (3) discussed above.

Rates for $\Gamma(\pi'^{\pm} \rightarrow \gamma \ell^{\mp} \nu_c + \gamma q\bar{q})$ are given in table 1. The rates vary over several orders of magnitude with $m_{\pi'}$, due to $m_{\pi'}^5$ behavior and the W-boson pole but for reasonable value of n_{ℓ} and n_c are still in the eV range. This corroborates our remarks earlier that unless the semiweak decays $\pi'^{\pm} \rightarrow \ell^{\mp} \bar{\nu}$ and the semileptonic decays (2) are absent, the branching ratios for the decays $\left(\begin{array}{l} \pi'^{\pm} \rightarrow \gamma \ell^{\mp} \nu_c \\ \rightarrow \gamma q\bar{q} \end{array} \right)$ would be miniscule.

In any case, the decay process (3) has a beautiful signature in e^+e^- annihilation [15]!

$$\begin{array}{l}
 e^+e^- \rightarrow \pi'^+ \pi'^- \\
 \quad \quad \quad \downarrow \\
 \quad \quad \quad \gamma(\ell^+ \nu_{\ell}, q\bar{q}) \\
 \quad \quad \quad \downarrow \\
 \quad \quad \quad \gamma(\ell^+ \nu_{\ell}, q\bar{q}), \quad (\ell = e, \mu, \tau),
 \end{array} \quad (32)$$

leading to events of the type

$$\begin{array}{l}
 e^+e^- \rightarrow 2\gamma + \text{hadrons} \\
 2\gamma + (\ell^-, \ell^+) + \text{hadrons} \\
 2\gamma + (\ell^+ \ell^-) + \nu's.
 \end{array} \quad (33)$$

The photons and leptons both would be very energetic which can be seen in fig. 8 which we have calculated for $m_{\pi'^{\pm}} = 15$ GeV and $\sqrt{s} = 40$ GeV. The background to

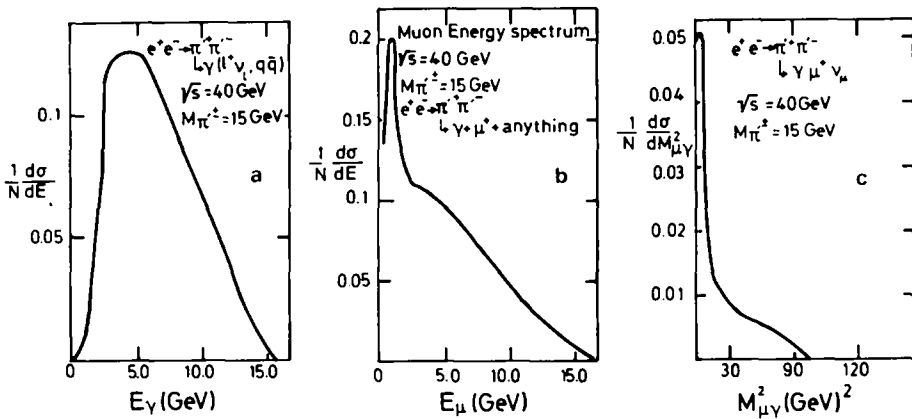


Fig. 8. Distributions from the second order electroweak process (32) involving technipions. (a) Photon energy distribution. (b) Inclusive muon (electron) energy distribution. (c) Invariant mass distribution $(1/\sigma)d\sigma/dm_{\mu\gamma}$.

the events in (28) comes from the third order QED process, and being proportional to $(\alpha/\pi)^3$ could be brought very much under control with appropriate cuts [14].

4. Discussion

In the preceding sections we discussed the production of a pair of charged color-singlet pseudo-Goldstone bosons π'^{\pm} in e^+e^- annihilation and their subsequent decays. While most of what we discussed in the previous sections is in the context of a technipion with $m_{\pi'} \sim O(10 \text{ GeV})$, the qualitative features of a heavier π' would still be very similar, namely:

(i) $\Delta R \sim \frac{1}{4}\beta^3$, (ii) $d\sigma/d\Omega \sim \sin^2\theta$, (iii) large sphericity and acoplanarity of the leptons and hadrons near threshold and (iv) the possibility of a not too small branching ratio for $\pi'^+ \rightarrow \pi'^0 + (\ell^+ \nu_{\ell, q\bar{q}})$, if $m_{\pi'^0} < m_{\pi'}$, resulting in events of the type $e^+e^- \rightarrow \ell^+\ell^- + 2 \text{ hadron jets}$.

There are two points we would like to record. The first one concerns the couplings of fermions, leptons and quarks to the technipions π'^{\pm}, π'^0 . We emphasize once again that these couplings are *not exactly* given by the masses of the ordinary fermions and their color representation. A consequence of this is that the ratios

$$\frac{\pi'^{\pm} \rightarrow \ell^{\pm} \nu_{\ell}}{\pi'^+ \rightarrow q\bar{q}'}, \quad \frac{\pi'^0 \rightarrow \ell^+ \ell^-}{\pi'^0 \rightarrow q\bar{q}}$$

are model dependent and hence *a priori unknown*. A more general search of the technipions is therefore needed than is normally advocated for a charged Higgs [8]. If it turns out that the estimates of the $\pi'f\bar{f}$ coupling is given by $\sim m_f/F_{\pi'}$ [1] which is true in most ETC models, then the most promising place to find a π'^{\pm} is in the decays of toponium $J_T \rightarrow \pi'^+ + b\bar{t}$; $\pi'^+\pi'^- + b\bar{b}$, etc., which would dominate the toponium decays [7].

The second point is about the signatures of $\pi'^+\pi'^-$ in the decays of Z^0 . For any of the technipions whose pair production thresholds lie below the Z^0 , one has [15]

$$\begin{aligned} \frac{\Gamma(Z^0 \rightarrow \pi'^+\pi'^-)}{\Gamma(Z^0 \rightarrow \nu\bar{\nu})} &= \frac{1}{2}(I_3 - 2Q\sin^2\theta_w)^2\beta^3 \\ &= \frac{1}{2}(1 - 2\sin^2\theta_w)^2\beta^3 \\ &\approx (0.15)\beta^3. \end{aligned}$$

The angular distribution is again given by $d\sigma/d\Omega \sim \sin^2\theta$. The $\pi'^+\pi'^-$ induced events can then be separated in exactly the same manner as discussed in sect. 2. The angular distribution of the lepton and/or the hadronic jet then should provide a

distinction from the usual heavy quark $Q\bar{Q}$ and heavy lepton L^+L^- pair production. This can be seen in fig. 5b, which, though drawn for the continuum production at $\sqrt{s} \simeq 100$ GeV, is also a useful guide for the process $e^+e^- \rightarrow Z^0 \rightarrow \pi'^+\pi'^-$.

Finally, we would like to draw attention to the semileptonic decays of the π'^{\pm} , which might not be as small as calculated for a charged Higgs ϕ^{\pm} decay. The signals (25), (26) and (28) associated with the semileptonic decays are so exciting that it would take quite an effort to miss them!

The spectroscopic structure of extended technicolor theories is quite rich but we have not belabored ourselves here with the task of studying the production of technihadrons with non-trivial color quantum numbers, for the obvious reason that they are unlikely to be produced at PETRA/PEP energies or even at LEP. By the same token we did not bother to look for technicolor signals in any other reaction.

The promise and potential of e^+e^- colliding beams is great. If there is going to be any experimental understanding of the phenomenon of spontaneous symmetry breaking, most probably it will come from present or future e^+e^- experiments.

One of us (A. Ali) would like to thank M.A.B. Bèg who encouraged us to undertake this study. Useful discussions with him, E. Farhi and L. Susskind are gratefully acknowledged. We would also like to thank our colleagues at DESY and in particular the members of the MARK-J Collaboration for discussion on experimental questions.

Note added

After this paper had been completed we received two preprints (i) J. Ellis, M.K. Gaillard, D. Nanopoulos and P. Sikivie, CERN-TH-2938 (Nucl. Phys. B182 (1981) 529), and (ii) S. Dimopoulos and J. Ellis, CERN-TH-2949 (Nucl. Phys. B182 (1981) 505), where some of the ideas pursued in this paper are studied. In particular, in (i) specific models are constructed to elucidate the differences between the hyperpion and Higgs couplings, exemplified through eqs. (11) and (12). We would like to thank J. Ellis for sending us advance copies of their papers and for his correspondence.

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* See ref. [16] for a Monte Carlo program written for the production $e^+e^- \rightarrow \pi'^+\pi'^-$ incorporating the decays discussed in this paper.

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