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DISTRIBUTIONS FOR ELECTRON-POSITRON ANNIHILATION INTO TWO AND THREE PHOTONS

F. A. BERENDS and R. KLEISS

Instituut-Lorentz, Leiden, The Netherlands
and
Deutsches Elektronen-Synchrotron DESY, Hamburg, W. Germany

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In this paper a procedure is outlined to simulate events for the reactions $e^+e^- \to \gamma\gamma$ and $\gamma\gamma\gamma$. Various distributions thus obtained are presented. The standard radiative corrections are also calculated in this way. Moreover, an analytical expression for the total annihilation cross section is given up to order α^3.

1. Introduction

Whereas mu pair production and Bhabha scattering are expected to undergo influences from weak interactions and hadronic vacuum polarization effects at higher energies, the electron-positron annihilation into two and three photons is expected to remain a pure QED reaction in this energy range. It may therefore play more and more the role of a reference reaction, which one should quantitatively know well.

A numerical evaluation of the radiative corrections to

$$e^+(p_+) + e^-(p_-) \to \gamma(q_1) + \gamma(q_2) \tag{1.1}$$

exists [1] for those experimental criteria which can be described in terms of an acollinearity angle ζ between $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ and in terms of threshold energies E_{th} for the two detected photons. When other cuts have to be made on the data the numerical integration programs have to be changed.

In practice it would be advantageous to be able to simulate numerically the events of reaction (1.1) and of the reaction

$$e^+(p_+) + e^-(p_-) \to \gamma(q_1) + \gamma(q_2) + \gamma(q_3) \tag{1.2}$$

at the same time. All kinds of selection criteria can then be applied to the sample of (q_1, q_2, q_3) four-momenta. The number of accepted events and the knowledge of the size of the cross section for the total number of events then provides the information needed to calculate the radiative correction. Of course, it will also often be useful specifically to generate events of reaction (1.2) when one wants to know the QED background to some kind of resonance decay.

The ingredients we will need in order to simulate events are virtual corrections, soft bremsstrahlung corrections and the hard bremsstrahlung cross section. These will be summarized in sect. 2. In sect. 3 the total cross section for the annihilation into two and three photons will be given. An approximate calculation for the same quantity is outlined in sect. 4. The relatively simple distributions originating from this evaluation serve as basis for the generation of events. This approximate distribution is then changed into the exact distribution by a selection procedure as outlined in sect. 5. Sect. 6 finally gives a number of distributions and a set of values for the standard type of radiative corrections.

2. Virtual corrections and bremsstrahlung

The lowest order cross section for reaction (1.1) is given by

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega_1} = \frac{\alpha^2}{s} \frac{1+c_1^2}{e^2 - c_1^2}, \tag{2.1}$$

where

$$e = p_{+0}/|\boldsymbol{p}_+|, \tag{2.2}$$

$$c_1 = \cos\theta_1, \tag{2.3}$$

where θ_1 is the angle between $\boldsymbol{q}_1$ and $\boldsymbol{p}_+$ and $s = 4p_{+0}^2 = 4E^2$. In eq. (2.1) terms of order m_e^2/E^2 have been neglected except in the denominator, since we also want to describe forward and backward scattering. The total lowest order cross section therefore reads, introducing a statistical factor of $\frac{1}{2}$,

$$\sigma_0 = \frac{2\pi\alpha^2}{s}\left(\ln\frac{s}{m_e^2} - 1\right) \equiv \frac{\pi^2\alpha}{s}\beta. \tag{2.4}$$

The cross section for reaction (1.1) including virtual corrections and soft bremsstrahlung, the energy of the third photon being limited by a value $q_{30}/|\boldsymbol{p}_+| = k \ll 1$, reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_1} = \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega_1}(1+\delta_\mathrm{A}), \tag{2.5}$$

where δ_A depends on E, c_1 and k. The subscript A denotes that this correction is known analytically. It is obtained by adding the virtual and soft bremsstrahlung corrections. The exact expressions were first obtained by Harris and Brown [2] and later recalculated by others [1,3]*. Here we shall use the extreme relativistic limit ($m_e/E \ll 1$) of the exact expression. Care must be taken not to overlook the narrow peaks in the extreme forward and backward directions. We thus obtain

$$\begin{aligned}\delta_A = -\frac{\alpha}{\pi}\Bigg\{ & 2(1-2v)(\ln k + v) + \tfrac{3}{2} - \tfrac{1}{3}\pi^2 + \frac{1}{2(1+c_1^2)} \\ & \times\Big[-4v^2(3-c_1^2) - 8vc_1^2 + 4uv(5+2c_1+c_1^2) \\ & \quad + 4wv(5-2c_1+c_1^2) - u(7-8c_1+c_1^2) - w(7+8c_1+c_1^2) \\ & \quad + \eta(c_1)(5+2c_1+c_1^2) + \eta(-c_1)(5-2c_1+c_1^2)\Big] \\ & + \frac{e^2-c_1^2}{2(1+c_1^2)}\Bigg[\frac{2u}{e+c_1-\frac{1}{2}m^2} + \frac{2w}{e-c_1-\frac{1}{2}m^2} \\ & \quad + \left(\eta(c_1)-\tfrac{1}{3}\pi^2\right)\frac{m^4}{(e+c_1)^3} + \left(\eta(-c_1)-\tfrac{1}{3}\pi^2\right)\frac{m^4}{(e-c_1)^3} \\ & \quad - \frac{4m^2u}{(e+c_1)^2} - \frac{4m^2w}{(e-c_1)^2} + \frac{2m^2}{(e-c_1)^2} + \frac{2m^2}{(e+c_1)^2}\Bigg]\Bigg\}, \end{aligned} \tag{2.6}$$

where

$$m = m_e/|\boldsymbol{p}_+|, \tag{2.7}$$

$$v = \tfrac{1}{2}\ln\frac{4}{m^2}, \tag{2.8}$$

$$u = \tfrac{1}{2}\ln\frac{2(e+c_1)}{m^2}, \tag{2.9}$$

$$w = \tfrac{1}{2}\ln\frac{2(e-c_1)}{m^2}, \tag{2.10}$$

$$\eta(c_1) = \mathrm{Li}_2\left(1 - \frac{2}{m^2}(e+c_1)\right) + \tfrac{1}{6}\pi^2, \tag{2.11}$$

* Refs. [2,3] contain some misprints (cf. ref. [1]).

the dilogarithm being denoted by Li_2. Outside the extreme forward and backward regions the triple and double pole terms in the last bracket may be omitted. Furthermore one can then take

$$\eta(\pm c_1) \simeq -2u^2, -2w^2, \tag{2.12}$$

respectively. The resulting expression is now

$$\begin{aligned}\delta_A = -\frac{\alpha}{\pi}\Big\{ & 2(1-2v)(\ln k + v) + \tfrac{3}{2} - \tfrac{1}{3}\pi^2 + \frac{1}{2(1+c_1^2)} \\ & \times\big[-4v^2(3-c_1^2) - 8vc_1^2 + 4uv(5+2c_1+c_1^2) \\ & + 4wv(5-2c_1+c_1^2) - u(5-6c_1+c_1^2) - w(5+6c_1+c_1^2) \\ & - 2u^2(5+2c_1+c_1^2) - 2w^2(5-2c_1+c_1^2)\big]\Big\}. \end{aligned} \tag{2.13}$$

This expression agrees with the ones given in the literature [4, 5].

Furthermore we need the cross section for the annihilation into three photons. Here we again take the extreme relativistic limit of the exact cross section. The latter was obtained by Mandl and Skyrme, the former was derived in refs. [5, 7]. It reads

$$\frac{d\sigma}{d\Gamma_{123}} = \frac{d\sigma}{d\Omega_1\, d\Omega_3\, dx_3} = \frac{\alpha^3}{8\pi^2 s}\omega_{123} F, \tag{2.14}$$

where

$$\omega_{123} = \frac{x_1 x_3}{y(z_2)}, \quad x_i = q_{i0}/|\boldsymbol{p}_+|, \tag{2.15}$$

$$y(z_2) = 2e - x_3 + x_3 z_2, \tag{2.16}$$

$$z_2 = \cos\alpha_2, \tag{2.17}$$

the angle α_2 being the angle between $\boldsymbol{q}_1$ and $\boldsymbol{q}_3$ and Ω_i being the solid angle of the ith photon, and

$$F=\sum_{\mathcal{P}}\left[-2m^2\frac{k_2'}{k_3^2k_1'}-2m^2\frac{k_2}{k_3'^2k_1}+\frac{2}{k_3k_3'}\left(\frac{k_2^2+k_2'^2}{k_1k_1'}\right)\right]$$

$$=\sum_{\mathcal{P}}M(1,2,3), \tag{2.18}$$

where $\mathcal{P}$ denotes all permutations of (1,2,3). The quantities k_i and k_i' are given by

$$k_i=\hat{p}_+\cdot\hat{q}_i=x_i(e-c_i),$$

$$k_i'=\hat{p}_-\cdot\hat{q}_i=x_i(e+c_i), \tag{2.19}$$

where

$$\hat{p}_\pm=p_\pm/|\boldsymbol{p}_+|,\qquad \hat{q}_i=q_i/|\boldsymbol{p}_+|. \tag{2.20}$$

3. The total cross section

The total cross section for the annihilation into two and three photons consists of

$$\sigma^{3\gamma}=\frac{1}{3!}\int \mathrm{d}\Gamma_{123}\frac{\mathrm{d}\sigma}{\mathrm{d}\Gamma_{123}}, \tag{3.1}$$

where the integral runs over all phase space except the regions where $x_i<k$. In these regions the integrated soft and virtual corrections contribute:

$$\sigma^{2\gamma}=\frac{1}{2!}\int \mathrm{d}\Omega_i\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_i}, \tag{3.2}$$

with the cross section given by (2.5).

The 3γ cross section can be written in terms of the quantities M:

$$\sigma^{3\gamma}=\frac{\alpha^3}{8\pi^2s}\frac{1}{3!}\int \mathrm{d}\Gamma_{123}\,\omega_{123}\sum_{\mathcal{P}}M(1,2,3) \tag{3.3}$$

$$=\frac{\alpha^3}{8\pi^2s}\int \mathrm{d}\Gamma_{123}\,\omega_{123}M(1,2,3), \tag{3.4}$$

where use has been made of

$$\mathrm{d}\Gamma_{123}\,\omega_{123}=\mathrm{d}\Gamma_{ijk}\,\omega_{ijk} \tag{3.5}$$

for any permutation of the photon indices. It should be noted that instead of $M(1,2,3)$ one can take any other function $N(1,2,3)$ which summed over all permutations again gives F [cf. eq. (2.18)].

It is convenient to choose N in such a way that in the integration over x_3 the infrared divergence only occurs for $x_3 \to 0$ and not for $x_3 \to e$, the latter infrared divergence being related to one of the other photons being soft.

Writing out $M(1,2,3)$ explicitly in the energies and angles we find for the last term of M (i.e. the term not proportional to m^2)

$$M'(1,2,3) = \frac{8e^2\left[x_3'^2 + x_1'^2\right]}{x_1^2 x_3^2 (e-c_1^2)(e-c_3^2)} - \frac{4}{x_3^2(e-c_3^2)} - \frac{4}{x_1^2(e-c_1^2)} + \frac{4}{x_1 x_3 (e-c_1)(e-c_3)} + \frac{4}{x_1 x_3(e+c_1)(e+c_3)}, \tag{3.6}$$

where

$$x_i' = e - x_i. \tag{3.7}$$

Since

$$x_1 = \frac{2ex_3'}{y(z_2)}, \tag{3.8}$$

we see that the third term in eq. (3.6) behaves like $1/x_3'^2$ which in combination with ω_{123} diverges for $x_3 \to e$. The part of M proportional to m^2 converges for $x_3 \to e$.

Instead of $M'(1,2,3)$ we shall use

$$N'(1,2,3) = \frac{16e^2 x_3'^2}{x_1^2 x_3^2 (e-c_1^2)(e-c_3^2)} - \frac{8}{x_3^2(e-c_3^2)} + \frac{4}{x_1 x_3(e-c_1)(e-c_3)} + \frac{4}{x_1 x_3 (e+c_1)(e+c_3)}. \tag{3.9}$$

This quantity only diverges for $x_3 \to 0$. Combined with the original m^2 terms, expression (3.9) gives a suitable expression $N(1,2,3)$ for the calculation of $\sigma^{3\gamma}$. Added to $\sigma^{2\gamma}$ it gives the total cross section for annihilation into two and three photons. It should be noted that both $N(1,2,3)$ and $M(1,2,3)$ are not positive definite expressions, but, of course, the quantity F is positive definite.

Integrating eq. (3.4) with the integrand N yields

$$\sigma^{3\gamma} = \frac{2\alpha^3}{s}\left[-(2v-1)^2(2\ln k + 1) + 3\right]. \tag{3.10}$$

Similarly the integration of (3.2), the integrand being given by (2.5) and (2.6) gives the expressions

$$\sigma^{2\gamma} = \sigma_0 + \frac{2\alpha^3}{s}\left[2(2v-1)^2 \ln k + \tfrac{4}{3}v^3 + 3v^2 + \left(\tfrac{2}{3}\pi^2 - 6\right)v - \tfrac{1}{12}\pi^2\right], \quad (3.11)$$

$$\sigma_{\mathrm{T}} = \sigma^{2\gamma} + \sigma^{3\gamma} = \sigma_0 + \frac{2\alpha^3}{s}\left[\tfrac{4}{3}v^3 - v^2 + \left(\tfrac{2}{3}\pi^2 - 2\right)v + 2 - \tfrac{1}{12}\pi^2\right]. \quad (3.12)$$

It should be noted that the logarithmic terms in (3.10) and (3.11) agree with those obtained in ref. [5], but not the constant terms. In eq. (3.11) we could trace the origin of this difference to the use of eq. (2.14) instead of (2.6) in ref. [5]. The disagreement in $\sigma^{3\gamma}$ could again lie in the treatment of the extreme forward and backward regions.

4. An approximate cross section

In this section we make an ansatz for an approximate 3γ cross section, which exhibits peaks at the same location as the exact cross section. Instead of the exact $\omega_{123} N(1,2,3)$ we take

$$\omega_{123}\tilde{N}(1,2,3) = \frac{8x_3'}{e x_3\left(e^2 - c_1^2\right)\left(e^2 - c_3^2\right)} + \frac{4}{y(c_1)(e-c_1)(e-c_3)} + \frac{4}{y(-c_1)(e+c_1)(e+c_3)}. \quad (4.1)$$

This means that the m^2 terms in N have been omitted and, moreover, the second term in (3.9), whereas in the last two terms in (4.1) $y(z_2)$ of the phase-space factor has been replaced by $y(\pm c_1)$.

The quantity $\tilde{N}(1,2,3)$ is positive definite and gives rise to a squared matrix element $\tilde{F}$, as in eq. (2.18). Since $\tilde{N}(i,j,k)$ gives rise to the same distribution as $\tilde{N}(1,2,3)$ except for a relabelling of the photons, it is sufficient to generate q_1, q_2, q_3 from $\tilde{N}(1,2,3)$. We use the approximate distribution

$$\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}k\,\mathrm{d}c_1\,\mathrm{d}c_3} = \frac{\alpha^3}{s}\left(\frac{A_-}{e-c_3} + \frac{A_+}{e+c_3}\right), \quad (4.2)$$

where

$$A_{\mp} = \frac{2x_3'}{x_3}\frac{1}{e^2 - c_1^2} + \frac{2}{y(\pm c_1)(e \mp c_1)}. \quad (4.3)$$

From this we obtain successively

$$\int \mathrm{d}c_3 \frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x_3\,\mathrm{d}c_1\,\mathrm{d}c_3} = \frac{\alpha^3}{s}\left[A_+ \ln(e+c_3) - A_- \ln(e-c_3)\right], \tag{4.4}$$

$$\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x_3\,\mathrm{d}c_1} = \frac{\alpha^3}{s} 2v(A_+ + A_-), \tag{4.5}$$

$$\int \mathrm{d}c_1 \frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x_3\,\mathrm{d}c_1} = \frac{4\alpha^3 v}{s}\left[\left(\frac{x_3'}{ex_3} + \frac{1}{\rho}\right)\ln\left(\frac{e+c_1}{e-c_1}\right) + \frac{1}{\rho}\ln\frac{y(c_1)}{y(-c_1)}\right], \tag{4.6}$$

with

$$\rho = 2e + \mu x_3, \qquad \mu = e - 1. \tag{4.7}$$

For the photon spectrum we obtain

$$\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x_3} = \frac{8\alpha^3 v}{s}\left[\frac{2x_3' v}{ex_3} - \frac{1}{\rho}\ln\left(1 - \frac{\rho}{\Delta e}\right)\right], \tag{4.8}$$

with

$$\Delta = e + 1, \tag{4.9}$$

and the integrated spectrum reads ($k \ll 1$)

$$\int_k^{x_3} \mathrm{d}x_3 \frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x_3} = \frac{4\alpha^3 v}{s}\left[4v\ln\frac{x_3}{k} - (2v-1)(x_3 - k) - k'\ln k' + x_3'\ln x_3'\right], \tag{4.10}$$

where k' is defined like x_3' in eq. (3.7). Thus,

$$\tilde{\sigma}^{3\gamma} = \frac{4\alpha^3}{s} v\left[-4v\ln k - (2v-1)k' - k'\ln k'\right]. \tag{4.11}$$

The leading logarithmic term in $\tilde{\sigma}^{3\gamma}$ is the same as in eq. (3.10).

5. Generating events

Choosing a k-value below which a photon energy can be neglected in an experiment, e.g. $k = 0.01$, one first uses $\sigma^{2\gamma}$ and $\tilde{\sigma}^{3\gamma}$ to decide whether one generates a 2γ or 3γ event. If it is a 2γ event one uses eq. (2.5) for generating c_1. If it is a 3γ event one uses successively the integrated distributions (4.10), (4.6) and (4.4) to obtain x_3, c_1 and c_3. This can be done for x_3 by generating a random number η in the interval $[0, \tilde{\sigma}^{3\gamma}]$ and solving for x_3 the equation

$$\int_k^{x_3} \mathrm{d}x_3 \frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x_3} = \eta . \tag{5.1}$$

For x_3 and c_1 these type of equations have to be solved numerically; in the case of (4.4) the two terms can be solved analytically. The azimuthal angles of $\boldsymbol{q}_1$ and $\boldsymbol{q}_3$ with respect to some plane through the beam axis are generated at random. In this way it is possible to generate quickly a large number of (q_1, q_2, q_3) momenta sets, which follow the distribution of the approximate distribution $\omega_{123}\tilde{F}$. It turns out that the real distribution $\omega_{123}F$ only deviates slightly from the approximate distribution, or, in other words, the weights

$$w(q_1, q_2, q_3) = F/\tilde{F} \tag{5.2}$$

of the events are around 1.

The set of n_1 events $\{(q_1, q_2, q_3)\}$ can be used either to calculate cross sections integrated over certain parts of phase space or to construct a restricted set of n_2 events with weights 1.

In the first case one sums the weights for those n_3 events which lie in the wanted part of the phase space and multiplies this with $\tilde{\sigma}^{3\gamma}/n_1$. For instance, one can then obtain the type of radiative corrections calculated in ref. [1].

In the second case one generates e.g. a random number η in the interval [0, 2]. If

$$\eta < w(q_1, q_2, q_3), \tag{5.3}$$

the event (q_1, q_2, q_3) is kept, otherwise not. The remaining events have weight 1, the number of events n_2 corresponds to a cross section $\sigma^{3\gamma}$, which one knows from eq. (3.10). It could also be obtained from summing all the weights and multiplying this number with $\tilde{\sigma}^{3\gamma}/n_1$.

Of course, in practice one mostly combines the soft and hard photon events into one set with different weights or in a restricted set with weight 1.

In the approach sketched here one generates *a priori* events in the full phase space. This is done because the symmetry of the problem can be exploited. Because of this the generation of events is fast so that it does not matter in practice that many events are experimentally uninteresting, being close to the beam.

6. Some results

In this section we give typical values both for the total cross section and for radiative corrections to the differential cross section. Moreover, histograms for some distributions are shown.

We express the total cross section σ_T, as given by eq. (3.12), in terms of the lowest order cross section σ_0 and a correction δ_T through

$$\sigma_T = \sigma_0(1+\delta_T). \tag{6.1}$$

Values for σ_0 and δ_T are listed in table 1. Although the leading logarithmic term in δ_T behaves like v^2 just as in the mu pair production case, the actual value of δ_T is much smaller, which is mainly due to the coefficient in front of v^2.

Radiative corrections to the differential cross section are also expressed in terms of a percentage correction

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega}(1+\delta_T), \tag{6.2}$$

where δ_T depends on E, θ, the acollinearity angle ζ and threshold energy E_{th}. Values for δ_T can be obtained by the numerical integration of ref. [1] or by the event generating approach described above. Typical results are given in table 2. Besides δ_T

TABLE 1
The lowest order total cross section σ_0 and the percentage correction δ_T for the total cross section, as given by eq. (6.1)

E(GeV)	3	5	10	15	25	50	100
σ_0(nb)	64.2	24.5	6.6	3.03	1.15	0.305	0.0806
δ_T(%)	13.8	15.3	17.4	18.8	20.5	23.0	25.7

TABLE 2
The radiative corrections to the lowest order differential cross section for various values of the beam energy E and scattering angle θ

E (GeV) / θ (deg)	3		15	
	δ_A	δ_T	δ_A	δ_T
20	−7.9	−0.3 ± 0.4	−9.5	−0.9 ± 0.5
40	−8.3	−5.0 ± 0.2	−9.9	−6.0 ± 0.2
60	−8.3	−6.8 ± 0.1	−9.9	−8.3 ± 0.1
90	−8.3	−7.8 ± 0.1	−9.9	−9.4 ± 0.1

For $E = 3$ GeV, $E_{th} = 0.2$ GeV for $E = 15$ GeV, $E_{th} = 3$ GeV. The acollinearity angle is in both cases $\zeta = 10°$.

the quantity δ_A [cf. (2.6) or (2.13)] is also given, evaluated for that k-value which represents the maximum photon energy for isotropic soft photon emission allowed for by the angle ζ (cf. ref. [1]).

Finally, a set of 154831 events was generated for a beam energy of 15 GeV. An acollinearity and acoplanarity histogram were made for a subset of events. These events were required to have at least two energies larger than 3 GeV and to have angles in the interval [10°, 170°]. Thus, e.g., the momenta q_i and q_j are such that

$$q_{i0}, q_{j0} > 3\ \text{GeV}, \tag{6.3}$$

$$\theta_i, \theta_j \in [10°, 170°]. \tag{6.4}$$

This subset contains 26520 events. The acollinearity and acoplanarity distributions are shown in figs. 1 and 2, using the definitions

$$\zeta = \arccos(-\hat{\boldsymbol{q}}_i \cdot \hat{\boldsymbol{q}}_j), \tag{6.5}$$

$$\psi = \arccos\left(- \frac{\boldsymbol{q}_i^{\mathrm{T}} \cdot \boldsymbol{q}_j^{\mathrm{T}}}{|\boldsymbol{q}_i^{\mathrm{T}}|\,|\boldsymbol{q}_j^{\mathrm{T}}|} \right), \tag{6.6}$$

where $\boldsymbol{q}_i^{\mathrm{T}}$ is the component of $\boldsymbol{q}_i$ transverse to the beam direction. Events for which also the third energy and angle satisfy (6.3) and (6.4) have three possible ζ and ψ

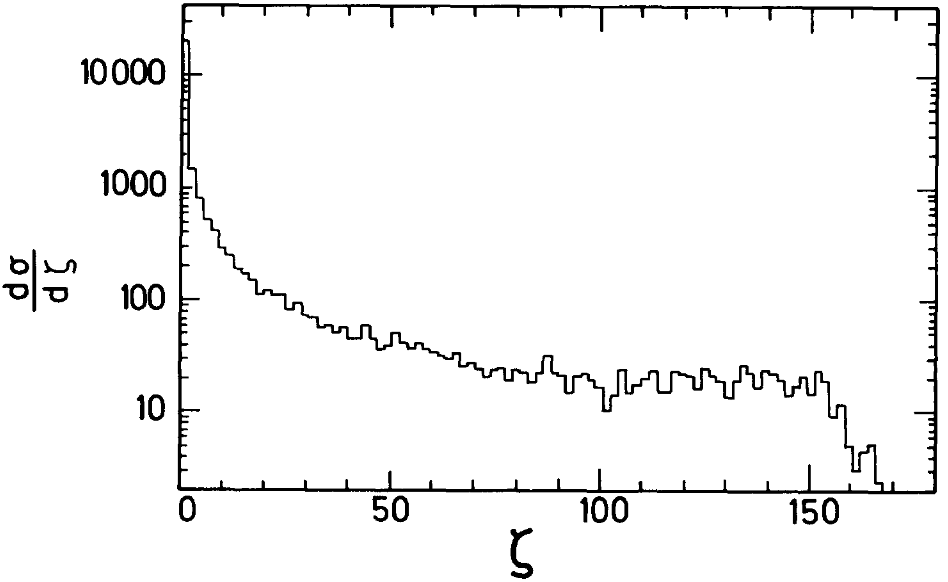

Fig. 1. The acollinearity distribution for a set of events at 15 GeV.

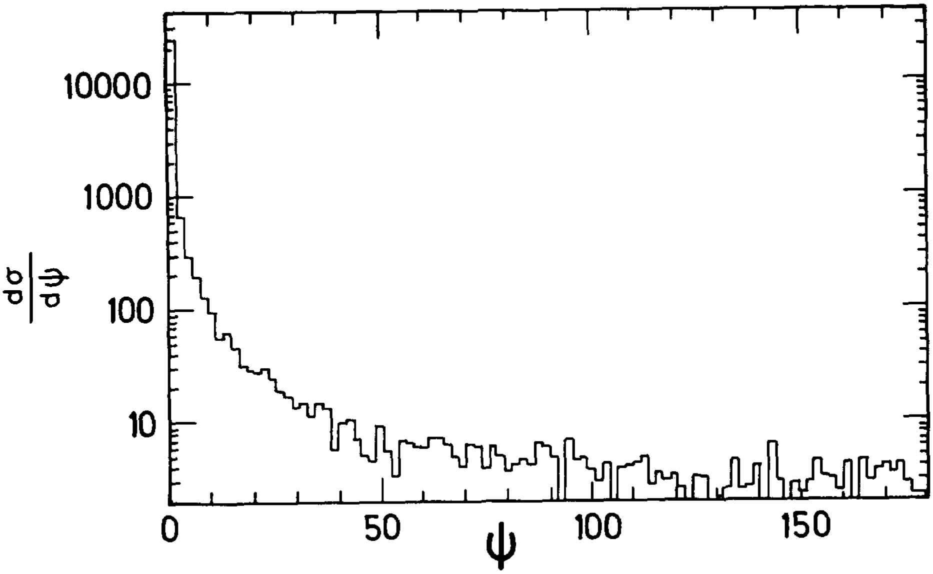

Fig. 2. The acoplanarity distribution for a set of events at 15 GeV.

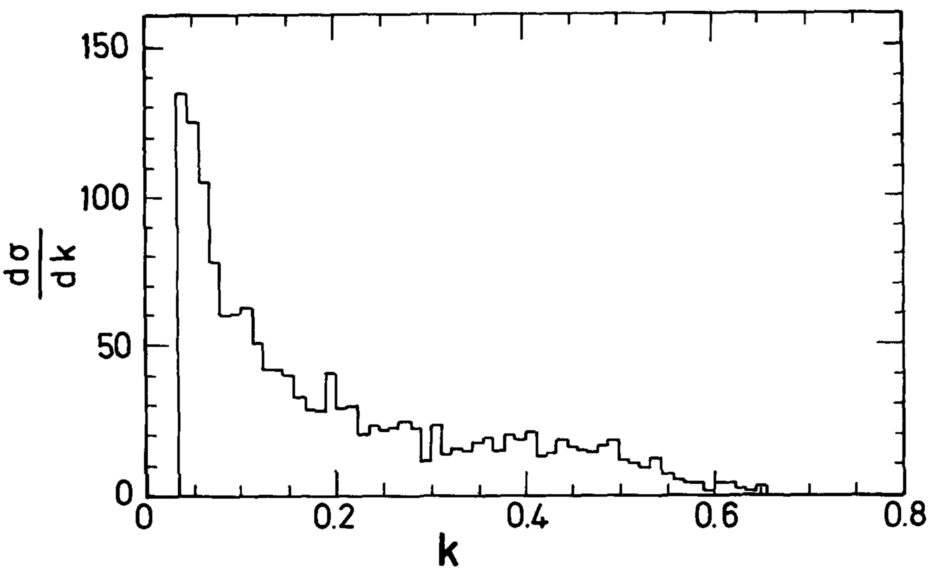

Fig. 3. The energy distribution of the softest photon for a set of events at 15 GeV.

values. They have been added in each of the appropriate bins of the histogram, but with weight $\frac{1}{3}$.

Finally, from the subset considered, another subset is made for which the remaining momentum satisfies

$$q_{l0} > 0.5 \text{ GeV}, \tag{6.7}$$

and θ_l satisfies (6.4). In this subset we denote the smallest photon energy of each event by k. A histogram of the k distribution thus obtained is shown in fig. 3. It contains 1522 events.

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