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A Simple Method of Four-Jet Analysis in e^+e^- Annihilation

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Abstract. For four-jet events in electron–positron annihilation at center-of-mass energies of 30 GeV or higher, a simple procedure is described to determine all four jet axes and jet energies. The result is insensitive to missing particles such as neutrals and the required computer time is quite moderate.

The observation and analysis of the three-jet events e^+e^- annihilation at PETRA by the TASSO Collaboration [1] was based on a method using the concept of generalized sphericity [2]. These three-jet events are naturally interpreted as gluon bremsstrahlung [3] $e^+e^- \to q\bar{q}g$. On the basis of the three-jet events, the gluon spin has been determined [4] to be 1 in agreement with quantum chromodynamics.

As applied to three-jet events, the method of generalized sphericity has the following desirable features:

1. All three jet axes are determined.
2. It is not necessary to include all particles in the analysis. For example, one can apply this procedure to charged particles only.
3. All measured momenta can be used; in other words, there is no need to introduce a cutoff for low momenta.
4. Computer time is moderate.

Since there are three-jet events, there must also be four-jet events [5, 6]. With the three-jet events interpreted as gluon bremsstrahlung, double gluon bremsstrahlung leads to four-jet events:

$$e^+e^- \to q\bar{q}gg.$$

Alternatively, the bremsstrahlung gluon may materialize into a $q\bar{q}$ pair:

$$e^+e^- \to q\bar{q}q\bar{q}.$$

The lowest-order QCD diagrams for these two types of four-jet events are shown respectively in Fig. 1a, b. Since two of the diagrams shown in Fig. 1a for double bremsstrahlung involve the coupling of three gluons, four-jet events give information about this gluon self-coupling, characteristic of a non-Abelian gauge theory [7].

In principle, four-jet analysis can be carried out by partitioning the observed tracks into four subsets and minimizing the sum of the sphericities (or maximizing the sum of the thrusts) of these four subsets. Similar to the case of the three-jet analysis, the fundamental difficulty of the four-jet analysis is due to the extremely large number of partitions. The number of ways to partition N observed tracks into four non-empty sets is

$$\tfrac{1}{6}(4^{N-1} - 3^N + 3.2^{N-1} - 1).$$

This number is ridiculously large even for moderate values of N; it is for example 1.8×10^{11} for $N = 21$. It is just about impossible to search over such a large number of combinations.

The basic idea of the three-jet analysis already mentioned is to project all the measured momenta into the event plane formed by the eigen-vectors $\hat{n}_2$ and $\hat{n}_3$ of the momentum tensor as explained in [1, 2]. The main point there is that the projections can be placed in a cyclic order according to their polar angles on the event plane and only contiguous partitions need to be studied. In the present case of four-jet analysis, there is no cyclic order, and hence a new idea is needed.

In Fig. 2, a four-jet event from TASSO [8] at PETRA is shown projected into the event plane. In this particular case, the four-jets are all clearly seen in this event plane. This is rather exceptional; in most cases two of the four jets are not separated in this projection. More precisely, when the three-jet analysis of [2] is applied to a four-jet event, then in most cases two of the three partitions consist of one

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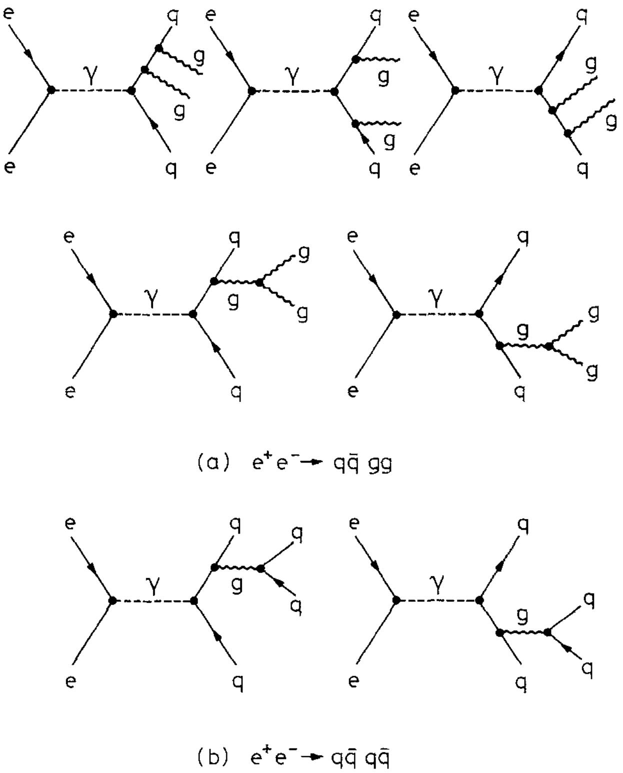

Fig. 1 a and b. Feynman diagrams for four-jet events

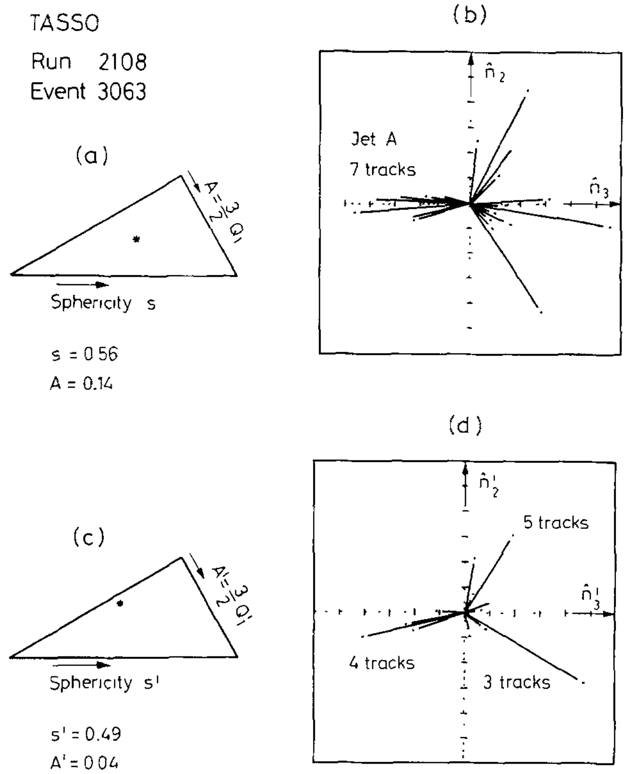

Fig. 3 a–b. Application of four-jet analysis to a TASSO event: **a** position of the event in the triangular plot; **b** the event as viewed in the event plane and the identification of jet A; **c** position of the event after the removal of jet A and Lorentz transformation to the center-of-mass of the remaining tracks; **d** these remaining tracks as viewed on the event plane of the boosted system

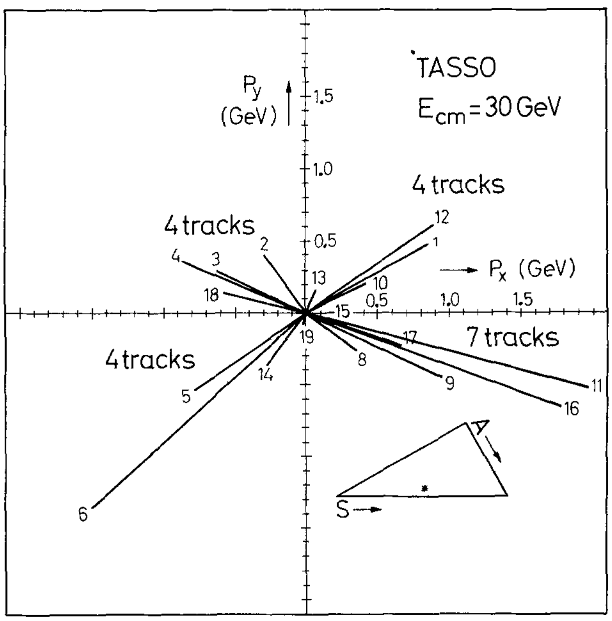

Fig. 2. A TASSO four-jet event viewed on the event plane

jet each, while the third partition contains two jets. Occasionally, one of the four jets is divided between two of the partitions; thus one of the partitions consists of one jet while the other three jets are split between the other two partitions. Only very rarely is one of the four jets divided between all three partitions. We ignore these very rare cases. By identifying the "thinnest" of the three partitions as one jet, its removal makes it possible to apply the three-jet analysis a second time to identify the other three jets. More precisely, the procedure followed for the four-jet analysis is as follows.

A) Use the program for three-jet analysis.

B) Determine the three jet axes by adding the momenta of the tracks in the jet. In general these three jet axes are not coplanar.

C) Find the average transverse momenta for the three jets with respect to their own jet axes as defined above. To avoid possible confusion with the average transverse momenta used in the three-jet analysis, let us call them $\langle PT' \rangle$.

D) Among the three jets being considered so far, define jet A as the one with the smallest value of $\langle PT' \rangle$.

E) Remove all the tracks of jet A, and study the remaining tracks, i.e. the tracks that are not in jet A.

F) Add up the energies and momenta of these remaining tracks to find the motion of the center of mass. Perform a Lorentz transformation to this center of mass of these remaining tracks.

G) Apply three-jet analysis to the transformed momenta for these remaining tracks. This gives the three new partitions. In particular, determine whether this is itself a three-jet event.

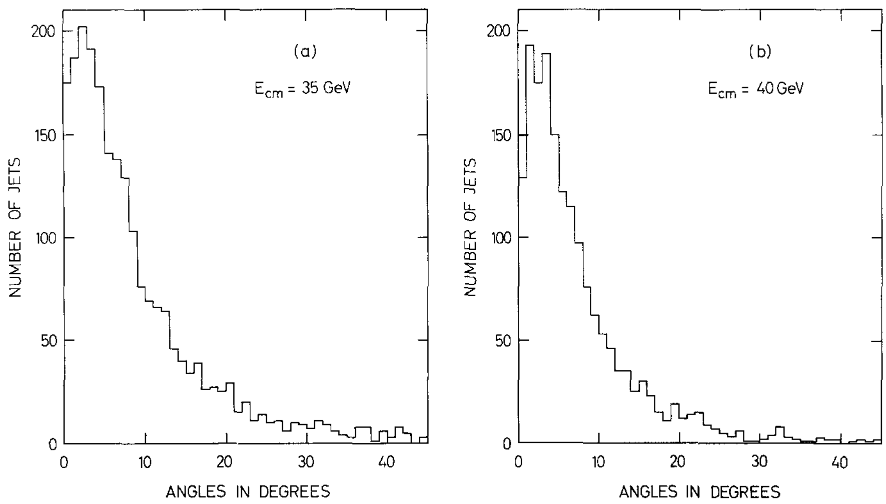

Fig. 4 a and b. The distribution for four-jet events of the angle between the reconstructed jet axis and the corresponding generated jet axis using both charged and neutral particles **a** at 35 GeV and **b** at 40 GeV of center-of-mass energy

H) If these remaining tracks form an acceptable three-jet event, go back to the original laboratory coordinate system.

I) In the laboratory coordinate system, determine the four jet axes by adding up the momenta of the tracks in each jet.

J) Find the angles between all the momentum directions and all the four jet axes. Each momentum should make the smallest angle with respect to the jet axis that it belongs to. If this is indeed the case, then the jet axes have been found properly.

K) If not, then such a momentum should be reassigned to the jet that gives the smallest angle. After reassignment, repeat steps I and J.

L) The energies of the four jets are determined under the assumption that the jet masses can be neglected. Let $\hat{n}_i$ with $i=1,2,3$, and 4 be the unit vectors in the directions of the jet axes, then the energies E_i of the jets are determined by the energy-momentum conservation

$$\sum_{i=1}^{4} E_i \hat{n}_i = 0$$

$$\sum_{i=1}^{4} E_i = E_{\mathrm{cm}}.$$

As an illustration, the method is applied to a TASSO event of large sphericity S and large aplanarity A. As shown in the triangular plot of Fig. 3a, $S=0.56$ and $A=0.14$ for this particular event. The first application of the three-jet analysis (step A) gives in the event plane the result of Fig. 3b. There are three groups with respectively 4, 7, and 8 charged tracks. Of these three, the one with 7 tracks has the smallest value of $\langle PT' \rangle$ and is hence designated as jet A (step D). After Lorentz transformation to the center-of-mass of the remaining 12 particles it is found, as shown in Fig. 3c, that the sphericity of the boosted system is still large ($S'=0.49$), while the aplanarity of the boosted system is small ($A'=0.04$). The second application of the three-jet analysis (step G) yields in the new event plane the three jets of Fig. 3d, consisting respectively of 3, 4, and 5 tracks. In this example, as in most cases, no reassignment of momenta (step K) is needed.

In order to have some notion of the accuracy of the present simple procedure, a Monte Carlo program [9] is used to generate four-jet events at E_{cm} of 35 and 40 GeV. The energy of each generated jet is required to be larger than 2 GeV, and a cut is intruded at $S \geqq 0.3$ and $A \geqq 0.1$. A comparison is made between the reconstructed jet axes as given by the present analysis and the generated jet axes defined as the direction of the sum of the momenta from the particles in the jet. In Fig. 4a, b histograms are shown for the angle between the generated and reconstructed jet axes, for 35 and 40 GeV respectively. In this result, both the charged and neutral momenta are used. The average angular deviation is found to be 9.5° for 35 GeV and 7.5° for 40 GeV. When the charged momenta alone are used, the result is worse by less than a factor of 2. Thus the present analysis is adequate for these energies.

This procedure of four-jet analysis has all the four advantages listed at the beginning of this paper for the three-jet analysis using the method of generalized sphericity. It has the further advantage that it requires very little additional programming on the computer. The computer time required is dominated by the two three-jet analysis of steps A and G. Since there are fewer momenta in step G than step A, the computer time of four-jet analysis is not much more than that of three-jet analysis for the same number of observed tracks. For example, with IBM 370/168 computer, this is about 2 s for 21 observed tracks, where the number of partitions is 1.8×10^{11}, and about 9s for 35 observed tracks, where the number of partitions is 5×10^{19}!

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